Certification Design with Common Values*

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January 21, 2024

Abstract

This paper studies certification design and its implications for information disclosure. Our model features a profit-maximizing certifier and the seller of a good of unknown quality. We allow for common values as the seller's opportunity cost may depend on the quality of the good. We compare certifier-optimal with transparency-maximizing certification design. Certifier-optimal certification design implements the evidence structure of Dye (1985) – a fraction of sellers acquire information while the remaining sellers are uninformed – and results in partial disclosure to the market. A transparency-maximizing regulator prefers a less precise signal which conveys more information to the market through a higher rate of certification and unraveling (Grossman, 1981; Milgrom, 1981) at the disclosure stage.

Keywords: Disclosure, Certification, strategic information transmission, information design

JEL Classification: D82, D83, L15

*For their valuable comments and advice, we sincerely thank three anonymous referees, Elchanan Ben-Porath, Alex Gershkov, Ilan Kremer, Motty Perry, Roland Strausz, Sebastian Schweighofer-Kodritsch, Boaz Zik, as well as audiences at Bar-Ilan University, ESSEC Business School, Hebrew University, University of Konstanz, University of Mannheim, the winter meeting of the econometric society in Berlin, and the annual conference of ASSET in Rethymnon, Crete. Weksler acknowledges support from the Israel Science Foundation (grant number 1728/23).

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1 Introduction

In many markets, buyers cannot readily observe the quality of the offered goods. Sellers often respond by voluntarily disclosing verifiable product information in the form of certificates, labels, or ratings. Sellers obtain such verifiable information from intermediaries – or certifiers – who evaluate the quality of the goods against a given standard.

The extent to which sellers disclose information to the market is of central interest to the analysis of quality certification. Viscusi (1978), Grossman and Hart (1980), Grossman (1981), and Milgrom (1981) provide a powerful *unraveling* argument for complete information disclosure. However, this result relies on the assumption that all sellers are informed about the quality of their goods. If some sellers are uninformed, Dye (1985) shows that disclosure remains partial as sellers of low quality conceal their information to pool with the uninformed sellers.

In this paper, we study how certification design affects the extent of certification and information disclosure. In practice, certification schemes are purposefully designed, and the design choices determine whether sellers buy certification and what they learn in case they do. Our analysis aims to answer the following questions: How does a monopolistic certifier optimally design the standard of certification? How many sellers obtain certification and what do they disclose under the certifier-optimal design? How do these outcomes compare to the case in which the certification standard is devised by a designer who aims to maximize transparency on the market?

These questions are of high relevance in practice where certification standards are designed by different types of institutions with different objectives. In some markets, the certifiers design the standard. This is the case in financial markets where credit rating agencies evaluate the risk of financial products. In other markets, the standard designer neither is the certifier nor shares the certifier's profit-maximizing objective. The International Organization of Standardization (ISO) – a non-governmental organization – provides a large variety of different standards such as the management quality standard ISO 9001. Standards may also be designed by governmental bodies such as the United States Environmental Protection Agency which authored the "Energy Star"-label as a standard for the energy efficiency of electronic products.[1]

We consider the following setup. A seller may sell a good on a market composed of homogeneous buyers. Initially, neither the seller nor the buyers know the quality of the good. We allow for common values in that the quality influences both the value of

[1]Independently of the type of the certification designer, sellers typically obtain certificates from a profit-maximizing certifier. For instance, the service of certification according to the quality management standard ISO 9001 is offered by the "Big Four" accounting firms Deloitte, Ernst & Young, KPMG, and PwC. Sellers who wish to obtain the "Energy Star" label are required to hire a "Professional Engineer" to review their application.

the buyers as well as the opportunity cost of the seller. Before entering the market, the seller may obtain a hard signal about the good's quality from a monopolistic certifier at a fee. The certifier incurs a cost from certifying the seller. Upon paying the fee, the seller observes a realization of the signal and decides whether or not to disclose the signal realization to the buyers. The buyers do not observe whether the seller obtains the signal and can therefore not differentiate between sellers who are uninformed and sellers who are informed but conceal their signal realization. We model certification design as the choice of a signal from a set of technologically feasible signals.

We obtain three main results. First, we provide an elegant and useful characterization of certifier-optimal certification design. In particular, the certifier's optimal signal maximizes the seller's willingness to pay across all feasible signals. This characterization holds for any arbitrary set of feasible signals and is irrespective of the probability at which the seller obtains the optimal signal in equilibrium. Second, we use the characterization to show that any certifier-optimal signal induces partial certification and, consequently, partial disclosure in equilibrium. Thus, the information structure of Dye (1985) arises endogenously. Third, we show that certifier-optimal certification design is suboptimal from the perspective of a regulator who aims to convey as much information to the market as possible. In particular, we show that by lowering the informativeness of a certifier-optimal signal, the regulator can increase the information on the market through higher equilibrium rates of certification and disclosure. Moreover, we provide conditions under which the regulator would always optimally choose a signal that induces full certification and unraveling as in Grossman (1981) and Milgrom (1981).

Two comments regarding these findings are in order. First, our results require the conditions of common values and costly certification. Common values between the seller and the buyers naturally arise in many markets in which certification is important.[2] In product markets, producers of higher quality often incur higher production costs. In markets for durable goods, owners of high quality goods derive higher utility from keeping the good. In financial markets, sellers of more profitable financial assets typically expect higher prices when selling their asset either in the future or through a different channel. Certification costs seem to be an equally natural condition as a certifier may incur some costs when signing a contract with the seller, reviewing the seller's documents, or providing access to the certificate for potential buyers. Moreover, our results hold even in the case where certification costs are strictly positive but arbitrarily small.[3] Our second

[2]With common values, the valuations of seller and buyers are determined by the same parameter, but need note be identical. This follows the terminology in Maskin and Tirole (1990, 1992).

[3]In the main model, we assume that the certifier incurs no costs beyond the aforementioned transaction costs, i.e., costs are uniform across signals. In Section 6.1, we extend our analysis to signal-dependent and fixed certification costs and discuss costless certification.

remark concerns the regulator's objective to increase the information on the market. The amount of information on the market is a key metric in the literature on information disclosure. However, it does not coincide with social welfare. Indeed, social welfare and market informativeness are perfectly misaligned in our model as full trade prevails independently of market information while certification costs are a waste from a social welfare perspective. However, this stark contrast is an artefact of the simplicity of our model. If information has social value either due to an ex-ante investment of the seller (Ben-Porath, Dekel and Lipman, 2018) or an ex-post investment of the buyer (Shavell, 1994), market informativeness and social welfare become much more closely aligned.

We start our analysis by making a case distinction regarding the signal which is central to our analysis. We say that a signal induces *irrelevant opportunity costs* (henceforward IOC) if the buyers' expected valuation after the worst signal realization lies weakly above the seller's unconditional expected opportunity costs. In this case, the opportunity cost does not matter as the seller always prefers to trade the good even if the buyers hold the most pessimistic belief regarding the quality of the good. By contrast, a signal induces *relevant opportunity costs* (henceforward ROC) if the buyers' expected valuation after the worst signal realization lies strictly below the seller's unconditional expected opportunity costs. In this case, the opportunity cost matters as the seller may prefer to keep the good if the buyers' belief is sufficiently pessimistic.

We find that the distinction between relevant and irrelevant opportunity costs determines whether the certifier's optimal fee implements full or partial disclosure. If a signal induces IOC, the certifier optimally sells the signal to the seller with probability one. The unraveling argument of Grossman (1981) and Milgrom (1981) applies and leads to full disclosure. If a signal induces ROC, the certifier optimally sells the signal with some positive probability strictly less than one, resulting in partial disclosure and an active market of uncertified goods as in Dye (1985).

We now provide an intuition for this result. If the seller always buys certification, unraveling ensues and the equilibrium price of uncertified goods deflates to the buyers' valuation after the worst signal realization. If the signal induces IOC, an uninformed seller prefers to sell at this price rather than keeping the good, and thus the seller is willing to pay a fee up to the difference between the unconditional expected valuation of the buyers and the lowest posterior mean. If the signal induces ROC, an uninformed seller prefers to keep the good. Thus, the willingness to pay for the signal equals the difference between the expectations of the buyers' and seller's valuations for the good, i.e., the expected gains from trade. Can the certifier attain a higher profit if the seller buys certification with less than probability one? As in Dye (1985), disclosure is then partial and the price for uncertified goods is decreasing in the probability of certification. A higher price for uncertified goods raises the expected payoff for a seller who obtains certification due to the option value from concealing bad signal realizations and selling without disclosure. If the signal induces IOC, the expected payoff for an uninformed seller rises more strongly as the seller always sells without disclosure. Thus, the seller's willingness to pay for being informed decreases and the certifier's profit falls. By contrast, if the signal induces ROC, the certifier can increase her profit. In particular, suppose the seller buys certification with a probability such that the price for uncertified goods equals the seller's expected opportunity costs. This increases the expected payoff from being informed – due to the option value from selling without disclosure – and leaves the expected payoff from being uninformed unchanged. If the certifier extracts the increased willingness to pay with her fee, neither the seller nor the buyers earn a positive rent. Thus, the certifier's profit is higher than under full unraveling as she extracts all gains from trade by inducing only a fraction of sellers to acquire certification.

The analysis of optimal signal fees paves the way toward a simple yet powerful characterization of the set of certifier-optimal signals. We show that the certifier's optimal profit from selling a given signal is a monotone increasing function of the seller's maximal willingness to pay for the signal, and does not depend on other properties of the signal or the equilibrium – such as the probability of certification. Thus, the certifier picks only signals that maximize the seller's maximal willingness to pay across all feasible signals. The seller's maximal willingness to pay for a signal inducing ROC is always higher than for a signal inducing IOC. Hence, the characterization directly implies that any optimal signal induces ROC and partial disclosure, as in Dye (1985), arises endogenously under certifier-optimal certification design.

Finally, we analyze regulator-optimal certification design in the case where the certifier retains the right to set the fee. First and seemingly paradoxical, we show that the regulator may increase the information on the market by lowering the informativeness of certification. Indeed, any signal that induces ROC can be garbled into a new signal that leads – under a certifier-optimal fee – to more information on the market in the sense of Blackwell (1951, 1953). The increase in information on the market is due to higher probabilities of certification and disclosure. If the certification costs lie below the expected gains from trade, the transparency-increasing garbling takes a simple form: posterior means below some cutoff are pooled into the same signal realization whereas the posterior means above the cutoff are perfectly revealed. The cutoff is chosen such that the signal induces IOC and is therefore sold to all sellers and fully disclosed. If some feasible signal Blackwell-dominates all other feasible signals, garbling the Blackwell-dominant signal as just described yields a transparency-maximal signal.

Our analysis sheds light on the subtle relationship between the precision of certification and market transparency. Very precise certification designs induce certifiers to set high fees at which only a fraction of sellers obtain certification. Less precise certificates induce lower fees from certifiers and may result in more transparency through more certification and higher disclosure activity. Credit rating agencies use a precise certification scheme.[4] Ali, Haghpanah, Lin and Siegel (2022) point out the US municipal bonds market as an exemplary case in which the sellers, i.e., the municipalities, are imperfectly informed about the relative quality of their good. They report that only about half of US municipal bonds are sold with a credit rating. This finding shows that – consistent with our theory – precise certification designs may be coupled with low rates of certification and, thus, low market transparency. The transparency-increasing effect of coarser certification schemes may also explain why the norm ISO 9001 and the "Energy Star"-label – designed by non-profit organizations – are much coarser than credit ratings.[5] Also in line with our theory, the Energy Star label attains high rates of market coverage.[6]

The paper is organized as follows. In the remainder of this section, we present the related literature. We present the model in Section 2. In Section 3, we provide a complete characterization of the seller's demand for certification. Section 4 presents the analysis of the certifier's optimal pricing of a given signal. In Section 5, we study optimal certification design from the perspectives of the certifier and the regulator. In Section 6, we discuss extensions of our model to more general certification costs and ex-ante information for sellers. Section 7 concludes. The proofs appear in the appendix.

1.1 Related Literature

Our paper relates to the literature on voluntary disclosure of verifiable information. This literature is concerned with the strategic disclosure of evidence by a sender to a receiver. Viscusi (1978), Grossman and Hart (1980), Grossman (1981), and Milgrom (1981) consider a perfectly informed sender who can disclose any evidence at no cost. In such an environment, information unravels as any sender type discloses in equilibrium. Verrecchia (1983) shows that unraveling breaks down if disclosure is costly as senders with relatively low values choose to conceal their evidence. Dye (1985) and Jung and Kwon (1988) show that the unraveling result also fails if the market is uncertain regarding whether or not the sender possesses evidence as senders with unfavorable evidence pool with uninformed senders by not disclosing their information. In all of these seminal contributions, as

[4]Credit rating agencies typically use scales ranging from AAA to C in around twenty steps.

[5]Certification according to ISO 9001 results in a binary outcome: either the certificate is awarded or not. Certification according to the "Energy Star" generates three possible grades as goods may either be certified or not and certified goods may receive the additional distinction "Most Efficient".

[6]For instance, in the US dishwasher market, the Energy Star has a market penetration rate of 96% (EPA, 2023).

well as in the more recent literature on disclosure games (Glazer and Rubinstein, 2008; Sher and Vohra, 2015; Hart, Kremer and Perry, 2017; Ben-Porath, Dekel and Lipman, 2019; Lichtig and Weksler, 2023), each sender's type is exogenously endowed with a set of evidence that the sender may disclose.

Our paper contributes to the growing literature that studies endogenous evidence structures. As opposed to our model, most of this literature deals with models that feature only a sender and a receiver (in a market context, a buyer and a seller). DeMarzo, Kremer and Skrzypacz (2019) study a buyer-seller model where the seller is choosing covertly from a given set of signals. They assume that the signals produce soft information with an exogenous positive probability, as in the information structure of Dye (1985). For any given set of signals, they characterize the equilibrium signal as the one that induces the minimal price in case the seller does not disclose. Shishkin (2019) analyzes optimal overt test design by a sender who wants to persuade a receiver to accept a project. Assuming that the test yields hard evidence only with a small exogenous probability, he shows that the optimal test has a simple cutoff structure. In our model, every signal yields a verifiable result with certainty.[7] The evidence structure of Dye (1985) emerges in our analysis as the result of a profit-maximizing pricing decision of a monopolistic information intermediary. Dasgupta, Krasikov and Lamba (2023) analyze a buyer-seller model, where the buyer overtly designs a signal about her utility from the object for sale. This signal can generate arbitrarily many hard signal realizations or a soft residual signal realization. They find that the buyer will optimally choose a test that generates the soft signal realization with positive probability, and, like in our model, a market where hard evidence is not presented endogenously emerges. Differently from our model, this results from the buyer's incentive to avoid full surplus extraction by the monopolistic seller.[8]

Our paper belongs to the strand of the literature that adds a strategic information intermediary to the basic disclosure setup with a seller (sender) and a competitive market (receiver). We contribute to this literature by studying the implications of common values on the incentive of the intermediary, an aspect which has previously not been studied.

One important strand of the literature analyzes the optimal behavior of a monopolistic information intermediary who offers the seller a signal at a fee. Lizzeri (1999) assumes that the seller is initially fully informed and finds that the intermediary's optimal signal is completely uninformative.[9] Ali et al. (2022) consider the same question with two major

[7]Thus, a seller who acquired a signal can always distinguish himself from a seller who did not.

[8]Also related are Onuchic (2023) and Whitmeyer and Zhang (2022). Both papers compare overt and covert certification design. Onuchic (2023) studies a model where the sender can commit to the disclosure probability for each piece of evidence. Whitmeyer and Zhang (2022) study a model where disclosure is costly as in Verrecchia (1983).

[9]In a recent paper, Ben-Porath, Dekel and Lipman (2021) consider a sender/receiver environment with an initially informed sender that chooses a signal that he can later voluntarily disclose. They take

differences. First, the seller is initially uniformed. Second, in case that the strategy of the intermediary induces multiple disclosure equilibria, the worst equilibrium from the intermediary's perspective is chosen. They find that the optimal signal is noisy and features a continuum of possible scores.[10] Our setting includes an initially uninformed seller as in Ali et al. (2022). Thus, our approach is also in the spirit of the literature on information design and Bayesian persuasion (Kamenica and Gentzkow, 2011) which it extends to a framework of verifiable disclosure. We differ from Ali et al. (2022) by assuming a favorable equilibrium selection from the intermediary's perspective as well as by allowing for common values.

Another strand of this literature considers a regulator who wants as much information as possible to be released to the market. Most of this literature deals with the comparison between two regulatory disclosure regimes: mandatory disclosure and voluntary disclosure (Shavell, 1994; Bar-Gill and Porat, 2020; Weksler and Zik, 2023) or the interplay between these two regimes (Friedman, Hughes and Michaeli, 2020; Bertomeu, Vaysman and Xue, 2021; Banerjee, Marinovic and Smith, 2021). An important exception is Harbaugh and Rasmusen (2018). They consider a model where the information intermediary is the regulator himself, the seller is initially fully informed, and creating a signal is costly. Harbaugh and Rasmusen (2018) find that, although the information intermediary's objective is to release as much information as possible, the optimal signal is not fully informative. By coarsening the signal, the information intermediary is able to induce more types of sellers to acquire the signal and thus release more information to the public. Specifically, they find that the optimal signal pools all values below some cutoff and fully reveals all values above this cutoff. This design induces a high willingness to pay for the low types while maintaining high transparency. We also consider a transparency-maximizing regulator whose signal choice we contrast with that of the profit-maximizing certifier. Importantly, in our framework, the regulator is only in charge of the signal choice while the profit-maximizing certifier remains in charge of selling the signal. However, our results do have some resemblance as our garbling procedure also pools low signal realizations and leaves high signal realizations unchanged. In our case, this garbling improves transparency as it changes the pricing behavior of the certifier to one that induces unraveling.

a mechanism design approach to the problem, i.e. the receiver has commitment power over his action after each disclosure.

[10]Faure-Grimaud, Peyrache and Quesada (2009) consider a seller who is initially partially informed and allow the intermediary to charge a testing fee and a disclosure fee that may depend on the signal realization. They assume that the signal is perfectly revealing and show that the intermediary is able to extract the entire surplus in this environment.

2 Model

2.1 Market

A risk-neutral seller seeks to sell a good in a competitive market to one of several risk-neutral buyers. The value of the good to the seller and the buyers depends on the state of the world $v \in V \subset \mathbb{R}$. We sometimes refer to the state as quality of the good. The state is initially unobservable to the seller and the buyers. The set V is compact and its convex hull is denoted by $[\underline{v}, \overline{v}]$. The state is the realization of a random variable ν with the commonly known cumulative distribution function $F(v) \equiv \Pr(\nu \leq v)$. For $v \in V$, each buyer values the good by v while the seller values the good by $\phi(v)$. Depending on the context, the opportunity cost of trade $\phi(v)$ can be understood as a production cost – such as with consumption goods –, the value of keeping the good – as with durable goods –, or the expected payoff from selling the good at a future date or through a different channel – as with financial assets.

2.2 Certification

Before going to the market, the seller can obtain hard information about the state of the world from a certifier. The certifier sets a fee $r \geq 0$ at which the seller can observe the realization of a signal σ. A signal σ is a random variable with generic realization s in the support S_σ that may be correlated with the state ν.[11] For a given signal σ, a signal realization $s \in S_\sigma$ induces the posterior mean $E_\sigma[\nu|s]$ of the buyers' value. The signal σ induces a distribution over the posterior means of the buyers' value of

$$G_\sigma(v) \equiv \Pr(E_\sigma[\nu|\sigma] \leq v).^{12}$$

The prior distribution $F(v)$ is a mean-preserving spread of $G_\sigma(v)$ for any signal σ, i.e.,[13]

$$\int_{\underline{v}}^{\overline{v}} v dG_\sigma(v) = E[\nu] \quad \text{and} \quad \int_{\underline{v}}^{v} G_\sigma(x)dx \leq \int_{\underline{v}}^{v} F(x)dx \quad \forall v \in [\underline{v}, \overline{v}].$$

Let V_σ be the set of posterior means induced by the signal σ. We assume that the set V_σ is a closed set in $\mathbb{R}$, and its convex hull is denoted by $[\underline{v}_\sigma, \overline{v}_\sigma]$. Denote the set of signals

[11] More formally, a signal consists of a Borel-measurable signal space S and a probability measure μ on the σ-algebra of $S \times V$ such that $\int_{S\times\{x\in V: x\leq v\}} d\mu = F(v)$.

[12] For any measurable function $h : S_\sigma \to \mathbb{R}$, we denote by $h(\sigma)$ the random variable which σ induces on the support $\{h(s)\}_{s\in S_\sigma}$.

[13] Let the random variable ε satisfy $\nu = E[\nu|\sigma] + \varepsilon$. Note that $E[\varepsilon|\sigma] = E[\nu|\sigma] - E[[\nu|\sigma]|\sigma] = 0$. Thus, $F(v)$ is a mean-preserving spread of $G_\sigma(v)$. The next line follows from Proposition 6.D.2 in Mas-Colell, Whinston and Green (1995).

that satisfy these conditions by Σ_F.

There is common knowledge regarding the distribution $G_\sigma(v)$ and the fee r. If the seller obtains a certificate, the certifier incurs a cost $c > 0$ and the seller observes a signal realization $s \in S_\sigma$. If the seller obtains certification and decides to disclose, the buyers observe the signal realization s. If the seller does not obtain certification or obtains certification and decides not to disclose, the buyers observe the signal realization $N \notin S_\sigma$.

2.3 Payoffs, seller's strategy, and market prices

The parties obtain the following payoffs in some state of the world $v \in V$. A buyer receives $v - p$ when buying the good at price $p \in \mathbb{R}$ and zero otherwise. The payoffs of the seller and the certifier for the price p and the fee r are given by

$$u = \phi(v) + \mathbf{1}(\text{sell})(p - \phi(v)) - \mathbf{1}(\text{cert})r \quad \text{and} \quad \pi = \mathbf{1}(\text{cert})(r - c),$$

where $\mathbf{1}(\text{sell}) = 1$ if the good is traded and $\mathbf{1}(\text{sell}) = 0$ otherwise, and $\mathbf{1}(\text{cert}) = 1$ if the seller obtains certification and $\mathbf{1}(\text{cert}) = 0$ otherwise.

Given a signal σ and a fee r, the seller faces the following sequence of decisions. First, the seller needs to decide whether to obtain certification or not. We denote the probability to buy certification by $a \in [0, 1]$. If the seller does not buy the certificate, she can decide whether to sell the good or not. We denote by $b_U \in [0, 1]$ the probability with which the seller sells the good in this case. If the seller obtains certification and observes $s \in S_\sigma$, she has three options: sell the good and disclose s, sell the good without disclosure, or keep the good. For any $s \in S_\sigma$, let $b_C^D(s) \in [0, 1]$ be the probability to sell with disclosure and $b_C^N(s) \in [0, 1]$ the probability to sell without disclosure. The total probability to sell satisfies $b_C^D(s) + b_C^N(s) \leq 1$. A (behavioral) strategy for the seller is therefore a collection

$$y = (a, b_U, b_C^D(\cdot), b_C^N(\cdot)).$$

If the seller decides to sell, the good is either traded uncertified or with some certificate $s \in S_\sigma$. Given a strategy y, the probability that the good is traded uncertified is

$$\Pr(N) \equiv aE[b_C^N(\sigma)] + (1 - a)b_U.$$

We say that the market for uncertified goods is *active* if $\Pr(N) > 0$ and *inactive* if $\Pr(N) = 0$. We denote by p^N the price on the market for uncertified goods. The function $p^D : S_\sigma \to \mathbb{R}$ assigns a market price to a good that is sold with the certificate $s \in S_\sigma$. Thus, the market prices are given by the collection

$$p = (p^N, p^D(\cdot)).$$

2.4 Equilibrium notion

Next, we define an equilibrium given a signal σ and a fee r. Denote the seller's expected payoff from a strategy y given the prices p, the signal σ and the fee r by

$$U(y,p,\sigma,r) \equiv a\Big(E\big[b_C^D(\sigma)p^D(\sigma) + b_C^N(\sigma)p^N + (1 - b_C^D(\sigma) - b_C^U(\sigma))E_\sigma[\phi(\nu)|\sigma]\big] - r\Big) + (1-a)\Big(b_U p^N + (1-b_U)E[\phi(\nu)]\Big).$$

Our equilibrium notion is the following.

Definition 1. *Given a signal σ and a fee r, an equilibrium is a combination of a strategy y for the seller and market prices p such that*

1. *y is optimal for the seller given p, i.e.,*

$$y \in \arg\max_{y'} U(y', p, \sigma, r), \tag{1}$$

2. *p is consistent with y, i.e.,*

$$p^D(s) = E_\sigma[\nu|s],\ \forall s \in S_\sigma, \quad p^N \in \begin{cases} \{E[\nu|N]\} & \text{if } \Pr(N) > 0, \\ [\underline{v}_\sigma, \bar{v}_\sigma] & \text{if } \Pr(N) = 0, \end{cases} \tag{2}$$

where

$$E[\nu|N] = \frac{aE[b_C^N(\sigma)E_\sigma[\nu|\sigma]] + (1-a)b_U E[\nu]}{\Pr(N)}.$$

The set of equilibria is denoted by $\mathcal{E}(\sigma, r)$.

The equilibrium notion consists of two conditions. Condition (1) requires that the seller plays an optimal strategy given the market prices. Condition (2) is a market-clearing condition. As the seller is on the short side of the market, the good is traded at the buyers' expected value. If the market for uncertified goods is active, the buyers' expected quality on this market is determined by Bayes' rule given the seller's strategy. If the market for uncertified goods is inactive, rational expectations do not pin down the buyers' expected value. Given the seller's information, any expectation in the set $[\underline{v}_\sigma, \bar{v}_\sigma]$

remains possible in this case.[14]

One may provide a game-theoretic foundation for this market equilibrium notion. Consider a game in which the seller first decides whether to offer the good and what to disclose, followed by the buyers bidding for the good in a second-price auction. Any (weak) Bayesian perfect equilibrium[15] of this game would then satisfy conditions (1) and (2).

2.5 Certifier's pricing problem

We now describe the certifier's problem of pricing a given signal σ. To this purpose, we define the demand $D_\sigma(r)$ for a given signal σ as a function of the fee r. We introduce this equilibrium object as it allows us to cast the certifier's problem in terms of standard monopoly analysis. We define the demand as the highest probability with which the seller acquires a signal σ in any equilibrium for a given fee r.

Definition 2. *The demand function for a signal σ is $D_\sigma(r) \equiv \max_{(y,p)\in\mathcal{E}(\sigma,r)} a$.*

The definition of the demand function implies certifier-preferred equilibrium selection – a standard assumption in contract theory and mechanism design.[16] For any given signal σ and fee $r \geq c$, the certifier's preferred equilibrium satisfies $a = D_\sigma(r)$. For the case $r < c$, the certifier might prefer an equilibrium with $a < D_\sigma(r)$. However, this is not problematic as the certifier can always set r sufficiently high to deter the seller from buying certification.

Thus, the certifier's optimal fee for a given signal σ solves the monopoly problem

$$\max_{r\geq 0}\ D_\sigma(r)(r-c). \tag{3}$$

2.6 Certification design

We capture certification design as the choice of a signal σ from a set of technologically feasible signals $\Sigma \subseteq \Sigma_F$. We first study the case in which the certifier chooses the signal from the set Σ. We then consider the case in which the signal is chosen by a regulator who seeks to maximize the information conveyed to the market. In both cases, the signal is then priced by the certifier according to the solution of problem (3) and sold to the seller. This assumption is motivated by the structure of certification markets in practice

[14] Note that the buyers form beliefs regarding the type of seller they are facing. A feasible belief must be a probability distribution over possible types of the seller. Given a signal σ, a seller could be either uninformed or informed by some realization of the signal σ. It follows that an expected quality outside of $[\underline{v}_\sigma, \bar{v}_\sigma]$ is not possible as it does not correspond to any probability distribution over the seller's types.

[15] As defined in Definition 9.C.3 of Mas-Colell et al. (1995)

[16] See Ali et al. (2022) for an analysis of certification design under adversarial equilibrium selection.

where – as described in the introduction – certification is typically provided by profit-maximizing certifiers while the standard may be designed by a transparency-maximizing institution.

2.7 Assumptions and discussion

We now specify three assumptions that we maintain throughout the analysis and discuss the two key conditions of common values and costly certification. Our first assumption ensures that there are always strictly positive gains from trade.

Assumption 1. *The function $\phi(\cdot)$ satisfies $\phi(v) < v$ for all $v \in V$.*

We denote the expected gains from trade by

$$\Delta \equiv E[\nu] - E[\phi(\nu)] = \int_{\underline{v}}^{\overline{v}} (v - \phi(v))dF(v).$$

Note that Assumption 1 puts very little restrictions on the seller's opportunity cost, e.g., $\phi(\cdot)$ does not have to be continuous nor monotone in the state of the world v. The second assumption is a technical condition on the set of feasible signals Σ.

Assumption 2. *The set $\Sigma \subseteq \Sigma_F$ satisfies the following conditions:*

i) the set $\{\underline{v}_\sigma\}_{\sigma\in\Sigma}$ is closed,

ii) the set $\{G_\sigma(\cdot)\}_{\sigma\in\Sigma}$ is closed under the weak topology.

The assumption ensures the existence of a certifier-optimal signal in the set of feasible signals Σ.[17] Note that conditions *i)* and *ii)* are satisfied if the set Σ is finite or equal to the set of all signals, i.e., $\Sigma = \Sigma_F$.

Our model features two natural conditions that drive our main results: common values and costly certification. We make the following assumption which specifies a minimal condition for common values to matter in equilibrium.

Assumption 3. *The function $\phi(\cdot)$ satisfies $E[\phi(\nu)] > \underline{v}$.*

The assumption implies that – under some sufficiently informative signal – the buyers' belief regarding the quality of uncertified goods may be so pessimistic – and thus the price for uncertified goods so low – that an uninformed seller prefers to keep the good rather than selling it without disclosure. In this case, the value of keeping the good influences the seller's willingness to pay for the signal and thereby becomes of relevance for the certifier.

[17] See the proof of Proposition 2 for details.

As our second key condition, we assume that certification is costly, i.e., $c > 0$. Costs of contracting, reviewing documents, or disseminating credible certificates suggest this to be a mild condition, especially as we allow c to be arbitrarily small.[18] In Section 6.1.2, we show how our results can be extended to the case where the certifier faces a signal-dependent cost of generating information in addition to the transaction cost c.

3 Demand for a signal

In this section, we take an intermediate step in our analysis by characterizing the demand $D_\sigma(r)$ for a given signal σ as a function of the fee r. We first introduce the following case distinction which is of relevance throughout the analysis.

Definition 3.

1. *A signal σ induces relevant opprtunity costs (ROC) if $\underline{v}_\sigma < E[\phi(\nu)]$.*
2. *A signal σ induces irrelevant opprtunity costs (IOC) if $\underline{v}_\sigma \geq E[\phi(\nu)]$.*

If $\underline{v}_\sigma < E[\phi(\nu)]$, the buyers' belief regarding the value of an uncertified good may be so pessimistic that the price on the uncertified market lies below the uninformed seller's expected value from keeping the good. Hence, the seller's opportunity cost $\phi(\cdot)$ may play an important role as self-consumption is an uninformed seller's best option in this case.

If $\underline{v}_\sigma \geq E[\phi(\nu)]$, keeping the good is a weakly dominated action for an uninformed seller as the price on the uncertified market always weakly exceeds the expected value from keeping the good. In this case, the specific opportunity cost function $\phi(\cdot)$ is irrelevant, and the analysis is the same as in the case where the seller has no opportunity cost, i.e., $\phi(\cdot) \equiv 0$.

Given a set of feasible signals Σ, we denote by Σ_{ROC} the subset of feasible signals that induce ROC, and by Σ_{IOC} the complementary subset of signals that induce IOC. Formally, we define

$$\Sigma_{ROC} \equiv \{\sigma \in \Sigma \mid \underline{v}_\sigma < E[\phi(\nu)]\} \quad \text{and} \quad \Sigma_{IOC} \equiv \{\sigma \in \Sigma \mid \underline{v}_\sigma \geq E[\phi(\nu)]\}.$$

The uninformative signal – if feasible – induces IOC as its support of posterior means is $\{E[\nu]\}$. If the fully informative signal is feasible, it induces ROC by Assumption 3 as the range of posterior means is $[\underline{v}, \bar{v}]$.

[18] We discuss the case of costless certification in Section 6.

3.1 Equilibrium analysis

In order to characterize the demand function, we make some important observations regarding equilibrium strategies and prices. Fix some signal σ. First, recall that the prices for certified goods need to reflect the information provided by the certificate both on and off the equilibrium path:

$$p^D(s) = E_\sigma[\nu|s], \quad \forall s \in S_\sigma. \tag{4}$$

Second, we characterize the equilibrium strategy of the seller for a given market price p^N for uncertified goods. First, suppose the seller has obtained certification. By Assumption 1, selling the good with disclosure strictly dominates keeping the good as $p^D(s) = E_\sigma[\nu|s] > E_\sigma[\phi(\nu)|s]$ for all $s \in S_\sigma$. Selling with disclosure is optimal if $p^D(s) = E_\sigma[\nu|s] \geq p^N$ and selling without disclosure is optimal if $E_\sigma[\nu|s] \leq p^N$. Formally, we have in any equilibrium

$$b_C^D(s) = 1 - b_C^N(s) \in \begin{cases} \{1\} & \text{if } E_\sigma[\nu|s] > p^N, \\ [0,1] & \text{if } E_\sigma[\nu|s] = p^N, \\ \{0\} & \text{if } E_\sigma[\nu|s] < p^N. \end{cases} \tag{5}$$

Next, suppose the seller has not obtained certification. The seller can then choose between keeping the good at an expected payoff of $E[\phi(\nu)]$ or selling the good without disclosure. Thus, the seller optimally plays

$$b_U \in \begin{cases} \{1\} & \text{if } p^N > E[\phi(\nu)], \\ [0,1] & \text{if } p^N = E[\phi(\nu)], \\ \{0\} & \text{if } p^N < E[\phi(\nu)]. \end{cases} \tag{6}$$

It follows that the seller's expected equilibrium payoff satisfies

$$U(y, p, \sigma, r) = a\Big(\int_{V_\sigma} \max\{v, p^N\} dG_\sigma(v) - r\Big) + (1-a) \max\{p^N, E[\phi(\nu)]\}.$$

The seller's willingness to pay for the signal σ is a function of the price for uncertified goods and given by

$$\Omega_\sigma(p^N) \equiv \int_V \max\{v, p^N\} dG_\sigma(v) - \max\{p^N, E[\phi(\nu)]\}. \tag{7}$$

The willingness to pay is the difference between the expected option value of an informed seller from choosing between selling with or without disclosure, and the option value of an uniformed seller who can choose between selling the uncertified good and keeping it.

Hence, the seller's optimal decision regarding certification satisfies

$$a \in \begin{cases} \{1\} & \text{if } r < \Omega_\sigma(p^N), \\ [0,1] & \text{if } r = \Omega_\sigma(p^N), \\ \{0\} & \text{if } r > \Omega_\sigma(p^N). \end{cases} \tag{8}$$

The conditions (5) to (8) characterize the equilibrium strategy of the seller for a given market price p^N for uncertified goods.

As the last step of our equilibrium characterization, we study the equilibrium price p^N given an optimal strategy for the seller. Consider an equilibrium with an active market for uncertified goods, i.e., $\Pr(N) > 0$. The market is populated by uninformed sellers and informed sellers of relatively low value goods. The expected value of an uncertified good to the buyers can be determined by Bayes' rule, giving us the equilibrium condition

$$\frac{(1-a)b_U E[\nu] + a \int_{\underline{v}}^{p^N} v dG_\sigma(v)}{(1-a)b_U + aG_\sigma(p^N)} = p^N.$$

Consider next an equilibrium with an inactive market for uncertified goods, i.e., $\Pr(N) = 0$. In such an equilibrium, informed sellers do not benefit from concealing their certificate. This requires $p^N \le \underline{v}_\sigma$. Moreover, uninformed sellers do not benefit from selling the good instead of keeping it, hence $p^N \le E[\phi(\nu)]$.

Thus, we obtain the following equilibrium condition for p^N given an optimal strategy of the seller:

$$p^N \in \begin{cases} \{\underline{v}_\sigma\} & \text{if } \Pr(N) = 0 \text{ and } a > 0, \\ \{v \in [\underline{v}_\sigma, \bar{v}_\sigma] : v \le E[\phi(\nu)]\} & \text{if } \Pr(N) = 0 \text{ and } a = 0, \\ \left\{\frac{(1-a)b_U E[\nu] + a \int_{\underline{v}}^{p^N} v dG_\sigma(v)}{(1-a)b_U + aG_\sigma(p^N)}\right\} & \text{if } \Pr(N) > 0. \end{cases} \tag{9}$$

This concludes the equilibrium characterization.

Lemma 1. *For any* $(\sigma, r) \in \Sigma \times \mathbb{R}$, $(y,p) \in \mathcal{E}(\sigma, r) \iff (y,p)$ *satisfies* (4) – (9).

Before proceeding to the characterization of demand, it is helpful to revisit the function $\Omega_\sigma(p^N)$ which captures the seller's willingness to pay for the signal σ as a function of the price p^N on the uncertified market.

Lemma 2. *The function* $\Omega_\sigma(\cdot)$ *is continuous and strictly quasi-concave on* $[\underline{v}_\sigma, \bar{v}_\sigma]$. *Its* *unique maximum is attained at* $p^N = \max\{E[\phi(\nu)], \underline{v}_\sigma\}$ *and given by*

$$\Omega^*_\sigma \equiv \max_{p^N \in [\underline{v}_\sigma, \overline{v}_\sigma]} \Omega_\sigma(p^N) = \begin{cases} \Delta + \int_{\underline{v}_\sigma}^{E[\phi(\nu)]} G_\sigma(v)dv & \text{if } \sigma \in \Sigma_{ROC}, \\ E[\nu] - \underline{v}_\sigma & \text{if } \sigma \in \Sigma_{IOC}. \end{cases}$$

If the signal σ induces ROC, the seller's willingness to pay first increases and then decreases in the price p^N on the segment $[\underline{v}_\sigma, \bar{v}_\sigma]$. An increase in p^N always benefits an informed seller who has the option to conceal the certificate and sell the good at p^N. If $p^N < E[\phi(\nu)]$, an increase in p^N does not affect the payoff of an uninformed seller as self-consumption remains more attractive than selling without disclosure. Thus, the willingness to pay for a signal increases in p^N as long as $p^N < E[\phi(\nu)]$. If $p^N > E[\phi(\nu)]$, an increase in p^N always benefits an uninformed seller as selling without disclosure is the uninformed seller's best choice. Moreover, the increase of p^N increases the uninformed seller's payoff by more than it increases the informed seller's payoff as the informed seller trades at p^N only with some probability. An increase in p^N therefore reduces the seller's willingness to pay in this case.

If the signal induces IOC, the seller's willingness to pay decreases in the price p^N as any equilibrium price p^N needs to exceed $E[\phi(\nu)]$. Thus, any increase in p^N increases the payoff of an uninformed seller more than the payoff of an informed seller, resulting in a lower willingness to pay for the signal.

3.2 Equilibrium demand for a signal

We are now in a position to state our characterization of the demand function $D_\sigma(r)$. To this purpose, define for any $\sigma \in \Sigma_{ROC}$ the function $\Omega_\sigma^{-1} : [\Delta, \Omega^*_\sigma] \to [E[\phi(\nu)], E[\nu]]$ as

$$\Omega_\sigma^{-1}(r) \equiv \max\{p^N \in [E[\phi(\nu)], E[\nu]] : \Omega_\sigma(p^N) \geq r\}.$$[19]

Lemma 3.

1. *Suppose* $\sigma \in \Sigma_{ROC}$. *Then*

$$D_\sigma(r) = \begin{cases} 1 & \text{if } r \leq \Delta, \\ \frac{E[\nu] - \Omega_\sigma^{-1}(r)}{r} & \text{if } r \in (\Delta, \Omega^*_\sigma], \\ 0 & \text{if } r > \Omega^*_\sigma, \end{cases}$$

and $D_\sigma(r)$ *is strictly increasing for all* r *with* $D_\sigma(r) \in (0, 1)$.

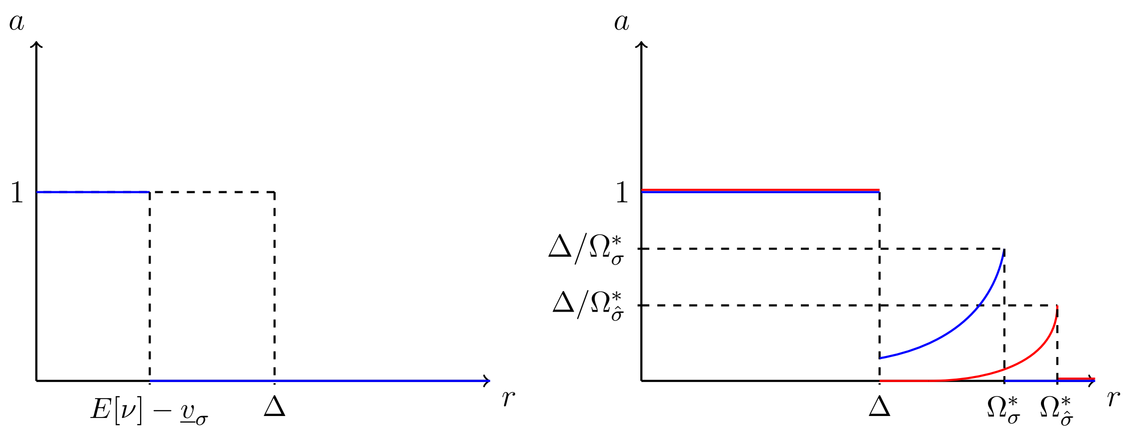
[19] Note that the function is the inverse of $\Omega_\sigma(\cdot)$ for $p^N \in [E[\phi(\nu)], E[\nu]]$.

2. Suppose $\sigma \in \Sigma_{IOC}$. Then

$$D_\sigma(r) = \begin{cases} 1 & \text{if} \quad r \le E[\nu] - \underline{v}_\sigma, \\ 0 & \text{if} \quad r > E[\nu] - \underline{v}_\sigma. \end{cases}$$

To construct the demand curve for a signal σ, we find for each fee $r \ge 0$ the equilibrium with the highest probability of certification. We relegate the formal steps of our analysis to the appendix.

Figure 1: Demand for a signal



The left panel depicts the demand functions for a signal σ which induces IOC. The right panel depicts the demand function for two signals σ (blue) and $\hat{\sigma}$ (red) with both inducing ROC.

The left panel of Figure 1 depicts the demand curve for a signal with IOC. The demand function is a simple step function. If the seller obtains certification with certainty, we obtain the information structure of Grossman (1981) and Milgrom (1981). Thus, unraveling occurs and pushes the price of uncertified goods to its lowest possible value, i.e., $p^N = \underline{v}_\sigma$. At this price, the seller's willingness to pay for the signal takes its maximal value $E[\nu] - \underline{v}_\sigma$. Thus, an equilibrium with full certification exists for $r \le E[\nu] - \underline{v}_\sigma$. Moreover, the seller is never willing to buy certification if the fee is higher, i.e., $r > E[\nu] - \underline{v}_\sigma$. Hence, the demand for certification is a step function, as it is typical for settings in which a monopolist sells a good to a group of homogeneous buyers.

The right panel of Figure 1 illustrates the shape of demand curves for signals that induce ROC. If the seller always buys the signal in equilibrium, unraveling collapses the uncertified price to $p^N = \underline{v}_\sigma$. At this price, the seller's willingness to pay is Δ. Thus, an equilibrium with full certification exists if $r \le \Delta$. If $r > \Delta$, there exists an equilibrium without certification in which the seller always self-consumes and the uncertified price $p^N = \underline{v}_\sigma$ is supported by the most pessimistic off-path belief following an offer of an uncertified good. The seller's willingness to pay does not peak at Δ if the signal induces ROC. Thus, equilibria with partial certification can arise if the market for uncertified goods is active. These equilibria are sustained by the option value brought about by the active market for uncertified goods. Partial certification requires indifference of the seller, i.e., $r = \Omega_\sigma(p^N)$. Moreover, the probability with which the seller becomes informed – which is an exogeneous parameter in Dye (1985) – needs to be such that the fraction of uninformed sellers sustains the price p^N. Uninformed sellers would self-consume if p^N falls below $E[\phi(\nu)]$. Hence, only the prices $p^N \in [E[\phi(\nu)], E[\nu]]$ – and the fees $r \in [\Omega_\sigma(E[\nu]), \Omega^*_\sigma]$ – can be supported in such equilibria. As the willingness to pay $\Omega_\sigma(p^N)$ is decreasing on $[E[\phi(\nu)], E[\nu]]$ and p^N is decreasing in the fraction of informed sellers, the equilibria with partial certification generate an increasing segment of the demand curve. As the red demand curve in Figure 1 shows, demand may even fall to zero for intermediate fees.[20]

4 Optimal pricing of a signal

In this section, we study how the certifier optimally prices a signal. We find the following solution to the certifier's pricing problem (3) given the demand curves illustrated in Figure 1.

Proposition 1.

1. *Suppose $\sigma \in \Sigma_{ROC}$. If $c < \Omega^*_\sigma$, the certifier attains the optimal profit $\Delta - (\Delta/\Omega^*_\sigma)c$ by certifying the seller with probability Δ/Ω^*_σ at the uniquely optimal fee $r = \Omega^*_\sigma$. If $c = \Omega^*_\sigma$, a fee is optimal if and only if $r \geq \Omega^*_\sigma$. If $c > \Omega^*_\sigma$, a fee is optimal if and only if $r > \Omega^*_\sigma$.*

2. *Suppose $\sigma \in \Sigma_{IOC}$. If $c < \Omega^*_\sigma$, the certifier attains the optimal profit $E[\nu] - \underline{v}_\sigma - c$ by certifying the seller with probability one at the uniquely optimal fee $r = E[\nu] - \underline{v}_\sigma$. If $c = \Omega^*_\sigma$, a fee is optimal if and only if $r \geq E[\nu] - \underline{v}_\sigma$. If $c > \Omega^*_\sigma$, a fee is optimal if and only if $r > E[\nu] - \underline{v}_\sigma$.*

The key insight generated by the proposition concerns the question whether the certifier prefers unraveling or partial disclosure on the market. We show that the certifier optimally induces an inactive market for uncertified goods through unraveling (Grossman, 1981; Milgrom, 1981) if the signal induces IOC. By contrast, the certifier induces partial disclosure as in Dye (1985) if the signal induces ROC.

[20]Formally, this arises for signals σ with $\Delta \leq \Omega_\sigma(E[\nu])$ and when $r \in (\Delta, \Omega_\sigma(E[\nu]))$.

In both cases of ROC and IOC, the certifier optimally sets the fee equal to the seller's maximal willingness to pay Ω^*_σ for the signal. Importantly, the optimal fee is independent of the certification cost c as long as the optimal fee covers the cost. With ROC, the optimal fee induces only a fraction of the sellers to acquire information, with the remaining fraction of uninformed sellers trading on the market for uncertified goods. With IOC, all sellers obtain certification and the market for uncertified goods is inactive.

With ROC, the certifier optimally accommodates an active market for uncertified goods as in Dye (1985) by inducing only a fraction of sellers to acquire certification. In particular, the certifier prefers this outcome over the one induced by certifying all sellers and shutting down the uncertified market. The latter outcome would be optimally induced by a fee $r = \Delta$ which would result in a profit of $\Delta - c$. By setting a fee $r \in (\Delta, \Omega^*_\sigma]$, the certifier can activate the uncertified market. As the demand function is increasing for these fees, the optimal fee with an active uncertified market is $r = \Omega^*_\sigma$. At this fee, the fraction Δ/Ω^*_σ of sellers is certified. Compared to the case of an inactive market for uncertified goods, the certifier obtains the same revenue of Δ for a strictly smaller certification cost of $(\Delta/\Omega^*_\sigma)c$. By maintaining an active market for uncertified goods, the certifier increases an informed seller's option value from concealing the certificate. At the same time, the active uncertified market does not increase the value of being uninformed as the price p^N on the uncertified market equals the expected value of self-consumption $E[\phi(\nu)]$.

It is noteworthy that the certifier may be active even if the costs of certification exceed the expected gains from trade. Indeed, the certifier charges a fee exceeding Δ and realizes a positive margin even if $\Delta < c < \Omega^*_\sigma$.

With IOC, it is optimal to shut down the market for uncertified goods through unraveling as in Grossman (1981) and Milgrom (1981). To prevent unraveling, the certifier would need to induce a fraction of sellers to remain uninformed and to implement a price for uncertified goods above $\underline{v}_\sigma$. However, the lost revenue could not be recouped from informed sellers. While increasing the price p^N above $\underline{v}_\sigma$ increases the payoff of informed sellers due to higher option value, the payoff of uninformed sellers increases more strongly. Thus, the signal's value to the seller shrinks.

5 Optimal certification design

In this section, we analyze optimal certification design as the choice of a signal σ. We first study the certifier's optimal choice from the set Σ and provide a characterization of certifier-optimal signals. In a second step, we consider a regulator who seeks to maximize transparency on the market and examine regulator-optimal certification design.

We assume that there exists a technologically feasible signal under which the certifier can make a positive payoff.

Assumption 4. *There exists a signal $\sigma \in \Sigma$ with $\Omega^*_\sigma > c$.*

By this assumption, which we maintain throughout the remainder of the analysis, we ensure that certification design is non-trivial.

5.1 Certifier-optimal certification design

We start by analyzing the certifier's optimal signal choice from the set Σ. We use the optimal pricing of signals in Proposition 1 to derive the following result.

Proposition 2. *The set of certifier-optimal signals is*

$$\Sigma^* = \arg\max_{\Sigma} \Omega^*_\sigma.$$

The proposition provides a simple and powerful characterization of the set of certifier-optimal signals: A signal is optimal if and only if the maximal willingness to pay for this signal is the highest across all feasible signals – independently of the probability with which this signal is sold in equilibrium. Therefore, the set of certifier-optimal signals can be identified using the statistic of the maximal willingness to pay Ω^*_σ only.[21] In particular, the set of certifier-optimal signals is independent of the certification cost c. The proof of the proposition builds on the characterization of optimal fees in Proposition 1 and shows that signals with higher maximal willingness to pay Ω^*_σ generate higher revenue and lower certification cost.

We highlight that the characterization of Proposition 2 holds for arbitrary sets of feasible signals satisfying the technical conditions of Assumption 2. We do not require further conditions such as all signals to be feasible, i.e., $\Sigma = \Sigma_F$, or the set Σ to be Blackwell-ordered or to contain a Blackwell-dominant signal.

Proposition 2 implies an important corollary: the evidence structure of Dye (1985) emerges endogenously under certification design.

Corollary 1. *If $\Sigma_{ROC} \neq \emptyset$, then $\Sigma^* \subseteq \Sigma_{ROC}$. Thus, the seller obtains certification with probability strictly less than 1 and partial disclosure, as in Dye (1985), prevails as a result of certifier-optimal certification design.*

The maximal willingness to pay Ω^*_σ for a signal inducing ROC exceeds the maximal willingness to pay for any signal which induces IOC. Thus, by Proposition 2, the set of

[21] Under part i) of Assumption 2, Ω^*_σ attains a maximum for $\Sigma = \Sigma_{IOC}$. Part ii) ensures this for $\Sigma \neq \Sigma_{IOC}$.

certifier-optimal signals Σ^* contains only signals that induce ROC, whenever at least one such signal is feasible.

The corollary further simplifies the procedure of finding the set of certifier-optimal signals if there exist feasible signals which induce ROC. In this case, one can restrict attention to the set Σ_{ROC} and identify all signals $\sigma \in \Sigma_{ROC}$ that maximize the statistic

$$\int_{\underline{v}}^{E[\phi(\nu)]} G_\sigma(v)dv.$$

This strengthening of our characterization is a result of the fact that for every $\sigma \in \Sigma_{ROC}$ the certifier is able to extract all the gains from trade Δ by charging the fee Ω^*_σ. Thus, it must be that the probability of selling a certificate is equal to $\frac{\Delta}{\Omega^*_\sigma}$. It follows that a signal that maximizes Ω^*_σ also minimizes the certifier's certification cost, and thereby maximizes the certifier's profit.

While the characterization of Proposition 2 applies to arbitrary sets of feasible signals, we obtain the following corollary if the set Σ features a most informative signal.

Corollary 2. *Suppose the set Σ contains a most informative signal $\bar{\sigma}$, i.e.,*

$$\int_{\underline{v}}^{v} G_\sigma(x)dx \leq \int_{\underline{v}}^{v} G_{\bar{\sigma}}(x)dx, \ \forall v \in [\underline{v}, \bar{v}], \ \ \forall \sigma \in \Sigma.$$

Then, $\bar{\sigma} \in \Sigma^*$

The corollary follows directly from Proposition 2 and Lemma 2 as the most informative signal of a set Σ induces the minimal lowest posterior mean $\underline{v}_\sigma$ and the largest value of the statistic $\int_{\underline{v}}^{E[\phi(\nu)]} G_\sigma(v)dv$ among all signals.

The logic behind the corollary allows us to make a stronger statement. Given any set of feasible signals Σ, at least one certifier-optimal signal is in the subset of signals $\Sigma_{info} \subseteq \Sigma$ that contains all maximally informative signals, i.e., all feasible signals that are not dominated by a different feasible signal in the sense of Blackwell's order. Thus, the optimizing certifier can restrict attention to the subset of signals Σ_{info} without any loss of optimality.

The certifier's optimal choice of signal and fee follows the principle of *maximizing the sellers' willingness to pay*, even if this requires inducing only a fraction of sellers to acquire information. If the set of possible signals Σ comprises only signals with IOC, the certifier induces unraveling. Moreover, the certifier's optimal signal minimizes the price on the uncertified market – the same metric as in the private value model of DeMarzo et al. (2019). By contrast, if there are signals with ROC, minimizing the nondisclosure price p^N is suboptimal. In this case, the certifier cannot increase revenue from pushing the nondisclosure price below $E[\phi(\nu)]$ as this would not affect the uninformed seller's payoff and lower the informed seller's payoff. Instead, the certifier activates the uncertified market to keep revenue constant and to lower the total cost of certification.

Example In order to illustrate the use of our results, we analyze a specific example of our model. Suppose the state ν is uniformly distributed on $V = [0, 1]$ and the seller's value is a fraction $\alpha \in (0, 1)$ of the buyers' value v, i.e., $\phi(v) = \alpha \cdot v$. We consider the set of feasible signals Σ to be the set of all binary threshold signals. In particular, for every $t \in [0, 1]$, denote by σ_t the signal that certifies whether the state is above or below t, and set $\Sigma = \{\sigma_t\}_{t\in[0,1]}$. Finally, suppose $c < \Delta = (1 - \alpha)E[\nu] = \frac{1-\alpha}{2}$.

Note that the set of binary threshold signals is not Blackwell-ordered, nor does it contain a Blackwell-dominant signal. Nevertheless, our results allow us to quickly find the set of certifier-optimal signals. We first identify the sets of signals inducing ROC and IOC. We have $\underline{v}_{\sigma_t} < E[\phi(\nu)] = \frac{\alpha}{2}$ if and only if $t < \alpha$. The sets of signals inducing ROC and IOC are therefore given by $\Sigma_{ROC} = \{\sigma_t\}_{t\in[0,\alpha)}$ and $\Sigma_{IOC} = \{\sigma_t\}_{t\in[\alpha,1]}$. Corollary 1 implies that all certifier-optimal signals must be in Σ_{ROC}. Using Proposition 2, we can now identify the optimal signals as those signals that lead to the largest maximal willingness $\Omega^*_{\sigma_t}$. The signal σ_t induces the posterior means $\frac{t}{2}$ and $\frac{1+t}{2}$ with probabilities t and $1 - t$, respectively. For any signal σ_t with $t < \alpha$, we therefore obtain

$$\Omega^*_{\sigma_t} = \Delta + \int_{\underline{v}_\sigma}^{E[\phi(\nu)]} G_{\sigma_t}(v)dv = \frac{1-\alpha}{2} + \int_{\frac{t}{2}}^{\frac{\alpha}{2}} t dv = \frac{1-\alpha}{2} + t\left(\frac{\alpha - t}{2}\right)$$

Clearly, this function is maximized at $t = \frac{\alpha}{2}$. Thus, the set of certifier-optimal signals in our example is the singleton $\Sigma^* = \{\sigma_{\frac{\alpha}{2}}\} = \{\sigma_{E[\phi(\nu)]}\}$.

The binary threshold signal $\sigma_{E[\phi(\nu)]}$ remains optimal in any set of feasible signals in which it is contained. Corollary 2 implies that the fully revealing signal $\tilde{\sigma}$ – which satisfies $G_{\tilde{\sigma}}(\cdot) = F(\cdot)$ – is certifier-optimal whenever it is feasible. We argue that the binary threshold signal $\sigma_{E[\phi(\nu)]}$ induces the same profit as the fully revealing signal. This follows from

$$\begin{aligned}\int_{\underline{v}}^{E[\phi(\nu)]} G_{\sigma_{E[\phi(\nu)]}}(v)dv &= F(E[\phi(\nu)])(E[\phi(\nu)] - E[\nu|\nu \leq E[\phi(\nu)]]) \\ &= \int_{\underline{v}}^{E[\phi(\nu)]} (E[\phi(\nu)] - v)dF(v) = \int_{\underline{v}}^{E[\phi(\nu)]} F(v)dv.\end{aligned}$$

This argument implies an even stronger result. Indeed, any signal which perfectly reveals whether the state lies above or below $E[\phi(\nu)]$ generates a maximal willingness to pay equal to that of the fully revealing signal, and is therefore certifier-optimal.

5.2 Regulator-optimal certification design

In this section, we consider a regulator who aims to maximize transparency on the market. We do not specify the regulator's preferences any further. Instead, we provide results based on Blackwell comparisons of the information on the market. In our analysis, the certifier retains the right to set the fee. This assumption is motivated by the structure of certification markets in practice as described in the introduction. In addition, this assumption allows us to identify differences in certifier- and regulator-optimal certification design independently from distortions that arise from the certifier's pricing power in the market for certifications.

We measure the information on the market using the equilibrium distribution over prices $H_\sigma(v)$. Given a signal σ and a certifier-optimal fee – as specified in Proposition 1 –, the equilibrium induces a distribution over disclosed signal realizations in the set $S_\sigma \cup \{N\}$. Any of these signal realizations results in a market price reflecting the buyers' expectation of the good's quality. The distribution over prices reflects the precision of the signal, the fraction of informed sellers, as well as the extent of disclosure. A signal σ leads to a better-informed market than another signal σ' if the distribution $H_\sigma(v)$ is a mean-preserving spread of the distribution $H_{\sigma'}(v)$.

The distribution over prices $H_\sigma(v)$ is determined as follows. Consider a signal σ with $c \leq \Omega^*_\sigma$. Assuming that the certifier sets the lowest optimal fee[22], the seller buys certification in equilibrium with the probability $\min\{\Delta/\Omega^*_\sigma, 1\}$. If the signal induces IOC, the certification is certain and full unraveling occurs. In this case, the distribution over prices equals the distribution $G_\sigma(v)$. If the signal induces ROC, the probability of certification is strictly interior and informed sellers disclose if the posterior mean weakly exceeds $E[\phi(\nu)]$ while they conceal their information otherwise. Thus, the distribution $H_\sigma(v)$ for an optimally priced signal σ is given by

$$H_\sigma(v) = \begin{cases} 0 & \text{if} \quad v < E[\phi(\nu)], \\ 1 - \min\{\Delta/\Omega^*_\sigma, 1\} + \min\{\Delta/\Omega^*_\sigma, 1\} G_\sigma(v) & \text{if} \quad v \geq E[\phi(\nu)]. \end{cases}$$

When σ induces ROC and the certifier prices the signal optimally, a seller can never gain from disclosing a signal realization that induces a posterior mean below $E[\phi(\nu)]$. Thus, no posterior means below this value arises in equilibrium. Signal realizations above $E[\phi(\nu)]$ are always disclosed in equilibrium. The posterior mean of $E[\phi(\nu)]$ is induced by the signal N which is generated if the seller is either uninformed or informed and refrains from disclosure.

Before we present the first result of this section, we recall the notion of a *garbling*

[22]This assumption is innocuous as the tie can be broken at almost no cost.

due to Marschak and Miyasawa (1968) as a tool to reduce the informational content of a signal. A signal σ can be garbled through a function $\gamma_\sigma : S_\sigma \to \Delta S$ which maps from the set of signal realizations of σ to the set of probability distributions over some set S of signal realizations. With slight abuse of notation, we denote the garbled signal by $\gamma(\sigma) \equiv \gamma_\sigma \circ \sigma$. Of course, the garbled signal $\gamma(\sigma)$ is less Blackwell informative than σ, i.e.,

$$\int_{\underline{v}}^{v} G_\sigma(x)dx \geq \int_{\underline{v}}^{v} G_{\gamma(\sigma)}(x)dx, \ \ \forall v \in [\underline{v}, \bar{v}].$$

In the following proposition, we show that garbling a signal with ROC can increase the information on the market.

Proposition 3. *For any $\sigma \in \Sigma_{ROC}$ with $\Omega^*_\sigma > c$, there exists a garbled signal $\gamma(\sigma) \neq \sigma$ which satisfies $\Omega^*_{\gamma(\sigma)} \geq c$, conveys weakly more information to the market than σ, i.e.,*

$$\int_{\underline{v}}^{v} H_{\gamma(\sigma)}(x)dx \geq \int_{\underline{v}}^{v} H_\sigma(x)dx \ \ \forall v \in [\underline{v}, \bar{v}],$$

and conveys strictly more information to the market than σ if $V_\sigma \cap (E[\phi(\nu)], \bar{v}_\sigma]$ is not a singleton. In this case,

$$\begin{aligned}\int_{\underline{v}}^{v} H_{\gamma(\sigma)}(x)dx \geq \int_{\underline{v}}^{v} H_\sigma(x)dx \ \ \forall v \in [\underline{v}, \bar{v}] \ \ \textit{and} \\ \int_{\underline{v}}^{v} H_{\gamma(\sigma)}(x)dx > \int_{\underline{v}}^{v} H_\sigma(x)dx \ \ \forall v \in (E[\phi(\nu)], \bar{v}_\sigma).\end{aligned}$$

The proposition shows that the regulator can increase the information on the market by reducing the informativeness of certification. Given some signal which induces ROC, the regulator can garble this signal and thereby increase the equilibrium rates of certification and disclosure. This results in a strict increase in market informativeness if the original signal features at least two posterior means above the unconditional expected payoff of keeping the good $E[\phi(\nu)]$.

Our proof of Proposition 3 is constructive. We show how any signal $\sigma \in \Sigma_{ROC}$ can be garbled into a signal $\gamma^*(\sigma)$ that satisfies the properties in the proposition.

The proposition builds on the observation that the price distribution $H_\sigma(v)$ results from a suboptimal garbling of the distribution of posterior means $G_\sigma(v)$. The price distribution $H_\sigma(v)$ can be obtained from the following procedure: draw posterior means from the distribution $G_\sigma(v)$, disclose posterior means above $E[\phi(\nu)]$ with probability Δ/Ω^*_σ, pool the remaining posterior means above $E[\phi(\nu)]$ with all the posterior means below $E[\phi(\nu)]$. The uniform treatment of posterior means above $E[\phi(\nu)]$ is not optimal from an informational perspective. Market information would increase if conditional on posterior means being above the threshold $E[\phi(\nu)]$ higher posterior means were disclosed with a larger probability than lower posterior means.

For $c \leq \Delta$, the garbling $\gamma^*(\cdot)$ improves upon the price distribution $H_\sigma(v)$ in the following way. Take the distribution $G_\sigma(v)$, pool all posterior means below a threshold into the same signal realization and perfectly disclose all posterior means above the threshold. The threshold is chosen such that the posterior mean induced by the pooling signal realization equals the expected value from keeping the good $E[\phi(\nu)]$. Thus, the garbled signal $\gamma^*(\sigma)$ induces IOC and results in full certification and unraveling. Moreover, the distribution of prices $H_{\gamma^*(\sigma)}(v)$ Blackwell-dominates $H_\sigma(v)$ as $\gamma^*(\sigma)$ pools only intermediate posterior means while perfectly revealing posterior means at the top whereas $H_\sigma(v)$ pools posterior means from the whole range and reveals the extreme posterior means only with some interior probability.

The garbled signal $\gamma^*(\sigma)$ strictly improves the market informativeness over σ whenever it changes the composition of posterior means above $E[\phi(\nu)]$ in the pooling signal realizations relative to $H_\sigma(v)$. This is possible if and only if σ induces at least two posterior means above the threshold, i.e., the set $V_\sigma \cap (E[\phi(\nu)], \bar{v}_\sigma]$ is not a singleton. If $V_\sigma \cap (E[\phi(\nu)], \bar{v}_\sigma]$ is a singleton, the garbled signal has to treat all posterior means above $E[\phi(\nu)]$ identically – a trivial statement as there is just one such posterior mean – and thus it leads to the same market informativeness as the original signal σ.

The garbling $\gamma^*(\cdot)$ is formally defined in equation (10) in the proof of Proposition 3. There, we also extends our argument to $c > \Delta$. In this case, only signals with ROC can cover certification costs. Nevertheless, the garbling $\gamma^*(\cdot)$ still improves upon all signals generating a positive margin to the certifier, i.e., $c < \Omega^*_\sigma$, by pooling intermediate posterior means into the same signal realization while fully revealing high and low posterior means. In the partial disclosure equilibrium for this signal, the pooling signal realization as well as the high posterior means are disclosed. By contrast, the low posterior means are concealed and pooled with the fraction of uninformed sellers.

We return to the case of $c \leq \Delta$ for which we obtain the following corollary.

Corollary 3. *Suppose $c \leq \Delta$, $V_\sigma \cap (E[\phi(\nu)], \bar{v}_\sigma]$ is not a singleton, and $\gamma^*(\sigma) \in \Sigma$ for all $\sigma \in \Sigma_{ROC}$. Any regulator-optimal signal induces IOC. Thus, full certification and unraveling as in Grossman (1981) and Milgrom (1981) prevails under regulator-optimal certification design.*

Corollaries 1 and 3 demonstrate a stark contrast between certifier- and regulator-optimal certification design. Under relatively mild conditions, any certifier-optimal signal induces the information structure of Dye (1985) and brings about partial certification and disclosure, whereas any regulator-optimal signal induces the information structure of Grossman (1981) and Milgrom (1981) and results in full certification and unraveling.

While Proposition 3 shows how the regulator can improve the information on the market by reducing the informativeness of a given signal, it does not specify a regulator-optimal signal. As we defined the regulator's preference only with respect to Blackwell's partial order, we have to add structure to the feasible set of signals Σ in order to get such a result. In the following corollary, we show that if the set of feasible signals contains a Blackwell-dominant signal, applying the garbling of Proposition 3 yields a regulator-optimal certification design.

Corollary 4. *Suppose the set Σ contains a most informative signal $\bar{\sigma}$, i.e.,*

$$\int_{\underline{v}}^{v} G_\sigma(x)dx \leq \int_{\underline{v}}^{v} G_{\bar{\sigma}}(x)dx, \ \ \forall v \in [\underline{v}, \bar{v}], \ \ \forall \sigma \in \Sigma,$$

and suppose the garbling $\gamma^*(\bar{\sigma})$ is also an element of Σ. Then, $\gamma^*(\bar{\sigma})$ is regulator-optimal, i.e.,

$$\int_{\underline{v}}^{v} H_{\bar{\gamma}(\bar{\sigma})}(x)dx \geq \int_{\underline{v}}^{v} H_\sigma(x)dx, \ \ \forall v \in [\underline{v}, \bar{v}], \ \ \forall \sigma \in \Sigma.$$

Corollaries 2 and 4 study the preferences of the certifier and the regulator regarding a most informative signal $\bar{\sigma}$. By Corollary 2, the most informative signal is always certifier-optimal. By Corollary 4, a garbling of the most informative signal is regulator-optimal. By Corollary 3, this garbling conveys strictly more information to the market than the most informative signal $\bar{\sigma}$, whenever $\bar{\sigma}$ induces ROC and features at least two posterior means above the threshold $E[\phi(\nu)]$.

A reduction in the informativeness of a signal with a single posterior mean above the threshold $E[\phi(\nu)]$ cannot improve the information on the market. However, this does not mean that the contrast between certifier- and regulator-optimal certification design is not relevant for such signals. In the following, we revisit our example of Section 5.1 in which the set of feasible signals consists of binary threshold signals that have a single posterior mean above the threshold $E[\phi(\nu)]$. We show that the conflict between certifier- and regulator-optimal certification design remains present in this class of signals.

Example Recall the setup of the example specified in Section 5.1. We argue that any regulator-optimal signal induces IOC. This stands in contrast to the certifier-optimal signal $\sigma_{\frac{\alpha}{2}}$ which induces ROC. In particular, we show that the signal σ_α – which provides evidence as to whether the state lies above or below α and induces IOC – is strictly preferred by the regulator over any signal that induces ROC.

Recall that $\Sigma_{ROC} = \{\sigma_t\}_{t\in[0,\alpha)}$ and $\sigma_\alpha \in \Sigma_{IOC}$ as $\underline{v}_{\sigma_\alpha} = \frac{\alpha}{2} = E[\phi(\nu)]$. We want to show that the regulator prefers the signal σ_α over any signal σ_t with $t < \alpha$. For every signal σ_t with $t \leq \alpha$, the distribution $H_{\sigma_t}(v)$ features two posterior means: $E[\phi(\nu)]$ and some

$\bar{\rho}_t > E[\nu]$. As the lower posterior mean is fixed, it is easy to see that the informativeness of $H_{\sigma_t}(v)$ is strictly monotone in $\bar{\rho}_t$. As $\bar{\rho}_t = E[\nu \mid \nu > t] = \frac{1+t}{2}$ is strictly increasing for $t \leq \alpha$, the distribution $H_{\sigma_\alpha}(v)$ is strictly more informative than $H_{\sigma_t}(v)$ for any $t < \alpha$. Thus, any regulator-optimal signal induces IOC.

The insight of the example generalizes. In particular, the binary threshold signal $\sigma_{\hat{t}}$ with $E[\nu|\nu \leq \hat{t}] = E[\phi(\nu)]$ is strictly preferred by the regulator over any signal which induces ROC and has a single posterior mean above $E[\phi(\nu)]$ whenever the prior distribution $F(v)$ is continuous on $[\underline{v}, \bar{v}]$. We formally prove this claim in Lemma 9 in the appendix.

6 Discussion

In this section, we consider two extensions of our model. First, we show that our main results are robust to different forms of certification costs. In a second step, we discuss the case in which the sellers possess some soft private information before certification.

6.1 Certification costs

Throughout the previous sections, we assume that the certifier incurs a strictly positive and signal-independent certification cost whenever the seller is certified. We now discuss the implications of different assumptions regarding the certification cost.

6.1.1 Costless certification and fixed certification costs

A certifier may be able to generate hard information at no cost, possibly after having incurred fixed costs due to hiring employees, designing a certification scheme or adhering to regulatory standards. We now want to argue that certifier-optimal certification design still endogenizes the information structure of Dye (1985) in these cases.

We start by considering the case of costless certification, i.e., $c = 0$. At first, we discuss how the absence of certification costs affects the optimal pricing of a given signal σ. The analysis leading to Proposition 1 can be straightforwardly extended to obtain the following results. If the signal induces IOC, the fee $r = E[\nu] - \underline{v}_\sigma$ remains uniquely optimal due to the demand curve being a simple step function. If the signal induces ROC, both fees $r = \Omega^*_\sigma$ and $r = \Delta$ are optimal as they lead to the same optimal profit Δ. However, based on our analysis of the case $c > 0$, one may argue that the fee $r = \Omega^*_\sigma$ is robustly optimal as it is remains optimal if the cost c is not exactly zero but slightly positive. Thus, probabilistic certification and partial disclosure are robustly optimal outcomes for the certifier.

Assuming that the certifier picks the robustly optimal fee $r = \Omega^*_\sigma$ for any $\sigma \in \Sigma_{ROC}$, we consider the problem of certifier-optimal certification design under $c = 0$. The certifier is indifferent between all feasible signals in Σ_{ROC} as they lead to an identical profit of Δ. All signals in Σ_{IOC} with $\underline{v}_\sigma > E[\phi(\nu)]$ lead to a strictly lower profit than Δ. Thus, the certifier never picks any of these signals whenever a signal inducing ROC is feasible. If a signal σ satisfies exactly the condition of $\underline{v}_\sigma = E[\phi(\nu)]$, this signal induces IOC and leads to the optimal profit Δ. Thus, we can conclude that certifier-optimal certification design also generates the information structure of Dye (1985) for costless certification whenever Σ does not contain a signal satisfying the knife-edge condition $\underline{v}_\sigma = E[\phi(\nu)]$.

Next, we consider the case in which all certification costs are fixed, i.e., the certifier incurs a cost $k > 0$ independently of whether or not the seller is certified. If the certifier enters the market and incurs the fixed cost, certifying the seller leads to no additional cost. Thus, our previous discussion still applies whenever the certifier is active. Thus, certifier-optimal certification design leads to the information structure of Dye (1985) if there exists a feasible signal that induces ROC, the certifier sets robustly optimal fees, the fixed cost is small enough, i.e., $k < \Delta$, and there exists no feasible signal σ with $\underline{v}_\sigma = E[\phi(\nu)]$.

6.1.2 Signal-dependent certification costs

In this section we extend our analyses of certifier-optimal signals and regulator-optimal signals to the case where the cost of certification depends on the signal. For every feasible signal $\sigma \in \Sigma$ we denote by c_σ the cost of certifying the seller. We maintain the assumption from our main model that the certification cost is strictly positive, i.e., $c_\sigma > 0$ for all $\sigma \in \Sigma$. We interpret the certification cost as consisting of a signal-independent, strictly positive transaction cost – which may arise from contracting with the seller, reviewing documents, or providing access to the certificates – and a signal-dependent cost of generating information. Our assumption of signal-independent certification costs from the previous sections may therefore be viewed as the case where information can be costlessly generated within the set of feasible signals Σ.

Our characterization of the certifier-optimal certification fee in Proposition 1 is unaffected by the signal-dependency of certification costs. Hence, we can use the analysis of Section 4 to obtain a characterization of certifier-optimal signals under signal-dependent certification costs. Let the set $\{(G_\sigma, c_\sigma)\}_{\sigma\in\Sigma} \subseteq \{G_\sigma\}_{\sigma\in\Sigma_F} \times \mathbb{R}$ be a compact subset under the induced product topology.[23] Additionally, assume there is a signal $\sigma \in \Sigma$ such that

[23]For instance, this holds if $c_\sigma = C(G_\sigma)$ where $C : \Sigma_F \to \mathbb{R}$ is continuous. By Assumption 2, the graph $\{G_\sigma, c_\sigma\}_{\sigma\in\Sigma}$ is then compact under the induced product topology.

$\Omega^*_\sigma > c_\sigma$.[24] Defining $\Delta_\sigma \equiv \min\{\Delta, E[\nu] - \underline{v}_\sigma\}$, we obtain the following characterization.

Proposition 4. *The set of certifier-optimal signals under signal-dependent certification cost is*

$$\Sigma^* = \arg\max_{\sigma\in\Sigma} \Delta_\sigma - \frac{\Delta_\sigma c_\sigma}{\Omega^*_\sigma}.$$

With signal-dependent costs, a new trade-off arises as a signal with high maximal willingness to pay Ω^*_σ may come at a high cost of certification c_σ. Within the set of signals inducing ROC, the probability of certification is weighed against the cost of certification. In particular, suppose all feasible signals induce ROC, i.e., $\Sigma = \Sigma_{ROC}$. In this case, the characterization of Proposition 4 takes the simple form:

$$\Sigma^* = \arg\max_{\sigma\in\Sigma} \frac{\Omega^*_\sigma}{c_\sigma}.$$

Thus, a signal is optimal if it leads to the highest maximal willingness to pay normalized by the certification cost. Within the set of signals inducing IOC, the trade-off is between revenue and cost of certification. For $\Sigma = \Sigma_{IOC}$, the set of certifier-optimal signals is

$$\Sigma^* = \arg\max_{\sigma\in\Sigma} \Omega^*_\sigma - c_\sigma.$$

Next, we argue that our analysis of regulator-optimal certification design extends to the case of signal-dependent certification cost. From the regulator's point of view, the cost of a signal $\sigma \in \Sigma$ matters only insofar as it determines whether this signal induces non-negative profits to the certifier, and is thus a viable option. Given a set of feasible signals Σ, the regulator determines the subset $\{\sigma \in \Sigma \mid \Omega^*_\sigma \geq c_\sigma\}$ and chooses from this set without taking the cost of the signals into account any further.

Proposition 3 carries over to the case of signal-dependent costs if we impose the condition of Blackwell monotonicity on the cost of signals.[25]

Assumption 5. *The cost of certification is Blackwell monotone, i.e.,*

$$\int_{\underline{v}}^{v} G_{\sigma'}(v)dv \leq \int_{\underline{v}}^{v} G_\sigma(v)dv, \ \ \forall v \in [\underline{v}, \bar{v}] \implies c_{\sigma'} \leq c_\sigma.$$

We then obtain the following extension of Proposition 3.

[24]These two assumptions naturally extend Assumptions 2 and 4 to the case of signal-dependent certification costs.

[25]For instance, the popular posterior-separable cost specification (Caplin and Dean, 2013) satisfies Blackwell monotonicity.

Proposition 5. *For any $\sigma \in \Sigma_{ROC}$ with $\Omega^*_\sigma > c_\sigma$, the garbled signal $\gamma^*(\sigma) \neq \sigma$ satisfies $\Omega^*_{\gamma^*(\sigma)} \geq c_{\gamma^*(\sigma)}$, conveys weakly more information to the market than σ, i.e.,*

$$\int_{\underline{v}}^{v} H_{\gamma^*(\sigma)}(x)dx \geq \int_{\underline{v}}^{v} H_\sigma(x)dx \;\; \forall v \in [\underline{v}, \bar{v}],$$

and conveys strictly more information than σ if $V_\sigma \cap (E[\phi(\nu)], \bar{v}_\sigma]$ is not a singleton. In this case,

$$\int_{\underline{v}}^{v} H_{\gamma^*(\sigma)}(x)dx \geq \int_{\underline{v}}^{v} H_\sigma(x)dx \;\; \forall v \in [\underline{v}, \bar{v}] \;\; \textit{and}$$
$$\int_{\underline{v}}^{v} H_{\gamma^*(\sigma)}(x)dx > \int_{\underline{v}}^{v} H_\sigma(x)dx \;\; \forall v \in (E[\phi(\nu)], \bar{v}_\sigma).$$

Proposition 3 implies that the garbled signal $\gamma^*(\sigma)$ satisfies the above mentioned informational properties with $\Omega^*_{\gamma^*(\sigma)} \geq c_\sigma$. Using Blackwell monotonicity, we obtain $\Omega^*_{\gamma^*(\sigma)} \geq c_\sigma \geq c_{\gamma^*(\sigma)}$.

The insights of Corollaries 3 and 4 carry over to the case of signal dependent certification costs as well. In particular, both corollaries hold under the condition that $c_\sigma \leq \Delta$ for all $\sigma \in \Sigma$. In this case, the regulator still induces full certification and unraveling as in Grossman (1981) and Milgrom (1981). Moreover, the garbling of the most informative signal remains a regulator-optimal certification design for any set of feasible signals.

As a next step, we revisit the example and show that our previous results extend to a natural case of signal-dependent costs.

Example Recall the example introduced in Section 5.1. Suppose that the binary threshold signals $\{\sigma_t\}_{t\in[0,1]}$ come with a certification cost $c(t)$. Let $c(t)$ be strictly positive, continuous, single peaked and symmetric around $\frac{1}{2}$ with $c(\frac{1}{2}) < \Delta$. It is easy to check that $c(t)$ is a positive, decreasing, and continuous transformation of the quadratic loss under a signal σ_t, a standard measure of the informativity of a signal.

We want to show that any certifier-optimal signal induces ROC and any regulator-optimal signal induces IOC. We first consider certifier-optimal certification design. Write the set of feasible signals as a set of pairs of feasible signals: $\Sigma = \{(\sigma_\tau, \sigma_{1-\tau})\}_{\tau\in[0,\frac{1}{2}]}$. By the symmetry of $c(t)$, each pair has the same certification cost. For any pair of signals that contains at least one signal which induces ROC, the certifier's preferred signal from the pair induces ROC. This observation follows directly from Corollary 1. If $\alpha > \frac{1}{2}$, any pair contains at least one signal which induces ROC.[26] It follows that any certifier-

[26]Recall that the set of signals inducing ROC is given by $\{\sigma_t\}_{t<\alpha}$.

optimal signal induces ROC for $\alpha > \frac{1}{2}$.[27] If $\alpha \leq \frac{1}{2}$, the pairs in $\{(\sigma_\tau, \sigma_{1-\tau})\}_{\tau \in [\alpha, \frac{1}{2}]}$ do not contain a signal which induces ROC. However, – due to $c(t)$ being maximal at $\frac{1}{2}$ – the certification cost of these pairs exceeds the cost of any signal in the set of signals inducing ROC $\{\sigma_t\}_{t<\alpha}$. As those signals are preferred by the certifier for a constant cost c, these signals remain preferred if they come at a lower cost of certification. Thus, any certifier-optimal signal induces ROC in the case of $\alpha \leq \frac{1}{2}$ as well.

Next we argue that any regulator-optimal signal induces IOC. From our analysis of the example in section 5.2, we know that the regulator cares about the signal cost only to the extent to which it makes some signals viable. Our assumption $c(\frac{1}{2}) < \Delta$ implies that all signals are viable. Thus, the set of regulator-optimal signals is still contained in $\Sigma_{IOC} = \{\sigma_t\}_{t \geq \alpha}$. It follows that our main insight regarding the difference between certifier- and regulator-optimal certification design remains valid: the former results in Dye's evidence structure and partial disclosure while the latter results in Grossman's and Milgrom's full certification and unraveling.

6.2 Ex-ante informed seller

In our main model, we assume that the seller and the buyers are initially uninformed about the state of the world $v \in V$. If the seller holds some information about the state v when deciding whether to obtain certification, two issues arise. First, the seller may earn information rents, thereby creating a screening problem for the certifier vis-à-vis the seller. Second, a signaling problem arises as the seller may try to convey information other than the certificate to the market. Due to these two complications, a full analysis of optimal certification design with ex-ante information is beyond the scope of this paper.

In this section, we argue that a key result of our analysis of certifier-optimal certification design continues to hold as long as the seller's initial information is not too precise: under certifier-optimal certification design, the seller is not always certified and disclosure is partial. Thus, important features of the information structure of Dye (1985) arise also with ex-ante information.

We consider the following extension of our main model. Before deciding whether to obtain certification, the seller observes the realization w of the random variable ω which is uniformly distributed on $[0, 1]$. Upon observing $w \in [0, 1]$, the seller updates the belief about the state to $J(v|w, q)$. The parameter $q \geq 0$ measures the informativeness of the ex-ante signal. We assume that (i) $J(v|w, q)$ is continuous in w and q for all v and (ii) as q becomes small, the informativeness of ω regarding v vanishes, i.e., $\lim_{q \to 0} J(v|w, q) = F(v)$ for all v and w.

[27] Note that we did not use the assumption of single-peakedness to derive this result.

Proposition 6. *Suppose the fully informative signal is feasible. If the seller's ex-ante information is sufficiently imprecise, the seller obtains certification with probability strictly less than 1 and disclosure is partial under certifier-optimal certification design.*

To prove the proposition, we first provide upper bounds on the certifier's profit in equilibria in which either the seller is certified with certainty or all certificates are disclosed. Consider at first an equilibrium in which the seller always obtains certification. We note that in any such equilibrium, the social surplus is bounded by $\Delta - c$ as the maximal gains from trade are given by Δ and the certification cost c is incurred with probability one. As certification is voluntary, the maximal social surplus of $\Delta - c$ is also an upper bound on the certifier's profit.

Consider next an equilibrium in which the seller obtains certification only for some values of w, but all certificates are disclosed. If a seller with ex-ante information w always discloses, the seller's gross payoff is $E[\nu|w, q]$.[28] As the seller has the outside option to keep the good, the seller is willing to pay at most $E[\nu|w, q] - E[\phi(\nu)|w, q]$ for certification. As the informativeness of the ex-ante signal decreases, this maximal willingness to pay for the certificate converges to Δ as $J(v|w, q)$ converges to $F(v)$ for all w. Thus, the certifier's profit converges to $a(\Delta - c)$ for some $a < 1$ and, therefore, falls below $\Delta - c$ if q is sufficiently small.

The proposition is then implied by the following lemma.

Lemma 4. *Suppose the fully informative signal $\tilde{\sigma}$ is feasible. There exists $\tilde{q} > 0$ such that for all $q < \tilde{q}$ there is a fee $\tilde{r}$ and an equilibrium for $(\tilde{\sigma}, \tilde{r})$ with probabilistic certification and partial disclosure in which the certifier's profit strictly exceeds $\Delta - c$.*

In the proof we construct an equilibrium under the fully informative signal in which the certifier's profit exceeds $\Delta - c$ as the ex-ante information becomes imprecise. Thus, the certifier is better off than in any equilibrium with either full certification or full disclosure for any signal σ. It follows that the certifier optimally induces probabilistic certification and partial disclosure for any certifier-optimal signal.

7 Conclusion

This paper studies how certification design is affected by the objective of the certification designer. Profit-maximizing certifiers prefer highly precise certification as it allows them to charge higher fees and to extract the gains from trade by inducing only a fraction of sellers to acquire certification. By contrast, transparency-maximizing regulators strike a

[28] Any signal induces a distribution of posterior means which is a mean-preserving contraction of $J(\cdot|w, q)$.

balance between the precision of certification and the extent of certification and disclosure. By reducing the precision of the certificate, the regulator induces the certifier to charge a lower fee, thereby certifying a higher share of sellers, which results in more information provision to the market.

As a consequence of this logic, profit-maximizing certification design endogenizes the information structure of Dye (1985) and results in partial disclosure whereas transparency-maximizing certification design endogenizes the information structure in Grossman (1981) and Milgrom (1981) and results in full disclosure through unraveling.

A Proofs

Proof of Lemma 1

Proof follows from the arguments in the main text. □

Proof of Lemma 2

Using integration by parts,

$$\begin{aligned}\Omega_\sigma(p^N) &= \int_V \max\{v, p^N\} dG_\sigma(v) - \max\{p^N, E[\phi(\nu)]\} \\ &= \int_V (v - \phi(v)) dG_\sigma(v) + \int_{\underline{v}}^{p^N} (p^N - v) dG_\sigma(v) - \max\{p^N - E[\phi(\nu)], 0\} \\ &= \Delta + \int_{\underline{v}}^{p^N} G_\sigma(v) dv - \max\{p^N - E[\phi(\nu)], 0\}.\end{aligned}$$

Clearly, the function $\Omega_\sigma(\cdot)$ is continuous on $[\underline{v}_\sigma, \bar{v}_\sigma]$ for any $\sigma \in \Sigma$. For $p^N = \underline{v}_\sigma$, the function takes the value $\min\{\Delta, E[\nu] - \underline{v}_\sigma\}$. If $E[\phi(\nu)] < \underline{v}_\sigma$, the function is strictly decreasing on $[\underline{v}_\sigma, \overline{v}_\sigma]$ as its derivative equals $-(1 - G_\sigma(p^N)) < 0$. For $E[\phi(\nu)] \geq \underline{v}_\sigma$, the function is strictly increasing on $(\underline{v}_\sigma, E[\phi(\nu)])$ as its derivative equals $G_\sigma(p^N)$. For $p^N \in (E[\phi(\nu)], \bar{v}_\sigma)$, the function is strictly decreasing as its derivative equals $-(1 - G_\sigma(p^N))$. Thus, $\Omega_\sigma(\cdot)$ is strictly quasi-concave and attains its unqiue maximum at $p^N = \max\{E[\phi(\nu)], \underline{v}_\sigma\}$. □

Proof of Lemma 3

We prove the Lemma with the help of the following Lemmata 5 to 8.

Lemma 5.

1. *Suppose $\sigma \in \Sigma_{ROC}$. There exists an equilibrium in which the seller always obtains certification if and only if $r \leq \Delta$.*

2. *Suppose $\sigma \in \Sigma_{IOC}$. There exists an equilibrium in which the seller always obtains certification if and only if $r \leq E[\nu] - \underline{v}_\sigma$.*

Proof. If $a = 1$ in equilibrium, condition (9) implies $p^N = \underline{v}_\sigma$. For $\Pr(N) = 0$, this follows directly from (9). For $\Pr(N) > 0$, the condition in (9) simplifies to

$$p^N G_\sigma(p^N) = \int_{\underline{v}_\sigma}^{p^N} v dG_\sigma(v) \iff p^N = \underline{v}_\sigma.$$

If $p^N = \underline{v}_\sigma$ in equilibrium, condition (8) implies that $a = 1$ if and only if $r \leq \Omega_\sigma(\underline{v}_\sigma)$. If $\underline{v}_\sigma < E[\phi(\nu)]$, $\Omega_\sigma(\underline{v}_\sigma) = \Delta$. If $\underline{v}_\sigma \geq E[\phi(\nu)]$, $\Omega_\sigma(\underline{v}_\sigma) = E[\nu] - \underline{v}_\sigma$. □

Lemma 6.

1. *Suppose $\sigma \in \Sigma_{ROC}$. For $r > \Delta$, there exists an equilibrium in which the seller obtains certification with an interior probability if and only if $r \in (\max\{\Delta, \Omega_\sigma(E[\nu])\}, \Omega^*_\sigma]$. For $r \in (\max\{\Delta, \Omega_\sigma(E[\nu])\}, \Omega^*_\sigma)$, the only equilibrium with $a > 0$ satisfies*

$$a = \frac{E[\nu] - \Omega_\sigma^{-1}(r)}{r}.$$

For $r = \Omega^*_\sigma$, there exists an equilibrium for any $a \in (0, \Delta/\Omega^*_\sigma]$.

2. *Suppose $\sigma \in \Sigma_{IOC}$. If $r > E[\nu] - \underline{v}_\sigma$, there is no equilibrium in which the seller obtains certification with an interior probability.*

Proof. First, consider a signal σ with ROC. Condition (8) implies that $r = \Omega_\sigma(p^N)$ in any equilibrium with $a \in (0, 1)$. If $p^N < E[\phi(\nu)]$ in such an equilibrium, we have $b_U = 0$ by (6). Due to condition (9), this implies $p^N = \underline{v}_\sigma$ and $r = \Omega_\sigma(\underline{v}_\sigma) = \Delta$. This is compatible with condition (9) for any $a \in (0, 1)$. If $p^N = E[\phi(\nu)]$ in such an equilibrium, we have $b_U \in [0, 1]$ by (6) and condition (9) implies $a = b_U\Delta/(b_U\Delta + \int_{\underline{v}_\sigma}^{E[\phi(\nu)]} G_\sigma(v)dv) \in [0, \Delta/\Omega^*_\sigma]$ and $r = \Omega^*_\sigma$. If $p^N > E[\phi(\nu)]$ in such an equilibrium, we have $b_U = 1$ by (6). Condition (9) implies $a = (E[\nu] - p^N)/\Omega_\sigma(p^N)$. The result follows from the definition of $\Omega_\sigma^{-1}(r)$.

Second, consider a signal σ with IOC. Condition (8) implies $r = \Omega_\sigma(p^N)$ in any equilibrium with $a \in (0, 1)$. By Lemma 2, $\Omega_\sigma(p^N) \leq \Omega_\sigma(\underline{v}_\sigma) = E[\nu] - \underline{v}_\sigma$. Thus, $r \leq E[\nu] - \underline{v}_\sigma$ in any equilibrium with $a \in (0, 1)$. □

Lemma 7.

1. *Suppose $\sigma \in \Sigma_{ROC}$. There exists an equilibrium in which the seller never obtains certification for $r > \Delta$.*

2. *Suppose $\sigma \in \Sigma_{IOC}$. There exists an equilibrium in which the seller never obtains certification for $r > E[\nu] - \underline{v}_\sigma$.*

Proof. If $a = 0$ in equilibrium, condition (9) implies that either $\Pr(N) > 0$ and $p^N = E[\nu]$ or $\Pr(N) = 0$ and $p^N \in [\underline{v}_\sigma, E[\phi(\nu)]]$. For any equilibrium with $\Pr(N) > 0$ and $p^N = E[\nu]$, condition (8) implies that we may have $a = 0$ if and only if $r \geq \Omega_\sigma(E[\nu])$. For any equilibrium with $\Pr(N) = 0$ and $p^N = \underline{v}_\sigma$, condition (8) implies that we may have $a = 0$ if and only if $r \geq \Omega_\sigma(\underline{v}_\sigma) = \Delta$. If $\underline{v}_\sigma < E[\phi(\nu)]$, the interval $[\underline{v}_\sigma, E[\phi(\nu)]]$ is nonempty and thus an equilibrium with $a = 0$ exists if and only if $r \geq \min\{\Delta, \Omega_\sigma(E[\nu])\}$. If $\underline{v}_\sigma \geq E[\phi(\nu)]$, $[\underline{v}_\sigma, E[\phi(\nu)]]$ is empty and an equilibrium with $a = 0$ exists if and only if $r \geq \Omega_\sigma(E[\nu])$. $\square$

Lemma 8. *The function $\frac{E[\nu]-\Omega_\sigma^{-1}(r)}{r}$ is increasing in r on $(\Omega_\sigma(E[\nu]), \Omega_\sigma^*)$.*

Proof. Note first that for $p^N \in (E[\phi(\nu)], E[\nu])$,

$$sign\left(\frac{\partial}{\partial r}\frac{E[\nu]-\Omega_\sigma^{-1}(r)}{r}\right) = sign\left(-\frac{\partial}{\partial p^N}\frac{E[\nu]-p^N}{\Omega_\sigma(p^N)}\right).$$

Moreover, basic manipulations yield for $p^N \in (E[\phi(\nu)], E[\nu])$

$$\begin{aligned}
\frac{\partial}{\partial p^N}\frac{E[\nu]-p^N}{\Omega_\sigma(p^N)} &= \frac{-\Omega_\sigma(p^N) - \frac{\partial \Omega_\sigma(p^N)}{\partial p^N}(E[\nu]-p^N)}{\Omega_\sigma(p^N)^2} \\
&= \frac{-\int \max\{v, p^N\} dG_\sigma(v) + p^N + (1-G_\sigma(p^N))(E[\nu]-p^N)}{\Omega_\sigma(p^N)^2} \\
&= \frac{\int_{\underline{v}}^{p^N}(v - E[\nu]) dG_\sigma(v)}{\Omega_\sigma(p^N)^2}
\end{aligned}$$

where the last term is strictly negative due to $p^N < E[\nu]$. $\square$

We argue that Lemma 3 follows directly from the above Lemmata. If $\sigma \in \Sigma_{ROC}$ we get from Lemma 5 that if $r \leq \Delta$ then $a = 1$ in the equilibrium with maximal probability of acquiring the signal. From Lemma 6, we get that for $r \in (\max\{\Delta, \Omega_\sigma(E[\nu])\}, \Omega_\sigma^*)$ there exists a unique equilibrium with strictly positive probability of acquiring the signal and thus this is the relevant equilibrium for the characterization of the demand in this range of fees. For the range of fees that are not covered by the previous cases, we get from Lemma 7 that the unique equilibrium in this range is an equilibrium with $a = 0$. Lemma 8 establishes that $D_\sigma(\cdot)$ is indeed increasing in r on the segment $(\max\{\Delta, \Omega_\sigma(E[\nu])\}, \Omega_\sigma^*)$. By using the part of Lemmas that corresponds to the case where $\sigma \in \Sigma_{IOC}$ in a similar way, we get our characterization for the case of $\sigma \in \Sigma_{IOC}$. $\square$

Proof of Proposition 1

With ROC, the demand function is weakly increasing for $r \in (\Delta, \Omega^*_\sigma)$. Thus, the only candidates for an optimal fee are $r = \Delta$, $r = \Omega^*_\sigma$, and $r > \Omega^*_\sigma$. At $r = \Delta$, the certifier obtains the profit $\Delta - c$. At $r = \Omega^*_\sigma$, the certifier's profit is $\Delta - (\Delta/\Omega^*_\sigma)c > \Delta - c$. At $r > \Omega^*_\sigma$, the certifier makes zero profit. If $c < \Omega^*_\sigma$, $r = \Omega^*_\sigma$ is strictly optimal. If $c = \Omega^*_\sigma$, $r = \Omega^*_\sigma$ and $r > \Omega^*_\sigma$ are optimal. If $c > \Omega^*_\sigma$, only $r > \Omega^*_\sigma$ are optimal. With IOC, demand is a step function and the only candidates for an optimal fee are $r = E[\nu] - \underline{v}_\sigma$ and $r > E[\nu] - \underline{v}_\sigma$. At $r = E[\nu] - \underline{v}_\sigma$, the certifier makes the profit $E[\nu] - \underline{v}_\sigma - c$. At $r > E[\nu] - \underline{v}_\sigma$, the profit is zero. If $c < E[\nu] - \underline{v}_\sigma$, $r = E[\nu] - \underline{v}_\sigma$ is strictly optimal. If $c = E[\nu] - \underline{v}_\sigma$, $r = E[\nu] - \underline{v}_\sigma$ and $r > E[\nu] - \underline{v}_\sigma$ are optimal. If $c > E[\nu] - \underline{v}_\sigma$, only $r > E[\nu] - \underline{v}_\sigma$ are optimal. $\square$

Proof of Proposition 2

Define $\Delta_\sigma \equiv \min\{\Delta, E[\nu] - \underline{v}_\sigma\}$. By Proposition 1, the certfier's profit can be expressed as a function of σ given by $\max\{\Delta_\sigma - (\Delta_\sigma/\Omega^*_\sigma)c, 0\}$. By compactness of V_σ and Assumption 2, the function attains a maximum. As any optimal signal induces strictly positive profit by Assumption 4, the nonempty set of certifier-optimal signals is

$$\Sigma^* = \arg\max_{\sigma\in\Sigma} \Delta_\sigma - (\Delta_\sigma/\Omega^*_\sigma)c.$$

The expression $\Delta_\sigma - (\Delta_\sigma/\Omega^*_\sigma)c$ is increasing in Δ_σ and Ω^*_σ for all signals that lead to strictly positive profit. Thus, the proposition follows from the observation that – due to Lemma 2 and the definition of Δ_σ – for any two signals $\sigma', \sigma'' \in \Sigma$ that lead to strictly positive profit, $\Omega^*_{\sigma'} > \Omega^*_{\sigma''}$ implies $\Delta_{\sigma'} \geq \Delta_{\sigma''}$. $\square$

Proof of Corollary 1

Proof follows from the arguments in the main text. $\square$

Proof of Corollary 2

Proof follows from the arguments in the main text. $\square$

Proof of Proposition 3

Consider a signal $\sigma \in \Sigma_{ROC}$ with $\Omega^*_\sigma > c$. Define the garbled signal $\gamma^*(\sigma)$ through the function

$$\gamma^*_\sigma(s) = \begin{cases} \delta_s & \text{if} \quad E_\sigma[\nu|s] \notin [v'_\sigma, v''_\sigma], \\ \xi'\delta_s + (1-\xi')\delta_{\hat{s}} & \text{if} \quad E_\sigma[\nu|s] = v'_\sigma, \\ \delta_{\hat{s}} & \text{if} \quad E_\sigma[\nu|s] \in (v'_\sigma, v''_\sigma), \\ \xi''\delta_s + (1-\xi'')\delta_{\hat{s}} & \text{if} \quad E_\sigma[\nu|s] = v''_\sigma. \end{cases} \tag{10}$$

where $\hat{s} \neq s$ for all signal realizations s in the support of σ, δ_s denotes the Dirac measure on the signal realization s, v'_σ and ξ' solve

$$\int_{\underline{v}_\sigma}^{E[\phi(\nu)]} \min\left\{G_\sigma(x), \xi' G_\sigma(v'_\sigma) + (1-\xi')G_\sigma(v'^{-}_\sigma)\right\} dx = \max\{c - \Delta, 0\},$$

and v''_σ and ξ'' solve – for given v'_σ and ξ' –

$$E_{\gamma^*(\sigma)}[\nu|\hat{s}] = E[\phi(\nu)].$$

From the definition of the garbled signal $\gamma^*(\sigma)$, we can determine the distribution $G_{\gamma^*(\sigma)}$ over posterior means as

$$G_{\gamma^*(\sigma)}(v) = \begin{cases} G_\sigma(v) & \text{if} \quad v < v'_\sigma, \\ \xi' G_\sigma(v'_\sigma) + (1-\xi')G_\sigma(v'^{-}_\sigma) & \text{if} \quad v \in [v'_\sigma, E[\phi(\nu)]), \\ \xi'' G_\sigma(v''_\sigma) + (1-\xi'')G_\sigma(v''^{-}_\sigma) & \text{if} \quad v \in [E[\phi(\nu)], v''_\sigma), \\ G_\sigma(v) & \text{if} \quad v \geq v''_\sigma. \end{cases}$$

Note that $\Omega^*_{\gamma^*(\sigma)} = \max\{\Delta, c\} < \Omega^*_\sigma$. Thus $\gamma^*(\sigma) \neq \sigma$. We next want to establish that

$$\int_{\underline{v}}^{v} H_{\gamma^*(\sigma)}(x)dx \geq \int_{\underline{v}}^{v} H_\sigma(x)dx \quad \forall v \tag{11}$$

and the inequality is strict for some v if and only if $V_\sigma \cap (E[\phi(\nu)], \bar{v}_\sigma]$ is not a singleton, or equivalently $G_\sigma(\bar{v}^-_\sigma) > G_\sigma(E[\phi(\nu)])$. Consider a generic signal $\hat{\sigma} \in \Sigma$ and let $a_{\hat{\sigma}} \equiv \min\{1, \Delta/\Omega^*_{\hat{\sigma}}\}$ be the equilibrium probability of certification given $\hat{\sigma}$ and the associated certifier-optimal fee. Using the formulation of $H_{\hat{\sigma}}(v)$ derived in the main text, we obtain

$$\int_{\underline{v}}^{v} H_{\hat{\sigma}}(x)dx = \begin{cases} 0 & \text{if} \quad v < E[\phi(\nu)], \\ (1-a_{\hat{\sigma}})(v - E[\phi(\nu)]) + a_{\hat{\sigma}} \int_{E[\phi(\nu)]}^{v} G_{\hat{\sigma}}(x)dx & \text{if} \quad v \geq E[\phi(\nu)]. \end{cases}$$

If $v < E[\phi(\nu)]$, the weak inequality in condition (11) is trivially satisfied. For $v \geq E[\phi(\nu)]$, we can further reformulate the expression as follows:

$$
\begin{aligned}
&(1-a_{\hat{\sigma}})(v-E[\phi(\nu)]) + a_{\hat{\sigma}} \int_{E[\phi(\nu)]}^{v} G_{\hat{\sigma}}(x)dx \\
&\qquad = v - E[\phi(\nu)] - a_{\hat{\sigma}} \left(v - E[\nu] + \Delta + \int_{\underline{v}}^{E[\phi(\nu)]} G_{\hat{\sigma}}(x)dx - \int_{\underline{v}}^{v} G_{\hat{\sigma}}(x)dx \right) \\
&\qquad = v - E[\phi(\nu)] - \Delta + \frac{\Delta\left(E[\nu] - v + \int_{\underline{v}}^{v} G_{\hat{\sigma}}(x)dx\right)}{\Delta + \int_{\underline{v}}^{E[\phi(\nu)]} G_{\hat{\sigma}}(x)dx} &(12)\\
&\qquad = v - E[\phi(\nu)] - \Delta + \frac{\Delta \int_{v}^{\bar{v}_{\hat{\sigma}}} (1 - G_{\hat{\sigma}}(x))dx}{\Delta + \int_{\underline{v}}^{E[\phi(\nu)]} G_{\hat{\sigma}}(x)dx}. &(13)
\end{aligned}
$$

We move from the second line to the third using the equality $a_\sigma = \frac{\Delta}{\Delta + \int_{\underline{v}}^{E[\phi(\nu)]} G_\sigma(x)dx}$ which follows from Proposition 1. The fourth line can be obtained from the third by integration by parts of the numerator in the fraction.

First, we compare the functions $\int_{\underline{v}}^{v} H_\sigma(x)dx$ and $\int_{\underline{v}}^{v} H_{\gamma^*(\sigma)}(x)dx$ for $v \in [v''_\sigma, \bar{v}_\sigma]$. Note that (i) the denominator in expression (13) for $\hat{\sigma} = \gamma^*(\sigma)$ equals $\max\{\Delta, c\}$ whereas the denominator for $\hat{\sigma} = \sigma$ strictly exceeds both Δ and c by $\sigma \in \Sigma_{ROC}$ and $\Omega^*_\sigma > c$, and (ii) the numerator in expression (13) is identical for $\hat{\sigma} = \sigma$ and $\hat{\sigma} = \gamma^*(\sigma)$ if $v \in [v''_\sigma, \bar{v}_\sigma]$, strictly positive for $v \in [v''_\sigma, \bar{v}_\sigma)$, and zero for $v = \bar{v}_\sigma$. The observations (i) and (ii) imply that $\int_{\underline{v}}^{v} H_{\gamma^*(\sigma)}(x)dx$ and $\int_{\underline{v}}^{v} H_\sigma(x)dx$ are identical for $v = \bar{v}_\sigma$ and that $\int_{\underline{v}}^{v} H_{\gamma^*(\sigma)}(x)dx$ strictly exceeds $\int_{\underline{v}}^{v} H_\sigma(x)dx$ for $v \in (v''_\sigma, \bar{v}_\sigma]$.

Second, we compare the functions $\int_{\underline{v}}^{v} H_\sigma(x)dx$ and $\int_{\underline{v}}^{v} H_{\gamma^*(\sigma)}(x)dx$ for $v \in [E[\phi(\nu)], v''_\sigma]$. Note that (iii) $\int_{\underline{v}}^{v} H_{\gamma^*(\sigma)}(x)dx$ is linear for $v \in [E[\phi(\nu)], v''_\sigma]$ while $\int_{\underline{v}}^{v} H_\sigma(x)dx$ is weakly convex, and (iv) both functions are zero at $v = E[\phi(\nu)]$. If $\int_{\underline{v}}^{v} H_{\gamma^*(\sigma)}(x)dx$ strictly exceeds $\int_{\underline{v}}^{v} H_\sigma(x)dx$ at $v = v''_\sigma$, the observations (iii) and (iv) imply that $\int_{\underline{v}}^{v} H_{\gamma^*(\sigma)}(x)dx$ strictly exceeds $\int_{\underline{v}}^{v} H_\sigma(x)dx$ for $v \in (E[\phi(\nu), v''_\sigma)$, and the two functions are identical for $v = E[\phi(\nu)]$. If $\int_{\underline{v}}^{v} H_{\gamma^*(\sigma)}(x)dx = \int_{\underline{v}}^{v} H_\sigma(x)dx$ at $v = v''_\sigma$, the observations (iii) and (iv) imply that $\int_{\underline{v}}^{v} H_{\gamma^*(\sigma)}(x)dx$ strictly exceeds $\int_{\underline{v}}^{v} H_\sigma(x)dx$ for $v \in (E[\phi(\nu), v''_\sigma)$ if there are posterior means in $(E[\phi(\nu), v''_\sigma)$, i.e., $G_\sigma(v''^{-}_\sigma) > G(E[\phi(\nu)])$. Otherwise, the two functions are identical for $v \in [E[\phi(\nu)], v''_\sigma]$.

We can conclude that the weak equality of conditions (13) always holds, and that the two functions are identical if and only if $v''_\sigma = \bar{v}_\sigma$ and $G_\sigma(v''^{-}_\sigma) = G(E[\phi(\nu)])$, i.e., the signal σ induces only a single posterior mean above $E[\phi(\nu)]$, which is equivalent to saying $G_\sigma(\bar{v}^{-}_\sigma) = G(E[\phi(\nu)])$. This concludes the proof. □

Proof of Corollary 3

If $c \leq \Delta$ then for every signal $\sigma \in \Sigma_{ROC}$ it holds that $\gamma^*(\sigma) \in \Sigma_{IOC}$. From the condition that for every $\sigma \in \Sigma$ there exist more than one posterior above $E[\phi(\nu)]$ we can deduce, according to Proposition 3, that $H_{\gamma^*(\sigma)}$ is a mean preserving spread of H_σ. It follows that for every $\sigma \in \Sigma_{ROC}$ the regulator strictly prefers the signal $\gamma^*(\sigma) \in \Sigma_{IOC}$ over the signal $\sigma \in \Sigma_{ROC}$ and so the regulator would always choose a signal that induces IOC. Thus, according to the certfier optimal pricing (Proposition 1) full certification and unraveling prevails. $\square$

Proof of Corollary 4

We want to prove that $\gamma^*(\bar{\sigma})$ is regulator-optimal. For any signal $\sigma \in \Sigma$ with $\Delta + \int_{\underline{v}}^{E[\phi(\nu)]} G_\sigma(v)dv < c$, the seller is never certified and $\gamma^*(\bar{\sigma})$ generates more information on the market. For any signal $\sigma \in \Sigma$ with $\Delta + \int_{\underline{v}}^{E[\phi(\nu)]} G_\sigma(v)dv \geq c$, the expression in equation (12) together with $\int_{\underline{v}}^{v} G_\sigma(x)dx \leq \int_{\underline{v}}^{v} G_{\bar{\sigma}}(x)dx$ for all v implies the upper bound

$$\int_{\underline{v}}^{v} H_\sigma(x)dx \leq \begin{cases} 0 & \text{if} \quad v < E[\phi(\nu)], \\ v - E[\phi(\nu)] - \Delta + \frac{\Delta\left(E[\nu] - v + \int_{\underline{v}}^{v} G_{\bar{\sigma}}(x)dx\right)}{c} & \text{if} \quad v \geq E[\phi(\nu)]. \end{cases}$$

Note that $\gamma^*(\bar{\sigma})$ attains the upper bound for $v < E[\phi(\nu)]$ and $v \geq v''_{\bar{\sigma}}$. For $v \in [E[\phi(\nu)], v''_{\bar{\sigma}}]$, $\int_{\underline{v}}^{v} H_{\gamma^*(\bar{\sigma})}(x)dx$ is linear. As $\int_{\underline{v}}^{E[\phi(\nu)]} H_\sigma(x)dx = 0$ and $\int_{\underline{v}}^{v} H_\sigma(x)dx$ weakly convex for all σ, we have $\int_{\underline{v}}^{v} H_{\gamma^*(\bar{\sigma})}(x)dx \geq \int_{\underline{v}}^{v} H_\sigma(x)dx$ for all $v \in [E[\phi(\nu)], v''_{\bar{\sigma}}]$ and $\sigma \in \Sigma$. $\square$

Proof of Proposition 4

Proposition 1 implies that the certifier's profit for the signal $\sigma \in \Sigma$ is a function of (G_σ, c_σ) given by $\max\{\Delta_\sigma - \frac{\Delta_\sigma c_\sigma}{\Omega^*_\sigma}, 0\}$. This function is clearly continuous and thus attains a maximum by the compactness of the domain $(G_\sigma, c_\sigma)_{\sigma \in \Sigma}$. The result follows then from our assumption that the certifier's profit is strictly positive for some signal. $\square$

Proof of Proposition 5

Proof follows from the arguments in the main text. $\square$

Proof of Lemma 4

The seller's willingness to pay for the fully revealing signal $\tilde{\sigma}$ given the ex-ante-information $w \in [0,1]$ and the price for the uncertified good p^N is

$$\Omega_{\tilde{\sigma}}(p^N|w,q) \equiv \int_{\underline{v}}^{\overline{v}} \max\left\{v, p^N\right\} dJ(v|w,q) - \max\left\{\int_{\underline{v}}^{\overline{v}} \phi(v) dJ(v|w,q), p^N\right\}.$$

Let $W^c \subseteq [0,1]$ denote the set of seller types who buy certification, and $W^u \subseteq [0,1]$ denotes the set of seller types who remain uncertified and sell without disclosure. Given the signal $\tilde{\sigma}$ and a certification fee r, (W^c, W^u, p^N) constitute an equilibrium if

i) $\Omega_{\tilde{\sigma}}(p^N|w,q) > r \implies w \in W^c$,

ii) $\Omega_{\tilde{\sigma}}(p^N|w,q) < r \wedge p^N \geq E[\phi(\nu)|w] \implies w \in W^u$,

iii) $\Omega_{\tilde{\sigma}}(p^N|w,q) < r \wedge p^N < E[\phi(\nu)|w] \implies w \notin W^c \cup W^u$, and

iv)
$$\frac{\int_{W^c} \int_{\underline{v}}^{p^N} v dJ(v|w,q) dw + \int_{W^u} \int_{\underline{v}}^{\overline{v}} v dJ(v|w,q) dw}{\int_{W^c} J(p^N|w,q) dw + I(W^u)} = p^N,$$

where $I(\cdot)$ denotes the uniform measure on $[0,1]$. We show that there exist a fee $\tilde{r}$ and an equilibrium for $(\tilde{\sigma}, \tilde{r})$ with $p^N = \bar{p}^N(q) \equiv \sup_{[0,1]}\{E[\phi(\nu)|w,q]\}$ if q is sufficiently small. $W^c \cup W^u = [0,1]$ is consistent with such an equilibrium by conditions $i)$ to $iii)$. Condition $iv)$ can be simplified to

$$\int_{W^c} \int_{\underline{v}}^{\bar{p}^N(q)} J(v|w,q) dv dw = \int_{[0,1]\setminus W^c} (E[\nu|w,q] - \bar{p}^N(q)) dw. \tag{14}$$

As q becomes small, $J(v|w,q)$ converges to $F(v)$ for all w and q. Thus, $\bar{p}^N(q)$ converges to $E[\phi(\nu)]$ and $E[\nu|w,q]$ converges to $E[\nu]$. It follows that $\int_{\underline{v}}^{\bar{p}^N(q)} J(v|w,q) dv > 0$ and $E[\nu|w,q] - \bar{p}^N(q) > 0$, and thus, as ω is continuously distributed, there exists a set $\hat{W}^c(q)$ which satisfies condition (14).

Finally, we show that the certifier's profit under this equilibrium exceeds $\max\{\Delta - c, 0\}$ for small q. The certifier's profit under the equilibrium above is $I(\hat{W}^c(q))(\hat{r}(q) - c)$ where $\hat{r}(q)$ is the unique fee which is consistent with $\hat{W}^c(q)$ in equilibrium. Recall that $J(v|w,q)$ is continuous in w and q. Thus, as q becomes small, the left hand side of equation (14) converges to $I(\hat{W}^c) \int_{\underline{v}}^{E[\phi(\nu)]} F(v) dv$ and the right hand side converges to $(1 - I(\bar{W}^c))\Delta$. Thus, $I(\hat{W}^c)$ converges to $\Delta/\Omega^*_{\tilde{\sigma}}$, and $\hat{r}(q)$ converges to $\Omega^*_{\tilde{\sigma}}$. The expected profit therefore converges to $\Delta - c\Delta/\Omega^*_{\tilde{\sigma}} > \max\{\Delta - c, 0\}$ as $\Omega^*_{\tilde{\sigma}} > c$. □

Proof of Proposition 6

The proof follows from the arguments in the main text and Lemma 4. □

Statement and Proof of Lemma 9

Lemma 9. *Suppose the prior F is continuous on $[\underline{v}, \bar{v}]$. Then, the binary threshold signal $\sigma_{\hat{t}}$ with $E[\nu|\nu \leq \hat{t}] = E[\phi(\nu)]$ leads to more market information than any other signal which induces ROC and features a single posterior mean strictly above $E[\phi(\nu)]$.*

Proof. First, we show the result for binary signals. Recall that any binary signal with posterior means of $\underline{\rho} \leq E[\phi(\nu)]$ and $\bar{\rho} > E[\phi(\nu)]$ generates disclosed posteriors means of $E[\phi(\nu)]$ and $\bar{\rho}$. Thus, the higher $\bar{\rho}$, the more information is conveyed to the market. The binary signal with the highest posterior mean from this class solves the optimization problem

$$\max_{\bar{\rho},\underline{\rho}} \bar{\rho} \quad \text{s.t.} \quad \underline{\rho} \leq E[\phi(\nu)], \quad 0 \leq \int_{\underline{v}}^{v} F(x)dx - \begin{cases} 0 & \text{if} \quad v < \underline{\rho}, \\ (v - \underline{\rho})\frac{\bar{\rho}-E[\nu]}{\bar{\rho}-\underline{\rho}} & \text{if} \quad v \in [\underline{\rho}, \bar{\rho}), \\ v - E[\nu] & \text{if} \quad v \geq \bar{\rho}. \end{cases}$$

The second constraint ensures that the prior distribution $F(v)$ is a mean-preserving spread of the binary distribution over $\underline{\rho}$ and $\bar{\rho}$, where the probability of $\underline{\rho}$ can be computed from $\Pr(\underline{\rho})\underline{\rho} + (1 - \Pr(\underline{\rho}))\bar{\rho} = E[\nu]$. Closer inspection of this second constraint reveals that it is never violated for $v < \underline{\rho}$ and $v > \bar{\rho}$ if it is satisfied for $v \in [\underline{\rho}, \bar{\rho}]$. In this intermediate range, the right-hand side of the constraint attains a unique minimum at $\hat{v}$ defined by the first-order condition $F(\hat{v}) = \frac{\bar{\rho}-E[\nu]}{\bar{\rho}-\underline{\rho}}$. As the choice of $\bar{\rho}$ is only limited by the second constraint, this constraint needs to bind and we obtain the condition $\int_{\underline{v}}^{\hat{v}} F(v)dv = (\hat{v} - \underline{\rho})F(\hat{v})$. Basic manipulations of this condition yield

$$\underline{\rho} = \frac{\int_{\underline{v}}^{\hat{v}} vdF(v)}{F(\hat{v})} = E[\nu|\nu \leq \hat{v}] \quad \text{and} \quad \bar{\rho} = \frac{\int_{\hat{v}}^{\bar{v}} vdF(v)}{1 - F(\hat{v})} = E[\nu|\nu > \hat{v}].$$

Both posterior means are strictly increasing in $\hat{v}$. Hence, the constraint $\underline{\rho} \leq E[\phi(\nu)]$ binds in any optimum, i.e., $\hat{v} = \hat{t}$. Thus, the binary threshold signal $\sigma_{\hat{t}}$ – which induces IOC – leads to a strictly higher posterior mean $\bar{\rho}$ than any binary signal that induces ROC.

Second, we argue that the binary threshold signal $\sigma_{\hat{t}}$ is also preferred by the regulator to any nonbinary signal σ' which induces ROC and generates a single posterior mean above $E[\phi(\nu)]$. This follows from the fact that the signal σ' conveys the same information to the market than the binary signal σ'_b which is obtained from σ' by pooling all posterior means below $E[\phi(\nu)]$ and therefore induces ROC. $\square$

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