B.1. **Full Implementation.** In this section, we study full implementation. We show that both strict sequential obedience and its reversed version are necessary and jointly sufficient for full implementation. We also show that, under the monotonicity assumption on V, the optimal information design problem under S-implementation is equivalent to that under full implementation.

To proceed, we add a symmetric dominance state assumption that there exists $\underline{\theta} \in \Theta$ such that $d_i(\mathbf{1}_{-i}, \underline{\theta}) < 0$ for all $i \in I$. We now allow an alternative interpretation of an ordered outcome as describing switches from action 1 to action 0. Thus, for a sequence $\gamma^0 \in \Gamma$, write $a^0(\gamma^0) = \mathbf{1} - a(\gamma^0) \in A$ for the action profile such that player i plays action 0 if and only if i is listed in γ^0 and $a^0_{-i}(\gamma^0) \in A_{-i}$ for the action profile such that only players before i in γ^0 play action 0. Thus an ordered outcome $\nu^0_\Gamma \in \Delta(\Gamma \times \Theta)$ *reverse induces* $\nu \in \Delta(A \times \Theta)$ if

$$\nu(a, \theta) = \sum_{\gamma^0 :\, a^0(\gamma^0) = a} \nu^0_\Gamma(\gamma^0, \theta)$$

for all $(a, \theta) \in A \times \Theta$.

Definition B.1. An ordered outcome $\nu^0_\Gamma \in \Delta(\Gamma \times \Theta)$ satisfies *reverse sequential obedience* (resp. *strict reverse sequential obedience*) if

$$\sum_{\gamma^0 \in \Gamma_i, \theta \in \Theta} \nu^0_\Gamma(\gamma^0, \theta) d_i(a^0_{-i}(\gamma^0), \theta) \leq \text{(resp. } <\text{) } 0 \tag{B.1}$$

for all $i \in I$ such that $\nu^0_\Gamma(\Gamma_i \times \Theta) > 0$. An outcome $\nu \in \Delta(A \times \Theta)$ satisfies *reverse sequential obedience* (resp. *strict reverse sequential obedience*) if there exists an ordered outcome $\nu^0_\Gamma \in \Delta(\Gamma \times \Theta)$ that reverse induces ν and satisfies reverse sequential obedience (resp. strict reverse sequential obedience).

Definition B.2. Outcome ν satisfies *two-sided grain of dominance* if $\nu(\mathbf{1}, \overline{\theta}) > 0$ and $\nu(\mathbf{0}, \underline{\theta}) > 0$.

Theorem B.1. (1) *If an outcome is fully implementable, then it satisfies consistency, strict sequential obedience, and strict reverse sequential obedience.*

(2) *If an outcome satisfies consistency, strict sequential obedience, strict reverse sequential obedience, and two-sided grain of dominance, then it is is fully implementable.*

Necessity (i.e., part (1)) follows immediately by applying Theorem A.1(1) in both directions. The proof for sufficiency (i.e., part (2)), given in Section B.1.1 below, is a simple adaption of the proof of Theorem A.1(2). It proceeds with two ordered outcomes satisfying strict sequential obedience and strict reverse sequential obedience, respectively. Then we construct an information structure analogous to that used in Theorem A.1(2), where an integer was drawn almost uniformly on the integers and a player observed a signal equal to that integer plus his rank in the sequence drawn from the ordered outcome establishing strict sequential obedience. But now two sequences are drawn independently from the two ordered outcomes (conditional on the recommended action profile and the state). A player's type will consist of an integer signal and an action recommendation, where the recommended action indicates which of the two sequences generates the integer signal. Then, an induction argument analogous to that for Theorem A.1(2) shows that there is a unique equilibrium (in fact, unique rationalizable strategy profile), which induces the target outcome.

Clearly, full implementability is stronger than S-implementability.[36] Yet, we show that *maximal* S-implementable outcomes (with respect to first-order stochastic dominance) must indeed be fully implementable.

Proposition B.1. *For any $\nu \in \overline{SI}$, there exists $\hat{\nu} \in \overline{FI}$ that first-order stochastically dominates ν.*

The proof is given in Section B.1.3. This proposition, in particular, implies that $FI \neq \emptyset$.

Consider, as in Section 1.3, the optimal information design problem but with full implementation:

$$\sup_{\nu \in FI} \sum_{a \in A, \theta \in \Theta} \nu(a,\theta) V(a,\theta) = \max_{\nu \in \overline{FI}} \sum_{a \in A, \theta \in \Theta} \nu(a,\theta) V(a,\theta).$$

By Proposition B.1, under the monotonicity assumption on V, solving this problem amounts to solving the problem with S-implementation: we have

$$\max_{\nu \in \overline{FI}} \sum_{a \in A, \theta \in \Theta} \nu(a,\theta) V(a,\theta) = \max_{\nu \in \overline{SI}} \sum_{a \in A, \theta \in \Theta} \nu(a,\theta) V(a,\theta),$$

and by Corollary 1, an optimal outcome of the information design problem with full implementation can be obtained by a maximal optimal solution to the problem $\max_{\nu \in \Delta(A \times \Theta)} \sum_{a,\theta} \nu(a,\theta) V(a,\theta)$ subject to consistency and sequential obedience.

B.1.1. *Proof of Theorem B.1(2).* In the following, for $S \subset I$, we denote by $\Gamma(S) \subset \Gamma$ the set of sequences of distinct players in S and by $\Pi(S) \subset \Gamma(S)$ the set of permutations of all players in S.

[36] For the game in Section 2, for example, if we reverse the order on actions to be "Invest < Not Invest", outcome (2.3) with $-\frac{1}{4} \leq \delta < 0$ satisfies strict sequential obedience but not strict reverse sequential obedience and hence is S-implementable but not fully implementable.

Let $\nu \in \Delta(A \times \Theta)$ satisfy consistency, strict sequential obedience, strict reverse sequential obedience, and two-sided grain of dominance, and let $\nu_\Gamma^+ \in \Delta(\Gamma \times \Theta)$ and $\nu_\Gamma^- \in \Delta(\Gamma \times \Theta)$ be ordered outcomes establishing strict sequential obedience and strict reverse sequential obedience, respectively. By two-sided grain of dominance, there exist $\overline{\gamma}, \underline{\gamma}$ containing all players such that $\nu_\Gamma^+(\overline{\gamma}, \overline{\theta}) > 0$ and $\nu_\Gamma^-(\underline{\gamma}, \underline{\theta}) > 0$ (where $\nu_\Gamma^+(\overline{\gamma}, \overline{\theta}) \leq \nu_\Gamma^-(\emptyset, \overline{\theta})$ and $\nu_\Gamma^-(\underline{\gamma}, \underline{\theta}) \leq \nu_\Gamma^+(\emptyset, \underline{\theta})$). For $\varepsilon > 0$ with $\varepsilon < \min\{\nu_\Gamma^+(\overline{\gamma}, \overline{\theta}), \nu_\Gamma^-(\underline{\gamma}, \underline{\theta})\}$, define $\tilde{\nu}_\Gamma^+, \tilde{\nu}_\Gamma^- \in \Delta(\Gamma \times \Theta)$ by

$$\tilde{\nu}_\Gamma^+(\gamma, \theta) = \begin{cases} \dfrac{\nu_\Gamma^+(\gamma, \theta) - \varepsilon}{1 - 2\varepsilon} & \text{if } (\gamma, \theta) = (\overline{\gamma}, \overline{\theta}), (\emptyset, \underline{\theta}), \\ \dfrac{\nu_\Gamma^+(\gamma, \theta)}{1 - 2\varepsilon} & \text{otherwise,} \end{cases}$$

and

$$\tilde{\nu}_\Gamma^-(\gamma, \theta) = \begin{cases} \dfrac{\nu_\Gamma^-(\gamma, \theta) - \varepsilon}{1 - 2\varepsilon} & \text{if } (\gamma, \theta) = (\underline{\gamma}, \underline{\theta}), (\emptyset, \overline{\theta}), \\ \dfrac{\nu_\Gamma^-(\gamma, \theta)}{1 - 2\varepsilon} & \text{otherwise,} \end{cases}$$

where we assume that ε is sufficiently small that $\tilde{\nu}_\Gamma^+$ and $\tilde{\nu}_\Gamma^-$ satisfy strict sequential obedience and strict reverse sequential obedience, respectively, i.e.,

$$\sum_{\gamma \in \Gamma_i, \theta \in \Theta} \tilde{\nu}_\Gamma^+(\gamma, \theta) d_i(a_{-i}(\gamma), \theta) > 0$$

for all $i \in I$ such that $\tilde{\nu}_\Gamma^+(\Gamma_i \times \Theta) > 0$, and

$$\sum_{\gamma \in \Gamma_i, \theta \in \Theta} \tilde{\nu}_\Gamma^-(\gamma, \theta) d_i(a_{-i}^0(\gamma), \theta) < 0$$

for all $i \in I$ such that $\tilde{\nu}_\Gamma^-(\Gamma_i \times \Theta) > 0$. Define also $\tilde{\nu} \in \Delta(A \times \Theta)$ by

$$\tilde{\nu}(a, \theta) = \begin{cases} \dfrac{\nu(a, \theta) - \varepsilon}{1 - 2\varepsilon} & \text{if } (a, \theta) = (\mathbf{1}, \overline{\theta}), (\mathbf{0}, \underline{\theta}), \\ \dfrac{\nu(a, \theta)}{1 - 2\varepsilon} & \text{otherwise.} \end{cases}$$

Observe that $\sum_{\gamma^+ : a(\gamma^+) = a} \tilde{\nu}_\Gamma^+(\gamma^+, \theta) = \sum_{\gamma^- : a^0(\gamma^-) = a} \tilde{\nu}_\Gamma^-(\gamma^-, \theta) = \tilde{\nu}(a, \theta)$ for all $(a, \theta) \in A \times \Theta$.

By the dominance state assumption, we can take a $\bar{q} < 1$ such that

$$\bar{q} d_i(\mathbf{0}_{-i}, \overline{\theta}) + (1 - \bar{q}) \min_{\theta \neq \overline{\theta}} d_i(\mathbf{0}_{-i}, \theta) > 0,$$
$$\bar{q} d_i(\mathbf{1}_{-i}, \underline{\theta}) + (1 - \bar{q}) \max_{\theta \neq \underline{\theta}} d_i(\mathbf{1}_{-i}, \theta) < 0$$

for all $i \in I$. Then let $\eta > 0$ be such that

$$\frac{\frac{\varepsilon}{|I|-1}}{\frac{\varepsilon}{|I|-1} + \eta} \geq \bar{q},$$

and

$$\sum_{\gamma\in\Gamma_i,\theta\in\Theta}(1-\eta)^{|I|-n(a_{-i}(\gamma))-1}\tilde{\nu}_\Gamma^+(\gamma,\theta)d_i(a_{-i}(\gamma),\theta)>0$$

for all $i\in I$ such that $\tilde{\nu}_\Gamma^+(\Gamma_i\times\Theta)>0$, and

$$\sum_{\gamma\in\Gamma_i,\theta\in\Theta}(1-\eta)^{|I|-n^0(a^0_{-i}(\gamma))-1}\tilde{\nu}_\Gamma^-(\gamma,\theta)d_i(a^0_{-i}(\gamma),\theta)<0$$

for all $i\in I$ such that $\tilde{\nu}_\Gamma^-(\Gamma_i\times\Theta)>0$, where $n^0(a^0_{-i}(\gamma))$ is the number of players playing action 0 in the action profile $a^0_{-i}(\gamma)$.

Now construct the type space (T,π) as follows. For each $i\in I$, let $T_i=\{1,2,\dots\}\times A_i$. Define $\pi\in\Delta(T\times\Theta)$ by the following: for each $t=(s_i,a_i)_{i\in I}\in T$ and $\theta\in\Theta$, let

$$\pi(t,\theta)$$
$$=\begin{cases}(1-2\varepsilon)\eta(1-\eta)^m\dfrac{\tilde{\nu}_\Gamma^+(\gamma^+,\theta)\tilde{\nu}_\Gamma^-(\gamma^-,\theta)}{\tilde{\nu}(a,\theta)} & \text{if } \tilde{\nu}(a,\theta)>0 \text{ and there exist } m\in\mathbb{N}, \gamma^+\in\Pi(S(a)), \text{ and } \gamma^-\in\Pi(I\backslash S(a)) \text{ such that } s_i=m+\ell(i,\gamma^+) \text{ for all } i\in S(a) \text{ and } s_i=m+\ell(i,\gamma^-) \text{ for all } i\in I\setminus S(a),\\ \dfrac{\varepsilon}{|I|-1} & \text{if } 1\le s_1=\cdots=s_{|I|}\le|I|-1 \text{ and } (a,\theta)=(\mathbf{1},\overline{\theta}),(\mathbf{0},\underline{\theta}),\\ 0 & \text{otherwise,}\end{cases}$$

where $\ell(i,\gamma)=\ell$ if $i=i_\ell$. Observe that π is consistent with μ: $\sum_t\pi(t,\theta)=\mu(\theta)$ for all $\theta\in\Theta$.

The rest of the proof is completed by mimicking the proof of Theorem A.1(2). A similar argument as in the proof of Theorem A.1(2) shows that action 1 (resp. 0) is uniquely rationalizable for all players of types $t_i=(s_i,a_i)$ with $a_i=1$ (resp. $a_i=0$). By construction, the unique rationalizable strategy profile, hence the unique equilibrium, induces ν, as desired.

B.1.2. *(Reverse) Sequential Obedience in Complete Information Games.* In this section, we report an important property of (reverse) sequential obedience in complete information games, which will be used in the proof of Proposition B.1 in Section B.1.3. A complete information BAS game is given by a profile of payoff difference functions $f_i\colon A_{-i}\to\mathbb{R}$, $i\in I$. Let $\overline{X}\subset\Delta(A)$ (resp. $\overline{X}^0\subset\Delta(A)$) denote the set of outcomes that satisfy sequential obedience (resp. reverse sequential obedience) in $(f_i)_{i\in I}$, which is endowed with the first-order stochastic dominance order.

Proposition B.2. *In any complete information BAS game, the following hold:*

(1) $\overline{X}$ *has a largest element, which is degenerate on some action profile and satisfies strict reverse sequential obedience.*

(2) $\overline{X}^0$ *has a smallest element, which is degenerate on some action profile and satisfies strict sequential obedience.*

In particular, from this proposition it follows that any complete information BAS game has an action profile (or an outcome degenerate on some action profile) that satisfies sequential obedience and strict reverse sequential obedience (by part (1)) and an action profile that satisfies strict sequential obedience and reverse sequential obedience (by part (2)).

Proof. By symmetry, we only prove (2). By the proof of Lemma 2(2) in Oyama and Takahashi (2019) along with the convexity, $\overline{X}^0$ has a smallest element, which is degenerate on some action profile, say $a^1 \in A$. Denote $S^1 = S(a^1) = \{i \in I \mid a_i^1 = 1\}$. Let $\mathbf{0}^0 \in \prod_{j \in I \setminus S^1} A_j$ denote the action profile of players in $I \setminus S^1$ such that all players in $I \setminus S^1$ play action 0, and for $i \in S^1$ and $\gamma^0 \in \Gamma(S^1)$, let $b_{-i}^0(\gamma^0) \in \prod_{j \in S^1 \setminus \{i\}} A_j$ denote the action profile of players in S^1 such that only the players in S^1 that appear before i in γ^0 play action 0.

Claim B.1. *There exists no ordered outcome $\rho^0 \in \Delta(\Gamma(S^1) \setminus \{\emptyset\})$ such that*

$$\sum_{\gamma^0 \in \Gamma(S^1) \setminus \{\emptyset\}} \rho^0(\gamma^0) f_i(b_{-i}^0(\gamma^0), \mathbf{0}^0) \leq 0 \tag{B.2}$$

for all $i \in S^1$.

Proof. Assume that there exists $\rho^0 \in \Delta(\Gamma(S^1) \setminus \{\emptyset\})$ that satisfies (B.2) for all $i \in S^1$. Let $\bar{\rho}^0 \in \Delta(\Pi(I \setminus S^1))$ be an ordered outcome that establishes reverse sequential obedience of a^1. Define $\hat{\rho}^0 \in \Delta(\Gamma)$ by

$$\hat{\rho}^0(\gamma^0) = \begin{cases} \bar{\rho}^0(\gamma_0^0)\rho^0(\gamma_1^0) & \text{if } \gamma^0 = (\gamma_0^0, \gamma_1^0) \text{ for some } \gamma_0^0 \in \Pi(I \setminus S^1) \text{ and } \gamma_1^0 \in \Gamma(S^1) \setminus \{\emptyset\}, \\ 0 & \text{otherwise.} \end{cases}$$

Then this $\hat{\rho}^0$ satisfies reverse sequential obedience and reverse induces an outcome that is strictly stochastically dominated by (the degenerate outcome on) a^1. The existence of such an ordered outcome contradicts the condition that a^1 is the smallest element of $\overline{X}^0$. □

By Claim B.1, it follows from a duality theorem (or from Oyama and Takahashi (2020, Lemma 2(1)) applied to the "subgame" $(f_i^1)_{i \in S^1}$ defined by $f_i^1(b_{-i}) = f_i(b_{-i}, \mathbf{0}^0)$ for

$b_{-i} \in \prod_{j\in S^1\setminus\{i\}}$) that there exists $(\lambda_i^1)_{i\in S^1} \in \mathbb{R}^{S^1}_{++}$ such that

$$\sum_{i\in S(\gamma^0)} \lambda_i^1 f_i(b^0_{-i}(\gamma^0), \mathbf{0}^0) > 0$$

for all $\gamma^0 \in \Gamma(S^1)\setminus\{\emptyset\}$. This is equivalent to the condition that for any $\gamma = (i_1, \ldots, i_{|S^1|}) \in \Pi(S^1)$,

$$\sum_{\ell=k}^{|S^1|} \lambda^1_{i_\ell} f_{i_\ell}(a_{-i_\ell}(\gamma)) > 0 \tag{B.3}$$

for all $k = 1, \ldots, |S^1|$. We want to show that the condition (A.5) in Proposition A.1 holds for the degenerate outcome on a^1 (with $|\Theta| = 1$). Fix any $(\lambda_i)_{i\in I} \in \mathbb{R}^I_+$ such that $\lambda_i > 0$ for some $i \in S^1$. Let $\gamma^\lambda = (i_1, \ldots, i_{|I|})$ be a permutation of all players such that $\{i_1, \ldots, i_{|S^1|}\} = S^1$ and $\frac{\lambda_{i_1}}{\lambda^1_{i_1}} \leq \cdots \leq \frac{\lambda_{i_{|S^1|}}}{\lambda^1_{i_{|S^1|}}}$. Then we have

$$\begin{aligned}(\text{LHS of (A.5)}) &\geq \sum_{i\in S^1} \lambda_i f_i(a_{-i}(\gamma^\lambda)) \\ &= \sum_{k=1}^{|S^1|} \left(\frac{\lambda_{i_k}}{\lambda^1_{i_k}} - \frac{\lambda_{i_{k-1}}}{\lambda^1_{i_{k-1}}}\right) \sum_{\ell=k}^{|S^1|} \lambda^1_{i_\ell} f_{i_\ell}(a_{-i_\ell}(\gamma^\lambda)) > 0\end{aligned}$$

by (B.3), where we set $\frac{\lambda_{i_0}}{\lambda^1_{i_0}} = 0$. Therefore, it follows from Proposition A.1 that a^1 satisfies strict sequential obedience. □

B.1.3. *Proof of Proposition B.1.* By Proposition B.2, we have the following:

Lemma B.1. *If $\nu \in \Delta(A\times\Theta)$ satisfies consistency, strict sequential obedience, and two-sided grain of dominance, then there exists $\hat{\nu} \in \Delta(A \times \Theta)$ that first-order stochastically dominates ν and satisfies consistency, strict sequential obedience, strict reverse sequential obedience, and two-sided grain of dominance.*

Proof. For each $S \subset I$ and $\theta \in \Theta$, apply Proposition B.2(1) to the complete information game $(d_i((\cdot, \mathbf{1}_{I\setminus S}), \theta))_{i\in S}$: let $a^*_{S,\theta} \in \prod_{i\in S} A_i$ be an action profile that satisfies sequential obedience and strict reverse sequential obedience in $(d_i((\cdot, \mathbf{1}_{I\setminus S}), \theta))_{i\in S}$ with $\rho_{S,\theta}$ and $\rho^0_{S,\theta}$, respectively, where $\rho_{S,\theta}(\Pi(S(a^*_{S,\theta}))) = 1$ and $\rho^0_{S,\theta}(\Pi(S \setminus S(a^*_{S,\theta}))) = 1$, and, by convention, $\rho_{\emptyset,\theta}(\emptyset) = \rho^0_{\emptyset,\theta}(\emptyset) = 1$. By construction, for any $S \subset I$ and $\theta \in \Theta$, we have

$$\sum_{\gamma\in\Gamma(S)\cap\Gamma_i} \rho_{S,\theta}(\gamma) d_i(a_{-i}((\gamma', \gamma)), \theta) \geq 0 \tag{B.4}$$

for all $\gamma' \in \Pi(I \setminus S)$ and all $i \in S(a^*_{S,\theta})$, and

$$\sum_{\gamma^0\in\Gamma(S)\cap\Gamma_i} \rho^0_{S,\theta}(\gamma^0) d_i(a^0_{-i}(\gamma^0), \theta) < 0 \tag{B.5}$$

for all $i \in S \setminus S(a^*_{S,\theta})$. Note, in particular, that $a^*_{I,\underline{\theta}} = \mathbf{0}$ and hence $\rho_{I,\underline{\theta}}(\emptyset) = 1$ by the dominance state assumption.

Let $\nu \in \Delta(A \times \Theta)$ satisfy consistency, strict sequential obedience with $\nu_\Gamma \in \Delta(\Gamma \times \Theta)$, and two-sided grain of dominance. Define $\hat{\nu}_\Gamma, \hat{\nu}^0_\Gamma \in \Delta(\Gamma \times \Theta)$ by

$$\hat{\nu}_\Gamma(\gamma, \theta) = \sum_{\gamma', \gamma'' : (\gamma', \gamma'') = \gamma} \nu_\Gamma(\gamma', \theta) \rho_{I \setminus S(\gamma'), \theta}(\gamma'')$$

and

$$\hat{\nu}^0_\Gamma(\gamma^0, \theta) = \sum_{a : S(a) \subset I \setminus S(\gamma^0)} \nu(a, \theta) \rho^0_{I \setminus S(a), \theta}(\gamma^0),$$

where for $\gamma \in \Gamma$, $S(\gamma)$ denotes the set of players that appear in γ. Observe that $\hat{\nu}_\Gamma(\gamma, \overline{\theta}) > 0$ for some $\gamma \in \Pi(I)$ and $\hat{\nu}_\Gamma(\emptyset, \underline{\theta}) > 0$ by two-sided grain of dominance and $\rho_{I,\underline{\theta}}(\emptyset) = 1$. Then define $\hat{\nu} \in \Delta(A \times \Theta)$ by

$$\begin{aligned} \hat{\nu}(a, \theta) &= \sum_{\gamma : a(\gamma) = a} \hat{\nu}_\Gamma(\gamma, \theta) \\ &= \sum_{a' : S(a') \subset S(a),\, S(a) \setminus S(a') = S\left(a^*_{I \setminus S(a'), \theta}\right)} \nu(a', \theta). \end{aligned}$$

One can verify that $\hat{\nu}$ satisfies consistency and two-sided grain of dominance and first-order stochastically dominates ν, and that $\hat{\nu}^0_\Gamma$ reverse induces $\hat{\nu}$.

Then, $\hat{\nu}_\Gamma$ satisfies strict sequential obedience, since for each $i \in I$, where $\nu_\Gamma(\Gamma_i \times \Theta) > 0$ and $\hat{\nu}_\Gamma(\Gamma_i \times \Theta) > 0$, we have

$$\begin{aligned} &\sum_{\gamma \in \Gamma_i, \theta \in \Theta} \hat{\nu}_\Gamma(\gamma, \theta) d_i(a_{-i}(\gamma), \theta) \\ &= \sum_{\theta \in \Theta} \sum_{\gamma' \in \Gamma_i} \nu_\Gamma(\gamma', \theta) d_i(a_{-i}(\gamma'), \theta) \\ &\quad + \sum_{\theta \in \Theta} \sum_{S \subset I \setminus \{i\}} \sum_{\gamma' \in \Pi(S)} \nu_\Gamma(\gamma', \theta) \sum_{\gamma'' \in \Gamma(I \setminus S) \cap \Gamma_i} \rho_{I \setminus S, \theta}(\gamma'') d_i(a_{-i}((\gamma', \gamma'')), \theta) > 0, \end{aligned}$$

where the inequality follows from the strict sequential obedience of ν_Γ and (B.4).

Finally, $\hat{\nu}^0_\Gamma$ satisfies strict reverse sequential obedience, since for each $i \in I$, where $\hat{\nu}^0_\Gamma(\Gamma_i \times \Theta) > 0$, we have

$$\begin{aligned} &\sum_{\gamma^0 \in \Gamma_i, \theta \in \Theta} \hat{\nu}^0_\Gamma(\gamma^0, \theta) d_i(a^0_{-i}(\gamma^0), \theta) \\ &= \sum_{\theta \in \Theta} \sum_{S \subset I \setminus \{i\}} \sum_{\gamma' \in \Pi(S)} \nu_\Gamma(\gamma', \theta) \sum_{\gamma^0 \in \Gamma(I \setminus S) \cap \Gamma_i} \rho^0_{I \setminus S, \theta}(\gamma^0) d_i(a^0_{-i}(\gamma^0), \theta) < 0, \end{aligned}$$

where the inequality follows from (B.5). □

From Lemma B.1 and Theorem B.1, we have the following:

Lemma B.2. *If an outcome ν satisfies consistency and sequential obedience, then there exists an outcome $\hat{\nu} \in \overline{FI}$ that first-order stochastically dominates ν.*

Proof. First, it follows from Lemma B.1 and Theorem B.1(2) that for any $\nu \in \Delta(A \times \Theta)$ that satisfies consistency, strict sequential obedience, and two-sided grain of dominance, there exists $\hat{\nu} \in FI$ that first-order stochastically dominates ν.

Now let $\nu \in \Delta(A \times \Theta)$ satisfy consistency and sequential obedience. Then, as in the proof of Theorem 1, there exists a sequence of outcomes $\nu^\varepsilon \in \Delta(A \times \Theta)$ converging to ν that satisfy consistency, strict sequential obedience, and two-sided grain of dominance. As noted above, for each ε, there exists an outcome $\hat{\nu}^\varepsilon \in FI$ that first-order stochastically dominates ν^ε. Then a limit point of $\hat{\nu}^\varepsilon$, which is contained in $\overline{FI}$, first-order stochastically dominates ν. □

Finally, Proposition B.1 follows from Theorem 1 and Lemma B.2.

B.2. **Alternative Assumptions.**

B.2.1. *Non-Supermodular Payoffs.* In this section, we consider general binary-action games with possibly non-supermodular payoffs and demonstrate that our arguments will still work if we employ rationalizability, rather than equilibrium, as a solution concept in implementing incomplete information games and strengthen the sequential obedience condition accordingly.

Let a base game $(d_i)_{i \in I}$ be given, which may not be supermodular. To simplify the argument, we focus on the "always play **1**" outcome, i.e., the outcome $\bar{\nu}$ such that $\bar{\nu}(\mathbf{1}, \theta) = \mu(\theta)$ for all $\theta \in \Theta$ (which satisfies consistency by construction). The outcome $\bar{\nu}$ is *fully implementable in rationalizable strategies* if there exists an information structure in which the profile of the "all types play action 1" strategies is the unique interim correlated rationalizable strategy profile. An ordered outcome $\nu_\Gamma \in \Delta(\Gamma \times \Theta)$ satisfies *strong sequential obedience* if

$$\sum_{\gamma \in \Gamma_i, \theta \in \Theta} \nu_\Gamma(\gamma, \theta) \min_{a_{-i} \geq a_{-i}(\gamma)} d_i(a_{-i}, \theta) > 0$$

for all $i \in I$ such that $\nu_\Gamma(\Gamma_i \times \Theta) > 0$; an outcome $\nu \in \Delta(A \times \Theta)$ satisfies strong sequential obedience if there exists an ordered outcome that induces ν and satisfies strong sequential obedience. Impose the dominance state assumption that there exists $\overline{\theta} \in \Theta$ such that for all $i \in I$, $d_i(a_{-i}, \overline{\theta}) > 0$ for all $a_{-i} \in A_{-i}$.

Now consider the BAS game $(\underline{d}_i)_{i\in I}$ defined by $\underline{d}_i(a_{-i},\theta) = \min_{a'_{-i}\geq a_{-i}} d_i(a'_{-i},\theta)$ for all $i \in I$ and $a_{-i} \in A_{-i}$ (where the dominance state assumption is satisfied). Then, clearly, $\bar{\nu}$ is fully implementable in rationalizable strategies in $(d_i)_{i\in I}$ if and only if it is fully (hence S-)implementable in $(\underline{d}_i)_{i\in I}$, and ν_Γ satisfies strong sequential obedience in $(d_i)_{i\in I}$ if and only if it satisfies strict sequential obedience in $(\underline{d}_i)_{i\in I}$. Therefore, by Theorem A.1, we have:

Proposition B.3. *Let $(d_i)_{i\in I}$ be any binary-action game. Then the outcome $\bar{\nu}$ is fully implementable in rationalizable strategies if and only if it satisfies strong sequential obedience.*

B.2.2. *Many Actions.* In this section, we consider games with many actions and present a generalized notion of sequential obedience that is necessary for S-implementability in these games. We also report a special case in which this notion is sufficient for S-implementability.

Let the action set A_i of player $i \in I$ be represented by a finite set of points in $[0,1]$ that contains 0 and 1 (so that $\min A_i = 0$ and $\max A_i = 1$), and let a base game $(u_i)_{i\in I}$ be given, where we assume supermodularity: for all $\theta \in \Theta$, $i \in I$, and $a_i, a'_i \in A_i$ with $a_i < a'_i$, $u_i((a'_i, a_{-i}), \theta) - u_i((a_i, a_{-i}), \theta)$ is nondecreasing in $a_{-i} \in A_{-i}$. The concept of smallest equilibrium implementability (S-implementability) is defined analogously to the case of binary actions.

A sequence $\gamma = (a^0, a^1, \ldots, a^k)$ of action profiles is a *unilateral deviation path* from $\mathbf{0}$ if $a^0 = \mathbf{0}$, and for each $\ell = 1, \ldots, k$, there exists $i_\ell \in I$ such that $a^\ell_{i_\ell} > a^{\ell-1}_{i_\ell}$ and $a^\ell_j = a^{\ell-1}_j$ for all $j \in I \setminus \{i_\ell\}$. Let Γ be the set of all unilateral deviation paths from $\mathbf{0}$. For $\gamma = (a^0, a^1, \ldots, a^k) \in \Gamma$, denote $a(\gamma) = a^k$. An ordered outcome $\nu_\Gamma \in \Delta(\Gamma \times \Theta)$ induces an outcome $\nu \in \Delta(A \times \Theta)$ if

$$\nu(a,\theta) = \sum_{\gamma: a(\gamma)=a} \nu_\Gamma(\gamma,\theta).$$

For $i \in I$ and $a_i, a'_i \in A_i$ with $a_i < a'_i$, let $\Gamma_i(a_i, a'_i) \subset \Gamma$ be the set of unilateral deviation paths along which player i switches from a_i to a'_i, and for $\gamma \in \Gamma_i(a_i, a'_i)$, let $a_{-i}(\gamma; a_i, a'_i)$ be the profile of the opponents' actions when player i switches from a_i to a'_i.

An ordered outcome $\nu_\Gamma \in \Delta(\Gamma \times \Theta)$ satisfies strict sequential obedience if

$$\sum_{\gamma\in\Gamma_i(a_i,a'_i),\theta\in\Theta} \nu_\Gamma(\gamma,\theta)(u_i((a'_i, a_{-i}(\gamma; a_i, a'_i)), \theta) - u_i((a''_i, a_{-i}(\gamma; a_i, a'_i)), \theta)) > 0 \qquad \text{(B.6)}$$

for all $i \in I$ and $a_i, a_i', a_i'' \in A_i$ with $a_i \le a_i'' < a_i'$ such that $\nu_\Gamma(\Gamma_i(a_i, a_i') \times \Theta) > 0$. An outcome $\nu \in \Delta(A \times \Theta)$ satisfies strict sequential obedience if there exists an ordered outcome $\nu_\Gamma \in \Delta(\Gamma \times \Theta)$ that induces ν and satisfies strict sequential obedience. If $|A_i| = 2$ for all $i \in I$, this coincides with the definition in the binary-action case.

Then, an argument almost identical with that in the proof of Theorem A.1(1) shows that if an outcome is S-implementable, then it satisfies this version of strict sequential obedience along with consistency and obedience: in an implementing information structure, consider the sequential best response process from the smallest strategy and for any pair of actions $a_i < a_i'$, aggregate the obedience conditions upon the switch from a_i to a_i' in the process.

For sufficiency, we report a special case in which the generalized strict sequential obedience condition above, along with consistency, implies S-implementability. We focus on the "always play $\mathbf{1}$" outcome, i.e., the outcome $\bar{\nu}$ such that $\bar{\nu}(\mathbf{1}, \theta) = \mu(\theta)$ for all $\theta \in \Theta$. Let $\Pi \subset \Gamma$ be the set of unilateral deviation paths $\gamma = (a^0, a^1, \ldots, a^{|I|})$ (of length $|I| + 1$) such that for each $\ell = 1, \ldots, |I|$, $a_{i_\ell}^{\ell-1} = 0$ and $a_{i_\ell}^{\ell} = 1$ for some $i_\ell \in I$. Now assume the dominance state assumption, that there exists $\overline{\theta} \in \Theta$ such that for all $i \in I$, $u_i((1, \mathbf{0}_{-i}), \overline{\theta}) - u_i((a_i, \mathbf{0}_{-i}), \overline{\theta}) > 0$ for all $a_i < 1$, and suppose that the outcome $\bar{\nu}$ is induced by some ordered outcome $\nu_\Gamma \in \Delta(\Pi \times \Theta)$ that satisfies the strict sequential obedience condition (B.6), i.e., for all $i \in I$,

$$\sum_{\gamma \in \Pi, \theta \in \Theta} \nu_\Gamma(\gamma, \theta)(u_i((1, a_{-i}(\gamma; 0, 1)), \theta) - u_i((a_i, a_{-i}(\gamma; 0, 1)), \theta)) > 0 \tag{B.7}$$

for all $a_i < 1$. Then, this condition can be thought of as $\nu_\Gamma \in \Delta(\Pi \times \Theta)$ satisfying strict sequential obedience in a game with *binary* actions. Formally, for each $(a_i)_{i \in I} \in \prod_{i \in I}(A_i \setminus \{1\})$, define $(d_i^{a_i})_{i \in I}$, $d_i^{a_i} \colon \{0,1\}^{I \setminus \{i\}} \times \Theta \to \mathbb{R}$, by

$$d_i^{a_i}(b_{-i}, \theta) = u_i((1, b_{-i}), \theta) - u_i((a_i, b_{-i}), \theta).$$

Thus, the ordered outcome ν_Γ satisfies the condition (B.7) in $(u_i)_{i \in I}$ if and only if it satisfies strict sequential obedience in the BAS game $(d_i^{a_i})_{i \in I}$ for every $(a_i)_{i \in I} \in \prod_{i \in I}(A_i \setminus \{1\})$, where Π is naturally identified with the set of permutations of all players. Hence, the construction in the proof of Theorem A.1(2) applies to this case, which shows that the outcome $\bar{\nu}$ is S-implementable in $(u_i)_{i \in I}$.

The condition discussed above is apparently very restrictive, but still broad enough to cover the result of Hoshino (2022) (which builds on the argument for Lemma 5.5 in

Kajii and Morris (1997)). Assume that for every state $\theta \in \Theta$, action profile $\mathbf{1} \in A$ is a $\mathbf{p}(\theta)$-dominant equilibrium for some $\mathbf{p}(\theta) = (p_i(\theta))_{i\in I} \in [0,1]^I$ with $\sum_{i\in I} p_i(\theta) \leq 1$, i.e., for any $i \in I$ and any $a_i < 1$,

$$\sum_{a_{-i}\in A_{-i}} q_i(a_{-i})(u_i((1,a_{-i}),\theta) - u_i((a_i,a_{-i}),\theta)) \geq 0$$

for any $q_i \in \Delta(A_{-i})$ with $q_i(\mathbf{1}_{-i}) \geq p_i(\theta)$. Note that this condition is equivalently written as: for any $i \in I$ and any $a_i < 1$,

$$\sum_{b_{-i}\in\{0,1\}^{I\setminus\{i\}}} \underline{q}_i(b_{-i}) \min_{a_{-i}\geq b_{-i}} (u_i((1,a_{-i}),\theta) - u_i((a_i,a_{-i}),\theta)) \geq 0$$

for any $\underline{q}_i \in \Delta(\{0,1\}^{I\setminus\{i\}})$ with $\underline{q}_i(\mathbf{1}_{-i}) \geq p_i(\theta)$. Then, there exists some ordered outcome $\nu_\Gamma \in \Delta(\Pi \times \Theta)$ that induces $\bar{\nu}$ and satisfies the strict sequential obedience condition (B.7). To see this, for each $i \in I$, let $\gamma^i = (a^0, a^1, \ldots, a^{|I|}) \in \Pi$ be any path such that player i is the last player who switches (i.e., such that $a_i^{|I|-1} = 0$ and $a_i^{|I|} = 1$). Define $\nu_\Gamma^{\mathbf{p}(\cdot)} \in \Delta(\Pi \times \Theta)$ by

$$\nu_\Gamma^{\mathbf{p}(\cdot)}(\gamma,\theta) = \begin{cases} \dfrac{p_i(\theta)}{\sum_{j\in I} p_j(\theta)}\mu(\theta) & \text{if } \gamma = \gamma^i,\ i \in I, \\ 0 & \text{otherwise,} \end{cases}$$

which induces $\bar{\nu}$. Then (B.7) is satisfied: for any $i \in I$ and any $a_i < 1$,

$$\begin{aligned} &\sum_{\gamma\in\Pi,\theta\in\Theta} \nu_\Gamma^{\mathbf{p}(\cdot)}(\gamma,\theta)(u_i((1,a_{-i}(\gamma;0,1)),\theta) - u_i((a_i,a_{-i}(\gamma;0,1)),\theta)) \\ &= \sum_{\theta\in\Theta}\mu(\theta) \sum_{b_{-i}\in\{0,1\}^{I\setminus\{i\}}} \underline{q}_i^{\mathbf{p}(\theta)}(b_{-i})(u_i((1,b_{-i}),\theta) - u_i((a_i,b_{-i}),\theta)) > 0, \end{aligned}$$

where $\underline{q}_i^{\mathbf{p}(\theta)} \in \Delta(\{0,1\}^{I\setminus\{i\}})$ is defined by $\underline{q}_i^{\mathbf{p}(\theta)}(b_{-i}) = \frac{\nu_\Gamma^{\mathbf{p}(\cdot)}(\{\gamma\in\Pi | a_{-i}(\gamma;0,1)=b_{-i}\}\times\{\theta\})}{\mu(\theta)}$, which satisfies $\underline{q}_i^{\mathbf{p}(\theta)}(\mathbf{1}_{-i}) = \frac{p_i(\theta)}{\sum_{j\in I} p_j(\theta)} \geq p_i(\theta)$, so that $\sum_{b_{-i}\in\{0,1\}^{I\setminus\{i\}}} \underline{q}_i^{\mathbf{p}(\theta)}(b_{-i})(u_i((1,b_{-i}),\theta) - u_i((a_i,b_{-i}),\theta)) \geq 0$ holds for all $\theta \in \Theta$, with strict inequality for $\theta = \overline{\theta}$.

In fact, even if $(u_i)_{i\in I}$ is not supermodular, a stronger form of (B.7) holds, under the dominance state assumption that for all $i \in I$, $u_i((1,a_{-i}),\overline{\theta}) - u_i((a_i,a_{-i}),\overline{\theta}) > 0$ for all $a_i < 1$ and all $a_{-i} \in A_{-i}$:

$$\begin{aligned} &\sum_{\gamma\in\Pi,\theta\in\Theta} \nu_\Gamma^{\mathbf{p}(\cdot)}(\gamma,\theta) \min_{a_{-i}\geq a_{-i}(\gamma;0,1)} (u_i((1,a_{-i}),\theta) - u_i((a_i,a_{-i}),\theta)) \\ &= \sum_{\theta\in\Theta}\mu(\theta) \sum_{b_{-i}\in\{0,1\}^{I\setminus\{i\}}} \underline{q}_i^{\mathbf{p}(\theta)}(b_{-i}) \min_{a_{-i}\geq b_{-i}} (u_i((1,a_{-i}),\theta) - u_i((a_i,a_{-i}),\theta)) > 0. \end{aligned}$$

Therefore, as argued in Section B.2.1, this implies that the outcome $\bar{\nu}$ is fully implementable in rationalizable strategies with or without supermodularity, which reproduces the result of Hoshino (2022, Theorem 1).

B.2.3. *Adversarial Information Sharing.* In this section, we formulate and study an optimal information design problem where the designer is also concerned that information might be shared among players in an adversarial way.

For an information structure $\mathcal{T} = ((T_i)_{i\in I}, \pi)$, an information structure $\mathcal{T}' = ((T_i')_{i\in I}, \pi')$ is an *information sharing* of $\mathcal{T}$ if there exist a profile $(Z_i)_{i\in I}$ of sets of "supplementary signals" and a signal generation rule $\phi\colon T \to \Delta(Z)$ such that $T_i' = T_i \times Z_i$ for each $i \in I$, and $\pi'(t, z, \theta) = \pi(t, \theta)\phi(t)(z)$ for any $t \in T$, $z \in Z$, and $\theta \in \Theta$. The value of information design under adversarial information sharing (and adversarial equilibrium selection) is then formulated as

$$V^\dagger = \sup_{\mathcal{T}} \inf_{\mathcal{T}'\colon \text{information sharing of } \mathcal{T}} \min_{\sigma\in E(\mathcal{T}')} \sum_{t'\in T', \theta\in\Theta} \pi'(t', \theta)V(\sigma(t'), \theta). \tag{B.8}$$

We first develop a useful alternative representation of this problem. A strategy profile $\sigma' = (\sigma_i')_{i\in I}$, $\sigma_i'\colon T_i \times Z_i \to \Delta(A_i)$, in the information sharing $\mathcal{T}' = ((T_i \times Z_i)_{i\in I}, \pi')$ of $\mathcal{T}$ induces an outcome $\xi \in \Delta(A \times T)$:

$$\begin{aligned}\xi(a, t) &= \sum_{z\in Z, \theta\in\Theta} \pi'(t, z, \theta) \prod_{i\in I} \sigma_i'(t_i, z_i)(a_i)\\ &= \pi(t) \sum_{z\in Z} \phi(t)(z) \prod_{i\in I} \sigma_i'(t_i, z_i)(a_i),\end{aligned}$$

where $\pi(t) = \sum_{\theta\in\Theta} \pi(t, \theta)$. Thus, the value of V under σ' is written as

$$\sum_{t'\in T', \theta\in\Theta} \pi'(t', \theta)V(\sigma'(t'), \theta) = \sum_{a\in A, \theta\in\Theta} \xi(a, t)V^{\mathcal{T}}(a, t),$$

where $V^{\mathcal{T}}(a, t) = \sum_{\theta\in\Theta} \pi(\theta|t)V(a, \theta)$ with $\pi(\theta|t) = \frac{\pi(t,\theta)}{\pi(t)}$. Say that an outcome $\xi \in \Delta(A \times T)$ is a *Bayesian solution* (Forges (1993)) of $\mathcal{T}$ if it satisfies consistency for $\mathcal{T}$: $\sum_{a\in A} \xi(a, t) = \pi(t)$ for all $t \in T$, and obedience for $\mathcal{T}$:

$$\sum_{a_{-i}\in A_{-i}, t_{-i}\in T_{-i}} \xi((a_i, a_{-i}), (t_i, t_{-i})) \left(u_i^{\mathcal{T}}((a_i, a_{-i}), (t_i, t_{-i})) - u_i^{\mathcal{T}}((a_i', a_{-i}), (t_i, t_{-i}))\right) \geq 0$$

for all $i \in I$, $t_i \in T_i$, and $a_i, a_i' \in A_i$, where $u_i^{\mathcal{T}}(a, t) = \sum_{\theta\in\Theta} \pi(\theta|t)u_i(a, \theta)$. One can show that an outcome $\xi \in \Delta(A \times T)$ is induced by some equilibrium of some information sharing of $\mathcal{T}$ if and only if it is a Bayesian solution of $\mathcal{T}$ (see Bergemann and Morris (2016)). Let $BS(\mathcal{T}) \subset \Delta(A \times T)$ denote the set of Bayesian solutions of $\mathcal{T}$. Thus, the original problem (B.8) is rewritten as

$$V^\dagger = \sup_{\mathcal{T}} \min_{\xi \in BS(\mathcal{T})} \sum_{a\in A, t\in T} \xi(a,t) V^{\mathcal{T}}(a,t). \tag{B.9}$$

We want to compare this problem with the constrained problem where the designer can only send public information to the players. We say that an information structure $\mathcal{T} = ((T_i)_{i\in I}, \pi)$ is *public* if $T_i = T_j$ for all $i, j \in I$, and $\sum_{t\in T: t_1 = \cdots = t_{|I|}, \theta\in\Theta} \pi(t,\theta) = 1$. Under the supermodularity of the payoffs and the monotonicity of V, the value of information design under S-implementation with public information structures is

$$V^*_{\text{public}} = \sup_{\mathcal{T}\colon \text{public}} \min_{\sigma\in E(\mathcal{T})} \sum_{t\in T, \theta\in\Theta} \pi(t,\theta) V(\sigma(t),\theta) \tag{B.10}$$

$$= \sup_{\mathcal{T}\colon \text{public}} \sum_{t\in T} \pi(t) V^{\mathcal{T}}(\underline{\sigma}(\mathcal{T})(t), t). \tag{B.11}$$

We immediately have $V^\dagger \leq V^*_{\text{public}}$ (compare the expressions (B.8) and (B.10)). On the other hand, if $\mathcal{T}$ is a public information structure, then for any $\xi \in BS(\mathcal{T})$ and for any $t \in T$, $\xi(\cdot|t) = \frac{\xi(\cdot,t)}{\pi(t)} \in \Delta(A)$ is a correlated equilibrium of $(u_i^{\mathcal{T}}(\cdot,t))_{i\in I}$, and by supermodularity, $BS(\mathcal{T})$ has a smallest element, which equals $\underline{\sigma}(\mathcal{T})$; thus we have $V^\dagger \geq V^*_{\text{public}}$ (compare (B.9) and (B.11)). Hence, by supermodularity and the objective monotonicity, we have:

Proposition B.4. $V^\dagger = V^*_{\text{public}}$.

Note that this result holds with any number of actions.

The set of outcomes that are S-implementable by public information structures can be characterized by the following strengthening of sequential obedience. Say that an ordered outcome $\nu_\Gamma \in \Delta(\Gamma\times\Theta)$ satisfies *public sequential obedience* (resp. *strict public sequential obedience*) if for every $\gamma \in \Gamma$ such that $\sum_{\theta\in\Theta} \nu_\Gamma(\gamma,\theta) > 0$,

$$\sum_{\theta\in\Theta} \nu_\Gamma(\gamma,\theta) d_i(a_{-i}(\gamma),\theta) \geq (\text{resp.} >)\ 0 \tag{B.12}$$

for all $i \in S(\gamma)$. An outcome $\nu \in \Delta(A\times\Theta)$ satisfies public sequential obedience (resp. strict public sequential obedience) if there exists an ordered outcome that induces ν and satisfies public sequential obedience (resp. strict public sequential obedience). It can readily be shown that an outcome is S-implementable by a public information structure if and only if it satisfies consistency, obedience, and strict public sequential obedience. Therefore, the problem (B.11) (hence (B.8)) can also be written as

$$V^*_{\text{public}} = \max_{\nu\in\Delta(A\times\Theta)} \sum_{a\in A, \theta\in\Theta} \nu(a,\theta) V(a,\theta)$$

subject to consistency and public sequential obedience. This also has a concavification form:

$$V^*_{\text{public}} = \sup_{Q \in \Delta_0(\Delta(\Theta))} \sum_{q \in \operatorname{supp} Q} Q(q) \sum_{\theta \in \Theta} q(\theta) V(\underline{a}(q), \theta)$$

subject to

$$\sum_{q \in \operatorname{supp} Q} Q(q) q(\theta) = \mu(\theta) \text{ for all } \theta \in \Theta,$$

where $\Delta_0(\Delta(\Theta))$ denotes the set of distributions over $\Delta(\Theta)$ with finite support (with $\operatorname{supp} Q$ denoting the support of $Q \in \Delta_0(\Delta(\Theta))$), and $\underline{a}(q) \in A$ denotes the smallest Nash equilibrium of the average game given by the common posterior $q \in \Delta(\Theta)$: $\sum_{\theta \in \Theta} q(\theta) u_i(a, \theta)$.

Let V^*_{private} denote the optimal value of the unconstrained problem (i.e., the problem under S-implementation with general information structures). Then $V^*_{\text{private}} \geq V^*_{\text{public}}$ trivially, and a strict inequality holds—or equivalently, the unconstrained optimal outcome is not S-implementable by a public information structure—for example under the assumptions in Section 4 if in addition there is nontrivial strategic interdependence among players in that $\Phi((1, \mathbf{0}_{-i}), \theta) < \frac{1}{|I|}\Phi(\mathbf{1}, \theta)$ for some $i \in I$ and some $\theta > \theta^*$. To see this, let ν^* be the optimal (perfect coordination) outcome given in (4.5) and let Π^i be the set of permutations of all players in which i appears first. Then for any ordered outcome ν_Γ that induces ν^*, we have

$$\begin{aligned}\sum_{i \in I, \gamma \in \Pi^i} \sum_{\theta \in \Theta} \nu_\Gamma(\gamma, \theta) d_i(a_{-i}(\gamma), \theta) &< \sum_{i \in I, \gamma \in \Pi^i} \sum_{\theta \in \Theta} \nu_\Gamma(\gamma, \theta) \frac{1}{|I|} \Phi(\mathbf{1}, \theta) \\ &= \frac{1}{|I|} \sum_{\theta \in \Theta} \nu^*(\mathbf{1}, \theta) \Phi(\mathbf{1}, \theta) = 0\end{aligned}$$

(the first equality holds since the sets Π^i form a partition of Π), so that public sequential obedience is violated for some $i \in I$ and $\gamma \in \Pi^i$.

In Section A.6, we illustrated the solutions under public information structures as well as under private (i.e., general) information structures for the example in Section 2.

B.2.4. *Finite Information Structures.* The construction in the proof of Theorem A.1(2) involves infinitely many types, but a similar construction with a finite number of types with a uniform—instead of geometric—distribution for the variable m can be used to S-implement the same outcome, where the number of types will be large enough depending on the probability $\mu(\overline{\theta})$ of the dominance state $\overline{\theta}$ and the degree of dominance at $\overline{\theta}$ relative to the payoffs at other states as well as the slackness of strict sequential obedience of the given outcome to be implemented. Specialized to a symmetric two-player two-state example, Mathevet et al. (2020) present an information structure with three types or less for each player that implements the optimal outcome. Our example in Section 2 also allows such a small information structure. Again specialized to a particular class of games (i.e., regime change games), Li et al. (2023) identify the unique optimal information structure when the number of types of each player is constrained by some upper bound K and show that their unconstrained optimal information structure is obtained as the limit of those finite information structures as $K \to \infty$.

While we do not need literally infinitely many types as argued above, our results rely on the assumption that there is no a priori bound on the number of types. To see what would happen otherwise, suppose that $\mathbf{0}$ is a strict equilibrium at every state $\theta \neq \overline{\theta}$. Let $SI^K(\mu)$ denote the set of outcomes that are S-implementable, under prior μ, by some information structure such that the number of types of each player is at most K. By Kajii and Morris (1997, Lemmas 5.2 and B), we immediately have the following.

Proposition B.5. *Suppose that* $\mathbf{0}$ *is a strict equilibrium at every* $\theta \neq \overline{\theta}$*. Then for any* $K < \infty$ *and* $\delta > 0$*, there exists* $\varepsilon > 0$ *such that if* $\mu(\overline{\theta}) \leq \varepsilon$*, then for any* $\nu \in SI^K(\mu)$*, we have* $\sum_{\theta \in \Theta} \nu(\mathbf{0}, \theta) \geq 1 - \delta$*.*

That is, if there is a bound on the number of types, then as the probability of the dominance state vanishes, the S-implementable outcomes tend only to be the trivial outcome "always play $\mathbf{0}$". Note that this result holds for general games (with any finite number of actions and possibly non-supermodular payoffs).

B.2.5. *Uncountable Information Structures.* In this section, we demonstrate that Theorem A.1(1) (and hence Theorem 1) will continue to hold with possibly uncountable type spaces.

Let the finite state space Θ with prior $\mu \in \Delta(\Theta)$ and the base game $(d_i)_{i \in I}$ be given as in Section 1.1. An information structure is defined as follows. For each player $i \in I$, the set T_i of types is a measurable space endowed with sigma-algebra $\mathcal{F}_i$, where we write $T = \prod_{i \in I} T_i$ and $\mathcal{F} = \bigotimes_{i \in I} \mathcal{F}_i$. The common prior π is a probability measure on $T \times \Theta$ (endowed with the product sigma-algebra $\mathcal{F} \otimes 2^{\Theta}$). Let π_X denote the marginal of π on $X = T_i, T, \Theta$, etc. We require π to be consistent with μ, i.e., $\pi_\Theta = \mu$.

In the incomplete information game induced by an information structure $((T_i, \mathcal{F}_i)_{i \in I}, \pi)$, a (pure) strategy for player $i \in I$ is an equivalence class of measurable functions from

T_i to A_i modulo being equal π_{T_i}-a.s. Let Σ_i be the partially ordered set of strategies of i, where $\sigma_i \leq \sigma_i'$ is understood as $\sigma_i(t_i) \leq \sigma_i'(t_i)$ for π_{T_i}-a.s. $t_i \in T_i$. Write $\Sigma = \prod_{i \in I} \Sigma_i$ and $\Sigma_{-i} = \prod_{j \neq i} \Sigma_j$ (endowed with the product partial orders, respectively). For $i \in I$, $t_i \in T_i$, and $\sigma_{-i} \in \Sigma_{-i}$, write

$$D_i(\sigma_{-i}|t_i) = \mathbb{E}[d_i(\sigma_{-i}(\cdot), \cdot)|\mathcal{F}_i](t_i),$$

which is measurable in t_i and nondecreasing in σ_{-i}, and let $\beta_i(\sigma_{-i})$ be the set of best responses to σ_{-i}, i.e., the set of strategies $\sigma_i \in \Sigma_i$ such that for π_{T_i}-a.s. $t_i \in T_i$, $D_i(\sigma_{-i}|t_i) \geq 0$ (resp. $D_i(\sigma_{-i}|t_i) \leq 0$) if $\sigma_i(t_i) = 1$ (resp. $\sigma_i(t_i) = 0$). A strategy profile $\sigma = (\sigma_i)_{i \in I} \in \Sigma$ is an equilibrium if for all $i \in I$, $\sigma_i \in \beta_i(\sigma_{-i})$. By the supermodularity (and the boundedness and the continuity of $d_i(a_{-i}, \theta)$ in a_{-i}), a smallest equilibrium exists and it is the limit of sequential best responses from the smallest strategy profile, as we now show below.

For $i \in I$, define $\underline{\beta}_i \colon \Sigma_{-i} \to \Sigma_i$ by

$$\underline{\beta}_i(\sigma_{-i})(t_i) = \begin{cases} 1 & \text{if } D_i(\sigma_{-i}|t_i) > 0, \\ 0 & \text{if } D_i(\sigma_{-i}|t_i) \leq 0 \end{cases}$$

for π_{T_i}-a.s. $t_i \in T_i$, which is well defined (measurable in t_i and unique up to π_{T_i}-a.s.) and nondecreasing in σ_{-i}. By construction, $\underline{\beta}_i(\sigma_{-i})$ is the smallest element of $\beta_i(\sigma_{-i})$. Then define the sequence of strategy profiles $\{\sigma^n\}$ as follows: let $\sigma_i^0(t_i) = 0$ for all $i \in I$ and $t_i \in T_i$, and for $n = 1, 2, \ldots$, let

$$\sigma_i^n = \begin{cases} \underline{\beta}_i(\sigma_{-i}^{n-1}) & \text{if } i \equiv n \pmod{|I|}, \\ \sigma_i^{n-1} & \text{otherwise.} \end{cases}$$

By the monotonicity of $\underline{\beta}_i$, this sequence is monotone increasing, $\sigma^0 \leq \sigma^1 \leq \cdots$, and converges as $n \to \infty$ to some $\underline{\sigma} \in \Sigma$ π_T-a.s. Since for each $i \in I$, $D_i(\underline{\sigma}_{-i}|t_i) = \lim_{n\to\infty} D_i(\sigma_{-i}^n|t_i)$ for π_{T_i}-a.s. by the dominated convergence theorem, $\underline{\sigma}$ is an equilibrium, and again by the monotonicity of $\underline{\beta}_i$, it is the smallest equilibrium.

Now we show the necessity of strict sequential obedience for S-implementability within this framework. Let $\nu \in \Delta(A \times \Theta)$ be S-implementable, and $((T_i, \mathcal{F}_i)_{i \in I}, \pi)$ be an information structure whose smallest equilibrium $\underline{\sigma}$ induces ν, i.e., $\nu(a, \theta) = \pi(\{t \in T \mid \underline{\sigma}(t) = a\} \times \{\theta\})$. Define the sequence of strategy profiles $\{\sigma^n\}$ as above, and let $\bar{T}_i \in \mathcal{F}_i$ be the set of types such that $\{\sigma_i^n(t_i)\}$ is monotone, where $\pi_{T_i}(\bar{T}_i) = 1$. On $\bar{T} = \prod_{i \in I} \bar{T}_i$, define $n_i(t_i)$ and $T(\gamma)$, $\gamma \in \Gamma$, as in the proof of Theorem A.1(1), where one can verify that

$T(\gamma) \in \mathcal{F}$. Then define $\nu_\Gamma \in \Delta(\Gamma \times \Theta)$ by $\nu_\Gamma(\gamma, \theta) = \pi(T(\gamma) \times \{\theta\})$, which induces ν. We want to show that ν_Γ satisfies strict sequential obedience.

Fix any $i \in I$ with $\nu_\Gamma(\Gamma_i \times \Theta) > 0$, where $\pi_{T_i}(\{t_i \in \bar{T}_i \mid n_i(t_i) = n\}) > 0$ for some $n \in \mathbb{N}$. Note that for all $n \in \mathbb{N}$ and all $t_i \in \bar{T}_i$ with $n_i(t_i) = n$, we have $D_i(\sigma_{-i}^{n-1}|t_i) > 0$. Hence, we have

$$\begin{aligned}
0 &< \sum_{n\in\mathbb{N}} \int_{\{t_i \in \bar{T}_i | n_i(t_i) = n\}} D_i(\sigma_{-i}^{n-1}|t_i) d\pi_{T_i}(t_i) \\
&= \sum_{n\in\mathbb{N}} \int_{\{t_i \in \bar{T}_i | n_i(t_i) = n\} \times \bar{T}_{-i} \times \Theta} d_i(\sigma_{-i}^{n-1}(t_{-i}), \theta) d\pi(t, \theta) \\
&= \sum_{n\in\mathbb{N}} \sum_{a_{-i}\in A_{-i}, \theta\in\Theta} d_i(a_{-i}, \theta) \pi(\{t \in \bar{T} \mid n_i(t_i) = n,\ \sigma_{-i}^{n-1}(t_{-i}) = a_{-i}\} \times \{\theta\}) \\
&= \sum_{a_{-i}\in A_{-i}, \theta\in\Theta} d_i(a_{-i}, \theta) \pi(\{t \in \bar{T} \mid n_i(t_i) < \infty,\ \sigma_{-i}^{n_i(t_i)-1}(t_{-i}) = a_{-i}\} \times \{\theta\}) \\
&= \sum_{a_{-i}\in A_{-i}, \theta\in\Theta} d_i(a_{-i}, \theta) \sum_{\gamma\in\Gamma_i : a_{-i}(\gamma) = a_{-i}} \pi(T(\gamma) \times \{\theta\}) \\
&= \sum_{\gamma\in\Gamma_i, \theta\in\Theta} d_i(a_{-i}(\gamma), \theta) \pi(T(\gamma) \times \{\theta\}) = \sum_{\gamma\in\Gamma_i, \theta\in\Theta} d_i(a_{-i}(\gamma), \theta) \nu_\Gamma(\gamma, \theta),
\end{aligned}$$

as desired.

B.2.6. *Dominance State Assumption.* The dominance state assumption, that there exists $\overline{\theta} \in \Theta$ such that $d_i(\mathbf{0}_{-i}, \overline{\theta}) > 0$ for all $i \in I$, is maintained throughout the analysis and used in Theorem A.1(2) (and other results that use Theorem A.1(2)). This exact form of the assumption, however, is stronger than needed and can be relaxed. Consider instead the following weakening, say the "sequential dominance states assumption": there exist a permutation γ of all players and states $\overline{\theta}^i \in \Theta$, $i \in I$, such that $d_i(a_{-i}(\gamma), \overline{\theta}^i) > 0$ for all $i \in I$. Under this condition, if ν satisfies consistency, obedience, strict sequential obedience, and grain of dominance with respect to $(\overline{\theta}^i)_{i\in I}$, i.e., $\nu(\mathbf{1}, \overline{\theta}^i) > 0$ for each $i \in I$, then one can construct an information structure, similar to, but more involved than, the one in the proof of Theorem A.1(2), that S-implements ν. Thus, we have:

Proposition B.6. *If the sequential dominance states assumption is satisfied with respect to $(\overline{\theta}^i)_{i\in I}$, and ν satisfies consistency, obedience, strict sequential obedience, and grain of dominance with respect to $(\overline{\theta}^i)_{i\in I}$, then $\nu \in SI$.*

Conversely, if there exists $\nu \in SI$ such that $\nu(\mathbf{1}, \theta) > 0$ for some $\theta \in \Theta$, then the sequential dominance states assumption must be satisfied.[37] Thus, it is weakest possible in this sense.

B.2.7. *Indispensability of Grain of Dominance.* This section presents an example demonstrating that the grain of dominance property is indispensable in Theorem A.1(2).

Consider the following game: Let $I = \{1, 2\}$ and $\Theta = \{\theta_1, \theta_2, \theta_3\}$, and let $\mu(\theta) = \frac{1}{3}$ for all $\theta \in \Theta$. The payoffs for each $i \in I$ are given by

$$d_i(a_j, \theta_1) = -2,\ d_i(a_j, \theta_3) = 1$$

for all $a_j \in A_j$, $j \neq i$, and

$$d_i(0, \theta_2) = -1,\ d_i(1, \theta_2) = 2.$$

The dominance state assumption is satisfied with $\overline{\theta} = \theta_3$.

Let $\nu \in \Delta(A \times \Theta)$ be defined by $\nu(\mathbf{0}, \theta) = \mu(\theta)$ for $\theta = \theta_1, \theta_3$ and $\nu(\mathbf{1}, \theta_2) = \mu(\theta_2)$ (and $\nu(a, \theta) = 0$ otherwise). It satisfies consistency, obedience, and strict sequential obedience (for example with $\nu_\Gamma \in \Delta(\Gamma \times \Theta)$ such that $\nu_\Gamma(\emptyset, \theta) = \nu(\mathbf{0}, \theta)$ for $\theta = \theta_1, \theta_3$ and $\nu_\Gamma(12, \theta_2) = \nu_\Gamma(21, \theta_2) = \frac{\nu(\mathbf{1}, \theta_2)}{2}$), but not grain of dominance. We claim that $\nu \notin SI$.

Let $\mathcal{T} = ((T_i)_{i \in I}, \pi)$ be any information structure that has an equilibrium σ that induces ν. We show that the smallest strategy profile σ^0, the strategy profile such that $\sigma_i^0(0|t_i) = 1$ for all t_i, is an equilibrium (hence the smallest equilibrium) in any such $\mathcal{T}$. Let

$$T_i^{a_i} = \{t_i \in T_i \mid \sigma_i(a_i|t_i) = 1\},$$

and $T^a = T_1^{a_1} \times T_2^{a_2}$. By the assumption that σ induces ν, we have $T_i = T_i^0 \cup T_i^1$ and

$$\pi(T^{\mathbf{0}} \times \{\theta\}) = \mu(\theta),\ \theta = \theta_1, \theta_3,$$
$$\pi(T^{\mathbf{1}} \times \{\theta_2\}) = \mu(\theta_2),$$

and hence $\pi(\theta_2|t_i) = 1$ for all $t_i \in T_i^1$. Therefore, for all $t_i \in T_i^1$, $a_i = 0$ is a best response against σ_j^0. For $t_i \in T_i^0$, $a_i = 0$, which is a best response against σ_j, continues to be a best response against σ_j^0 by supermodularity. This shows that σ^0 is the smallest equilibrium for any information structure that partially implements ν, which implies that $\nu \notin SI$.

[37]If such an S-implementable outcome exists, then for any $S \subsetneq I$, there exist $i \in I \setminus S$ and $\theta \in \Theta$ such that $d_i(\mathbf{1}_S, \theta) > 0$ (if there is no such pair of $i \in I \setminus S$ and $\theta \in \Theta$, then for any $\nu \in SI$, we would have $\nu(a, \theta) = 0$ for all $\theta \in \Theta$ whenever $a_i = 1$ for some $I \setminus S$). Then inductively apply this condition to construct γ and $\overline{\theta}^i$ in the sequential dominance state assumption.

B.3. **Perfect Coordination in Generalized Regime Change Games.** In this section, we show that the perfect coordination property holds—i.e., an optimal outcome is found among perfect coordination outcomes—in generalized regime change games.

The base game $(d_i)_{i\in I}$ is a *generalized regime change game* if there exists a function $r\colon A\times\Theta\to\{0,1\}$ such that for all $i\in I$,

$$d_i(a_{-i},\theta)\begin{cases}>0 & \text{if } r((0,a_{-i}),\theta)=1,\\ \leq 0 & \text{if } r((0,a_{-i}),\theta)=0.\end{cases}$$

By supermodularity, $r(a,\theta)$ must be nondecreasing in a. To be consistent with the dominance state assumption, we assume that for some $\overline{\theta}\in\Theta$, $r(\mathbf{0},\overline{\theta})=1$ for all $i\in I$. This game is a generalization of the regime change game in Example A.2, where $r(a,\theta)=1$ if and only if $n(a)\geq|I|-k(\theta)$, and corresponds to the game considered in Inostroza and Pavan (2022, Additional Material, Section AM3). The following proposition is a version of their Theorem AM3-1 restricted to our setting, where our concise proof appeals to our sequential obedience characterization of Theorem 1.

Proposition B.7. *Let $(d_i)_{i\in I}$ be a generalized regime change game. For any outcome $\nu\in\overline{SI}$, there exists a perfect coordination outcome $\hat{\nu}\in\overline{SI}$ such that $\sum_{a\in A:r(a,\theta)=1}\hat{\nu}(a,\theta)=\sum_{a\in A:r(a,\theta)=1}\nu(a,\theta)$ for all $\theta\in\Theta$.*

Proof. Let $(d_i)_{i\in I}$ be a generalized regime change game, and let $\nu\in\overline{SI}$. By Theorem 1, ν satisfies consistency, obedience, and sequential obedience. Thus, by Proposition A.1, it satisfies condition (A.5). Define $\hat{\nu}\in\Delta(A\times\Theta)$ by

$$\hat{\nu}(a,\theta)=\begin{cases}\sum_{a'\in A:r(a',\theta)=1}\nu(a',\theta) & \text{if } a=\mathbf{1},\\ \sum_{a'\in A:r(a',\theta)=0}\nu(a',\theta) & \text{if } a=\mathbf{0},\\ 0 & \text{otherwise,}\end{cases}$$

which by construction satisfies consistency and perfect coordination. It also satisfies, for each $\theta\in\Theta$, $\sum_{a\in A:r(a,\theta)=1}\nu(a,\theta)=\sum_{a\in A:r(a,\theta)=1}\hat{\nu}(a,\theta)$ since by the monotonicity of $r(a,\theta)$ in a, $\{a\in A\mid r(a,\theta)=1\}\neq\emptyset$ if and only if $r(\mathbf{1},\theta)=1$. We want to show that $\hat{\nu}$ satisfies obedience and sequential obedience.

First, for sequential obedience, we show that $\hat{\nu}$ satisfies condition (A.5) in Proposition A.1. Indeed, for any $(\lambda_i)_{i\in I}\in\mathbb{R}^I_+$, we have

$$\sum_{a\in A,\theta\in\Theta}\hat{\nu}(a,\theta)\max_{\gamma:a(\gamma)=a}\sum_{i\in S(a)}\lambda_i d_i(a_{-i}(\gamma),\theta)$$

$$
\begin{aligned}
&= \sum_{\theta\in\Theta} \hat{\nu}(\mathbf{1},\theta) \max_{\gamma:a(\gamma)=\mathbf{1}} \sum_{i\in I} \lambda_i d_i(a_{-i}(\gamma),\theta) \\
&= \sum_{\substack{a\in A,\theta\in\Theta\\ r(a,\theta)=1}} \nu(a,\theta) \max_{\gamma:a(\gamma)=\mathbf{1}} \sum_{i\in I} \lambda_i d_i(a_{-i}(\gamma),\theta) \\
&\geq \sum_{\substack{a\in A,\theta\in\Theta\\ r(a,\theta)=1}} \nu(a,\theta) \max_{\gamma:a(\gamma)=a} \sum_{i\in I} \lambda_i d_i(a_{-i}(\gamma),\theta) \\
&\geq \sum_{\substack{a\in A,\theta\in\Theta\\ r(a,\theta)=1}} \nu(a,\theta) \max_{\gamma:a(\gamma)=a} \sum_{i\in S(a)} \lambda_i d_i(a_{-i}(\gamma),\theta) \\
&\geq \sum_{a\in A,\theta\in\Theta} \nu(a,\theta) \max_{\gamma:a(\gamma)=a} \sum_{i\in S(a)} \lambda_i d_i(a_{-i}(\gamma),\theta) \geq 0,
\end{aligned}
$$

where the first inequality holds by supermodularity (for $i \notin S(\gamma)$, $a_{-i}(\gamma)$ denotes the action profile of i's opponents such that player j plays action 1 if and only if $j \in S(\gamma)$), the second inequality holds since $d_i(a_{-i},\theta) > 0$ when $r((0,a_{-i}),\theta) = 1$, the third inequality holds since $d_i(a_{-i},\theta) \leq 0$ when $r((1,a_{-i}),\theta) = 0$, and the last inequality holds by condition (A.5) for ν. Therefore, by Proposition A.1, $\hat{\nu}$ satisfies sequential obedience.

Second, for lower obedience (i.e., condition (1.1) with $a_i = 0$), for each $i \in I$ we have

$$
\begin{aligned}
&\sum_{a_{-i}\in A_{-i},\theta\in\Theta} \hat{\nu}((0,a_{-i}),\theta) d_i(a_{-i},\theta) = \sum_{\theta\in\Theta} \hat{\nu}(\mathbf{0},\theta) d_i(\mathbf{0}_{-i},\theta) = \sum_{\substack{a\in A,\theta\in\Theta:\\ r(a,\theta)=0}} \nu(a,\theta) d_i(\mathbf{0}_{-i},\theta) \\
&\leq \sum_{\substack{a\in A,\theta\in\Theta:\\ r(a,\theta)=0}} \nu(a,\theta) d_i(a_{-i},\theta) \\
&= \sum_{\substack{a_{-i}\in A_{-i},\theta\in\Theta:\\ r((1,a_{-i}),\theta)=0}} \nu((1,a_{-i}),\theta) d_i(a_{-i},\theta) + \sum_{\substack{a_{-i}\in A_{-i},\theta\in\Theta:\\ r((0,a_{-i}),\theta)=0}} \nu((0,a_{-i}),\theta) d_i(a_{-i},\theta) \\
&\leq \sum_{\substack{a_{-i}\in A_{-i},\theta\in\Theta:\\ r((0,a_{-i}),\theta)=0}} \nu((0,a_{-i}),\theta) d_i(a_{-i},\theta) \\
&\leq \sum_{a_{-i}\in A_{-i},\theta\in\Theta} \nu((0,a_{-i}),\theta) d_i(a_{-i},\theta) \leq 0,
\end{aligned}
$$

where the first inequality holds by supermodularity, the second inequality holds since $d_i(a_{-i},\theta) \leq 0$ when $r((1,a_{-i}),\theta) = 0$, the third inequality holds since $d_i(a_{-i},\theta) > 0$ when $r((0,a_{-i}),\theta) = 1$, and the last inequality holds by the lower obedience of ν. Therefore, $\hat{\nu}$ satisfies lower obedience.

Hence, we have $\hat{\nu} \in \overline{SI}$ by Theorem 1. □

For a generalized regime change game $(d_i)_{i\in I}$, the objective V is a *generalized regime change objective* with respect to $(d_i)_{i\in I}$ if it is written as

$$V(a,\theta) = \begin{cases} > 0 & \text{if } r(a,\theta) = 1, \\ = 0 & \text{if } r(a,\theta) = 0, \end{cases}$$

where we maintain the assumption that $V(a,\theta)$ is nondecreasing in a. By Proposition B.7, we immediately have the following.

Proposition B.8. *Let $(d_i)_{i\in I}$ be a generalized regime change game, and V a generalized regime change objective with respect to $(d_i)_{i\in I}$. Then there exists an optimal outcome of the adversarial information design problem that satisfies perfect coordination.*

Like Inostroza and Pavan (2022), we are not able to obtain explicitly the solution to the problem at this level of generality. Under the assumption of the existence of a convex potential (which covers symmetric regime change games), we derived an explicit expression of the optimal perfect coordination outcome in Theorem 2 in Section 4.