SUPPLEMENT TO "LOCATION AS AN ASSET"
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APPENDIX C: EXTENSIONS WITH AMENITIES AND VARIABLE HOUSING

C.1. *Proof of Lemma 6*

THE OPTIMALITY CONDITIONS from the extended model of Section 2.6 are

$$\frac{c_1}{\beta c_0} \geq R \quad \text{with equality iff} \quad a^* > \underline{a}, \quad \frac{c_1}{\beta c_0} = \frac{s}{q'(z)} + \frac{Ac_1}{\beta q'(z)}.$$

We impose $\beta = R = 1$ to simplify the exposition, but the results generalize to $\beta < 1 < R$.

Unconstrained Individuals. Combine both budget constraints to obtain $c_1 = y_1 + a + sz = y_1 + sz + y_0 - c_0 - q(z)$. Using the Euler equation $c_1 = c_0$, obtain $c_0 = c_1 = \frac{1}{2}I(s, y_0, y_1, z)$, where $I(s, y_0, y_1, z) \equiv y_0 - q(z) + y_1 + sz \equiv \bar{I}(y_0, y_1) + sz - q(z)$. The location decision then writes $q'(z) = s + Ac_1$. Rearrange to $s = \frac{2q'(z) + Aq(z) - A\bar{I}(y_0, y_1)}{2 + Az}$, implying $q'(z) - s = \frac{A}{2 + Az}[zq'(z) - q(z) + \bar{I}(y_0, y_1)]$.

The location response of an unconstrained individual to a y_0 shock, $D_U = \partial_{y_0} z^*$, is then $2q''(z)D_U + Aq'(z)D_U - A = AsD_U$. Rearranging, $D_U = \frac{A}{2q''(z) + \frac{A^2}{2 + Az}[zq'(z) - q(z) + \bar{I}(y_0, y_1)]}$.

Proportional Income Shock. We now compare the location response of two unconstrained individuals P, R who would locate in the same location z absent the shock. For any individual $j \in \{P, R\}$, where $y_0^R > y_0^P$, $\frac{(1+\tau)A}{2+Az}y_0 D_U \equiv \frac{\tilde{y}_0}{X(A,z)+\tilde{y}_0}$, where $\tilde{y}_0 = \frac{(1+\tau)A^2 y_0}{2+Az}$ and $X(A, z) = 2q''(z) + \frac{A^2}{2+Az}[zq'(z) - q(z)]$. $\frac{\tilde{y}_0}{X(A,z)+\tilde{y}_0}$ is an increasing function of $\tilde{y}_0$ as long as $X(A, z) > 0$, which we show below. Therefore, $D^R = y_0^R D_U^R > y_0^P D_U^P = D^P$.

We now prove that that $X(A, z) > 0$. Suppose first that q is convex. When $\underline{a} = -\infty$, every populated location has an unconstrained individual, and so the last populated location has $q(z) = 0$. Convexity of $q(z)$ then implies that $q(z)/z \equiv p(z)$ is an increasing function. Then $q'(z) = p(z) + zp'(z)$ and so $zq'(z)/q(z) = 1 + z^2 p'(z)/q(z) \geq 1$, and $X(A, z) > 0$.

We now prove that that q is convex in equilibrium. We know that q must be convex when $A = 0$. Increasing A continuously keeps q convex because market clearing conditions are continuous in A. As we increase A, q can become locally concave only if q' becomes constant some z^*. As in the baseline case, this implies only unconstrained individuals that satisfy $s + Ac_1 = q'(z^*)$ optimally locate in a neighbourhood of z^* of infinite Radon–Nikodym derivative relative to the Lebesgue measure dz. In contrast, individuals in a comparable slice of the distribution of (y_0, y_1, s) locates in a neighborhood of any other z

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with a finite Radon–Nikodym derivative relative to the Lebesgue measure. As long as the population distribution of (y_0, y_1, s) is absolutely continuous, the infinite Radon–Nikodym derivative violates land market clearing as it implies zero housing prices, a contradiction. Thus, q must be convex for any $A > 0$.

Constrained Individuals. Constrained individuals have $a^* = \underline{a}$. Impose $\underline{a} = 0$ for notational simplicity but without loss of generality. Their location choice is given by $s + Ac_1 = q'(z)\mathcal{R}(s, y_0, y_1, z)$ where $\mathcal{R}(s, y_0, y_1, z) \equiv \frac{y_1 + sz}{y_0 - q(z)}$. Express s as $s = \frac{\frac{q'(z)}{y_0 - q(z)} - A}{1 + Az - \frac{zq'(z)}{y_0 - q(z)}} y_1$. As a result, $c_1 = y_1 + sz = \frac{y_1}{1 + Az - \frac{zq'(z)}{y_0 - q(z)}}$ and $c_0 = y_0 - q(z)$, where $\mathcal{R} = \frac{y_1}{(y_0 - q(z))(1 + Az) - zq'(z)}$ Differentiate the location FOC: $(\mathcal{R}q''(z) + q'(z)\mathcal{R}_z)D_C + q'(z)\mathcal{R}_{y_0} = AsD_C$. Rearranging and after some algebra: $D_C = \frac{q'(z)}{q''(z)(y_0 - q(z)) + 2q'(z)^2 - 2q'(z)[A(y_0 - q(z))] + [A(y_0 - q(z))]^2}$. Now, D_C is increasing in A iff $\partial_A(-2q'(z)[A(y_0 - q(z))] + [A(y_0 - q(z))]^2) < 0$, that is, $2(y_0 - q(z))^2 A < 2q'(z)(y_0 - q(z))$, that is, $A < \frac{q'(z)}{y_0 - q(z)}$. But from the location FOC, we know that $0 \leq \frac{s}{c_1} = \frac{q'(z)}{y_0 - q(z)} - A$, and so the inequality above is always satisfied. Therefore, D_C is always strictly increasing in A: a constrained individual also downgrade more when receiving a negative income shock when amenities are valued.

Downgrading of Constrained Relative to Unconstrained Individuals. When $A \to 0$, $y_0^R D_U^R \approx y_0^R \frac{A}{2q''(z)}$. Similarly, $y_0^P D_C^P \approx d(y_0^P, z) + \frac{q'(z)}{q''(z)(y_0^P - q(z)) + 2(q'(z))^2} \cdot \frac{2q'(z)(y_0 - q(z))}{q''(z)(y_0^P - q(z)) + 2q'(z)^2} A$. Therefore, for a level income shock, $D_U^R - D_C^R \approx d(y_0^P, z) + \frac{1}{2q''(z)} \cdot [\frac{XY}{(X+Y)^2} - 1]A$, where $X = q''(z)(y_0^P - q(z))$ and $Y = 2q'(z)^2$. Now, $(X + Y)^2 = X^2 + Y^2 + 2XY = (X - Y)^2 + 4XY \geq 4XY > XY$. Therefore, $D_U^R - D_C^R \approx d(y_0^P, z) - f(y_0^P, z)A$, which is a decreasing function of A since $f(y_0^P, z) > 0$.

For a proportional income shock, $y_0^R D_U^R - y_0^P D_C^R \approx y_0^P d(y_0^P, z) + \frac{1}{2q''(z)} \cdot [\frac{XY}{(X+Y)^2} y_0^P - y_0^R]A$ and the term that multiplies A is still negative. Therefore, $y_0^R D_U^R - y_0^P D_C^R$ is decreasing in A to a first order.

C.2. *Generalized Two-Period Model*

We enrich the extended model of Section 2.6 with variable housing choice, amenities, and city income in both periods. Suppose that individuals indexed by (y, s) solve the following problem:

$$V(y_0, y_1, s) = \max_{c_0, c_1, h_0, h_1, a, z} \log\left(A(z) \cdot h_0^\alpha c_0^{1-\alpha}\right) + \beta \log\left(A(z) \cdot h_1^\alpha c_1^{1-\alpha}\right)$$

$$\text{s.t. } c_0 + a + q(z) + p(z)\ell_0 = y_0 + \tau\Phi(s, z),$$

$$c_1 + \theta\left[q(z) + p(z)\ell_1\right] = y_1 + Ra + \Phi(s, z), \tag{8}$$

$$a \geq \underline{a},$$

where, relative to the model in the main text, τ governs how much of the mobility returns individual receive immediately, and θ governs how much housing costs must be paid in the second period. When $\Phi(s, z) = sz$, $\tau = \theta = p(z) = 0$, $A(z) = 1$, and $\alpha = 0$, we obtain the model in the main text.

Maximizing out the housing choice, we obtain

$$V(y_0, y_1, s) = \max_{c_0, c_1, a, z} \log\big(B(z) \cdot c_0\big) + \beta \log\big(B(z) \cdot c_1\big)$$

$$\text{s.t. } \frac{c_0}{1-\alpha} + a + q(z) = y_0 + \tau\Phi(s, z),$$

$$\frac{c_1}{1-\alpha} + \theta q(z) = y_1 + Ra + \Phi(s, z),$$

$$a \geq \underline{a},$$

where $B(z) = \frac{A(z)}{p(z)^\alpha}$ are perceived amenities after variable housing consumption has been internalized. The financial Euler equation is unchanged. The mobility Euler equation becomes

$$\frac{c_1}{\beta c_0} = \frac{\Phi_z(s, z) - \theta q'(z)}{q'(z) - \tau\Phi_z(s, z)}.$$

LEMMA 7: *Suppose the following assumptions hold*: (i) $A(z)$ *is continuously differentiable and nondecreasing in* z; (ii) $p(z) = p_0$ *is constant across locations (materials for construction)*; (iii) *the housing production technology results in land prices* $q(z) = Q(L(z))$ *where* $L(z)$ *is total population in* z, *and* Q *is an increasing function such that* $Q(0) = 0$ *and* $\lim_{L \to +\infty} Q(L) = +\infty$; (iv) *the supply of land exceeds population*: $\int h(z)\,dz \geq 1$, *where* $h(z)$ *is the density of land of quality* $z \geq 0$; (v) *individual income is of the form* $I(y, s, z) = y + \Phi(s, z)$, *where* Φ *is continuously differentiable and* $\Phi(s, z) > 0$, $\Phi_z(s, z) > 0$, $\Phi_{sz}(s, z) > 0$; (vi) *there are no credit constraints*: $\underline{a} = -\infty$.

Consider two individuals A and B who solve Problem (8), *with the same future income and location choice. Namely, they have*: (a) *the same period-1 income*: $y_1^A = y_1^B$; (b) *different period-0 incomes*: $y_0^A < y_0^B$. *A is initially lower-income than B*; (c) *he same location choice*: $z^A = z^B = z^*$.

Suppose that they both receive a negative income shock in period 0, *such that both individuals loose income down to* $y_0'^A = y_0'^B < y_0^A$. *Then the initially high-income individual* (B) *downgrades location more than the initially low-income* (A):

$$z'^B < z'^A < z^*.$$

This result holds under the less restrictive, single assumption of positive sorting between individuals and locations, which is implied by assumptions (i)–(v).

COROLLARY 8: *Replace assumptions* (i)–(v) *in Lemma* 7 *by*: *primitives are such that individuals choose location according to a matching function* $Z(\mathbf{y}, s)$, *where* $\mathbf{y}$ *is permanent income, and such that* $Z_y(y, s) > 0$ *and* $Z_s(y, s) > 0$. *Then the implications of Lemma* 7 *continue to hold*.

We now prove Lemma 7 and Corollary 8.

Equivalence With a Static Problem. Using the Euler equation and combining both budget constraints into the intertemporal budget constraint, we obtain: $V(y_0, y_1, s) = \max_z B(z)[\mathbf{y} + \mathbf{\Phi}(s, z) - \mathbf{q}(z)]$, where $\mathbf{\Phi}(s, z) = (\tau + R^{-1})\Phi(s, z)$, $\mathbf{y} = y_0 + R^{-1}y_1$, and $\mathbf{q}(z) = (1 + \theta R^{-1})q(z)$. The FOC is $\nu(z) + \frac{\mathbf{\Phi}(s,z) - z q'(z)}{\mathbf{y} + \mathbf{\Phi}(s,z) - \mathbf{q}(z)} = 0$, where $\nu(z) = \frac{z B'(z)}{B(z)}$ is the

elasticity of perceived amenities. Rearrange it as $y = \frac{(\nu(z)+\eta(z))\mathbf{q}(z)-(\phi(s,z)+\nu(z))\Phi(s,z)}{\nu(z)}$, where $\phi(s,z) = \frac{z\Phi(s,z)_z(s,z)}{\Phi(s,z)(s,z)}$. Define $G(z,s) = \frac{(\nu(z)+\eta(z))\mathbf{q}(z)-(\phi(s,z)+\nu(z))\Phi(s,z)}{\nu(z)}$.

PROOF OF COROLLARY 8: Suppose that there is positive sorting, that is, that there exists a unique solution $Z(y,s)$ to $y = G(Z(y,s),s)$, with $Z_y(y,s), Z_s(y,s) > 0$. The implicit function theorem implies that $G_z > 0$ and $G_s < 0$. In particular, $\nu(z) > 0$. Now consider individuals A and B, before and after the shock. Then $G(z'^A, s^A) = G(z'^B, s^B)$. Because $G_s < 0 < G_z$, it must be that $z'^A > z'^B$. Thus, the initially high-income individual downgrades more.   Q.E.D.

PROOF OF LEMMA 7: We show that positive sorting obtains in equilibrium under Assumptions (i)–(vi). That $A(z)$ is increasing and $p(z) = p_0$ ensure that $\nu(z) = \frac{zB'(z)}{B(z)} \geq 0$. Denote $u(z; y, s) = B(z)[y + \Phi(s,z) - q(z)]$. Then $\frac{zu_z(z;y,s)}{u} \cdot [y + \Phi(s,z) - q(z)] = \nu(z)y + \nu(z)\Phi(s,z) + z\Phi_z(s,z) - \nu(z)q(z) - zq'(z)$. The assumption of excess land together with $Q(0) = 0$ ensures that there is always a worst city which is empty with zero land price. Thus, in equilibrium, $y + \Phi(s,z) - \mathbf{q}(z) \geq 0$. Finally, denote again the optimal location choice $Z(y,s)$. If $u_z(Z(y,s); y, s) = 0$, then $u_z(Z(y,s); y, s') > 0$ for $s' > s$. Therefore, it must be that $Z(y,s)$ is weakly increasing in s: $Z_s(y,s) \geq 0$. If the matching function is locally flat (i.e., the derivative is zero), then the assumption that $\lim_{L\to+\infty} Q(L) = +\infty$ implies that prices are locally infinite. This cannot be an equilibrium, and hence the matching function is strictly increasing. The same logic applies for $Z_y(y,s) > 0$. The conclusion follows from the proof of Corollary 8.   Q.E.D.

C.3. *Infinite-Horizon Extension Without Credit Constraints*

We now extend the previous results in our two-period model to an infinite-horizon model without credit constraints. Individuals solve

$$V(a_0, y_0, z_{-1}, s) = \max_{c,a,h,z} \sum_{t=0}^{\infty} \beta^t \log\left(A(z_t)c_t^{1-\alpha}h_t^{\alpha}\right) \tag{9}$$

$$\text{s.t. } c_t + q(z_t) + p(z_t)h_t + a_{t+1} = Ra_t + s(z_{t-1} + \tau z_t) + y_t,$$

where a_t are assets, z_t is location, c_t is consumption of a perishable good, h_t is housing consumption. s is a permanent skill that governs returns to location. τ governs the fraction of location-specific income that accrues upon arrival in a location. y_t is an exogenous income stream. $R \geq 1$ is an exogenous interest rate on financial assets. $A(z)$ are amenities, $p(z)$ is the price of variable housing, and $q(z)$ is the price of the fixed component of housing. Lemma 7 extends as follows. Assume perfect foresight for simplicity.

COROLLARY 9: *Impose either assumptions* (i)–(vi) *of Lemma* 7, *or assumptions of Corollary* 8 *for period-0 location choice. Consider two individuals A and B solving Problem* (9) *in a stationary equilibrium, with the same initial assets, past location and future income, and location choice. Namely, they have* (a) *the same income after period* 1: $y_t^A = y_t^B$ *for all* $t \geq 1$; (b) *the same asset holdings* $a_0^A < a_0^B$; (c) *the same past location* $z_{-1}^A = z_{-1}^B$, (d) *different period-0 income* $y_0^A < y_0^B$: *A is initially lower-income than B*; (e) *the same location choice in period* 0: $z_0^A = z_0^B = z_0^*$.

Suppose that they both receive a negative income shock in period 0, *such that both individuals loose* $y_0'^A = y_0'^B < y_0^A$.

Then the initially high-income individual (B) downgrades location more than the initially low-income (A): $z_0'^B < z_0'^A < z_0^*$.

PROOF: As in the two-period model, (i) maximize out variable housing choice, (ii) use the Euler equation to link consumption across time periods, (iii) iterate forward on the budget constraints and use the transversality condition to rewrite problem (9) in present-value terms:

$$V(a_0, \{y_t\}_t, z_{-1}, s) = \max_{\{z_t\}_{t \geq 0}} B(\{z_t\}_t)[Y(a_0, \{y_t\}_t) + \Psi(s, \{z_t\}_t) - Q(\{z_t\}_t)],$$

where

$$B(\{z_t\}_t) = \exp\left[\sum_{t=0}^{\infty} \beta^t \log \frac{A(z_t)}{p(z_t)^\alpha}\right], \qquad Y(a_0, \{y_t\}_t) = R\left[a_0 + \sum_{t=0}^{\infty} R^{-t} y_t + \Phi(s, z_{-1})\right],$$

$$\Psi(s, \{z_t\}_t) = \sum_{t=0}^{\infty} R^{-t}(\Phi(s, z_t) + \tau\Phi(s, z_{t+1})), \qquad Q(\{z_t\}_t) = \sum_{t=0}^{\infty} R^{-t} q(z_t).$$

$\beta R = 1$ must hold in a stationary equilibrium. Then the utility value of amenities decays at the same rate as income. Taking the FOC with respect to z_t, we obtain $\nu(z_t) + \frac{(\tau + R^{-1})\Phi_z(s, z_t) - q'(z_t)}{Y_0 + \Psi(s, \{z_t\}_t) - Q(\{z_t\}_t)} = 0$ where ν is the elasticity of $\frac{A(z)}{p(z)^\alpha}$. Further, denote

$$B(z) = \frac{A(z)}{p(z)^\alpha}, \qquad \Phi(s, z) = (\tau + R^{-1})\Phi(s, z) \quad z_0^* = Z(Y_0, \{y_t\}_{t \geq 1}, s),$$

$$\mathbf{Y}(a_0, \{y_t\}_{t \geq 1}, z_{-1}) = R\left[a_0 + \sum_{t-1}^{\infty} R^{-t} y_t + \Phi(s, z_{-1})\right] - \sum_{t-1}^{\infty} R^{-t} q(Z(Y_t, \{y_r\}_{r \geq t}, s))$$

$$+ \sum_{t-1}^{\infty} R^{-t}(\Phi(s, Z(Y_t, \{y_r\}_{r \geq t}, s)) + \tau\Phi(s, Z(Y_t, \{y_r\}_{r \geq t+1}, s))).$$

The problem for solving for z_0 as a function of (y_0, s) given $(a_0, \{y_t\}_{t \geq 1}, z_{-1})$ is now equivalent to solving the static problem

$$V(y_0, s; a_0, \{y_t\}_{t \geq 1}, z_{-1}) = \max_z B(z)[y_0 + \mathbf{Y}(a_0, \{y_t\}_{t \geq 1}, z_{-1}) + \Phi(s, z) - q(z)].$$

The result then follows from the proof of Lemma 7. *Q.E.D.*

C.4. *Two-Period Model With Idiosyncratic Preferences for Locations*

We now present an extension of our two-period model that features idiosyncratic preferences for locations. Since the goal of this section is to contrast the predictions of a model in which spatial sorting arises solely due to taste for amenities, we shut down any borrowing and saving between time periods. Individuals then solve

$$\max_{c_t, z_t} \log(c_t) + A z_t + \varepsilon_{z_t} \quad \text{s.t. } c_t + q(z_t) = y_t, \quad t = 0, 1.$$

The preference shocks ε_z are assumed to be independently drawn across locations z and follow a Gumbel distribution with shape parameter θ.

We show numerically below that, while this model may deliver that low-y individuals downgrade relatively more than high-y individuals in response to an adverse income shock if preference heterogeneity is large enough, we also highlight that such a configuration comes with additional stark implications that are at odds with what we find empirically in Section 4.

To numerically investigate the location responses of individuals to income shocks, we must also specify whether idiosyncratic preferences for locations ε_z are a permanent attribute of the individual, or whether they are re-drawn every period. Since our empirical analysis follows individuals over no more than four years, we find the assumption of permanent shocks more plausible. Nevertheless, we also present results when shocks are drawn anew every period.

We choose standard parameter values. θ is set to 3, a common value in the literature. To obtain a housing-to-income ratio close to a third, we set $q(z) = z^{1.1}$. Finally, we split disposable income into assets and wages. We consider three types of individuals: poor ($a_0 = 0$), medium ($a_0 = 1$) and rich ($a_0 = 2$). All individuals receive a wage of $w_0 = 1$. We then consider individuals in the second period who earn a lower wage $w_1 = 0.5$. We choose $A = 2$. The results of our simulations are presented in Figure 10 below.

While the rightmost panels of Figure 10(a) and (b) reveal that our calibration indeed predicts that poor individuals may downgrade their location relative more than rich in-

(a) Location changes: permanent preference shocks.
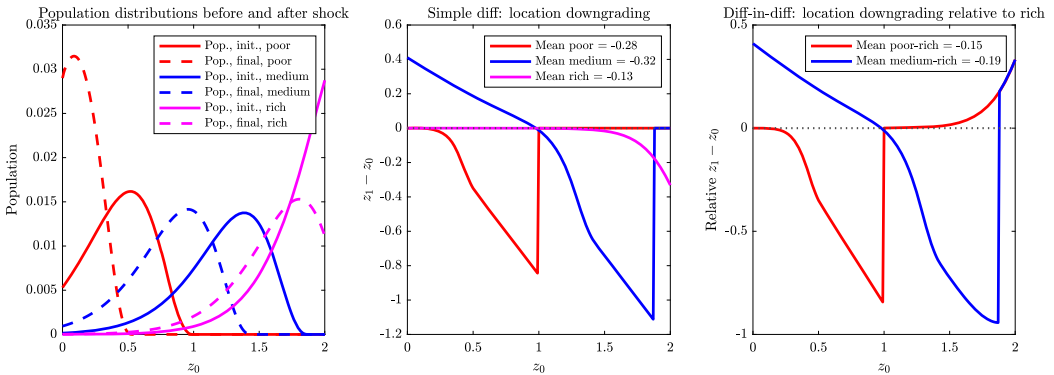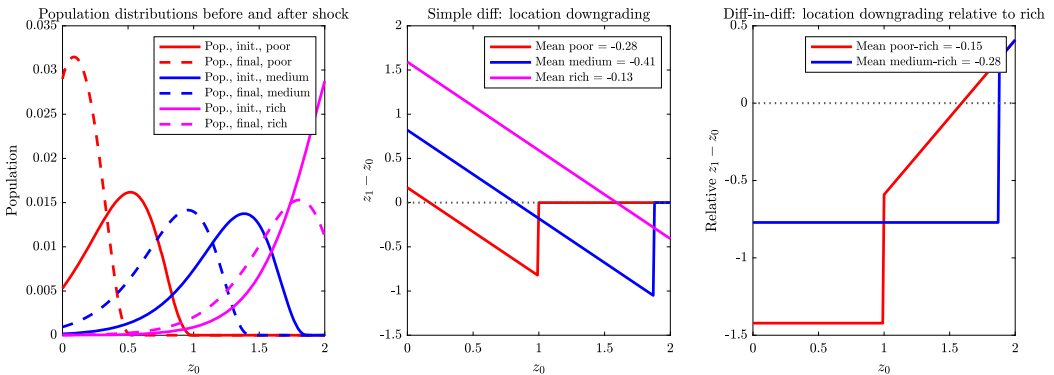

(b) Location changes: i.i.d. preference shocks.

FIGURE 10.—Location decisions following an income shock with amenities only.

dividuals on average, it also predicts the following counterfactual observations. First, medium and rich individuals downgrade their location after receiving the negative income shock (middle panels). Second, medium individuals downgrade their location relative to rich individuals on average (right panels).

APPENDIX D: CALIBRATION AND ADDITIONAL EXERCISES

D.1. *Calibration*

We calibrate our infinite horizon economy to an annual level with two income states $N = 2$ for CRRA utility $u(c) = \frac{c^{1-\frac{1}{\sigma}}-1}{1-\frac{1}{\sigma}}$. We choose the parameter values in Table VIII.

Most of those values are standard. For instance, if we interpret the low income state y_1 as unemployment and the high income state y_2 as employment, we can compute the stationary unemployment rate in this economy through the invariant distribution of the Markov chain transition matrix Λ'. At our current values, we obtain a stationary nonemployment rate of 14%, consistent with the prime-age male nonemployment rate in France.

Our value of the Intertemporal Elasticity of Substitution σ (IES) is within the accepted range. The median skill we use is $s_0 = 1$. Given our house rent schedule and the equilibrium city choice, this implies that the idiosyncratic component of income y_t represents between 5% and 15% of total labor income $y_t + s_0 z_t$ depending on where individuals are in the state space. Persistent income $s_0 z_t$ thus represents between 85% and 95%. This reflects the large observed differences in wages across cities. The differences in location between the best city 1 and the lowest city 0.5 individuals locate in, imply an income change of 0.5, which is of the order of magnitude of the high idiosyncratic income state.

Finally, our house rents schedule is constructed in such a way that unconstrained individuals of skill $s = 1.2$ locate at the best available city, and are free to downgrade as much

TABLE VIII

CALIBRATION PARAMETERS

Parameter	Notation	Value
Preferences		
Discount Factor	β	0.95
Intertemporal Elasticity of Substitution	σ	0.2
Idiosyncratic Income		
Reference skill	s	1
Skill distribution		Uniform on [0.8, 1.2]
Low Income State	y_1	0.05
High Income State	y_2	0.2
Transition Probability From Low to High	Λ_{12}	0.6
Transition Probability From High to Low	Λ_{21}	0.1
Financial Markets		
Risk-Free Rate	R	1.04
Credit Constraint	$\underline{a}$	0.00
Cities		
Best City	$\bar{z}$	1.00
Worst City	$\underline{z}$	0.3
House Rents Slope	$q'(z)$	$0.29 + 0.87 \cdot ((z-0.3)/0.7)^{1.5}$
House Rents	$q(z)$	$\int_{\underline{z}}^{z} q'(x)\,dx$

as they like. It also implies housing expenses of about one-third of total labor income, consistent with its empirical counterpart reported in Davis and Ortalo-Magne (2011).

To solve the model numerically, we adapt the method of endogenous grid points of Carroll (2006).

D.2. *Additional Exercises and Results*

In Figure 11, we present the percentage gain in consumption, and in consumption equivalent welfare from using the location asset. The values are calculated starting from the ideal city for unconstrained individuals, $\mathcal{Z}^U(s)$, and we keep the skill of the individual fixed, as in Figure 3. Figure 11 then plots the relative consumption and welfare from using the location asset as a function of the starting asset level, as well as the invariant distribution of assets in the right panel.[43] It presents the gains for agents with a current high or low income realization. Clearly, because we are not estimating the parameters of the model for a particular circumstance, the level of the gains provides only a rough indication of what is at stake from using the location asset. In contrast, the qualitative patterns are more interesting. Most consumption gains happen close to the constraint for low-income individuals who are dis-saving. These consumption gains quickly fade away as we consider individuals with higher levels of assets. However, because those consumption gains occur precisely in the high marginal utility states, they translate into welfare gains of 0.5% close to the constraint.[44] The figure also shows that agents in the low income state
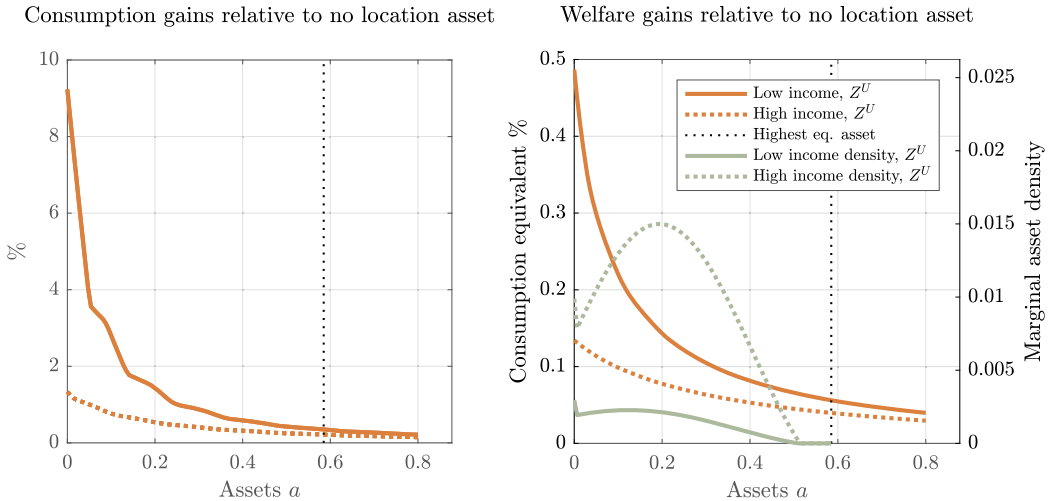

FIGURE 11.—Consumption and welfare gains from the use of the Location Asset.

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[43]Because financially constrained individuals borrow with the location asset, our two-asset model predicts more individuals at the financial constraint than the standard one-asset formulation. This has interesting implications for macroeconomic policy. For instance, tax rebate shocks would be partly saved by financially constrained individuals by upgrading location.

[44]Gains in flow consumption can be as high as 10% for low-income individuals close to the constraint that live in their unconstrained preferred location. These gains are larger than the ones depicted in Figure 3. This is the case because individuals usually start downgrading location one period before they hit the constraint. Note also that the small kinks in the consumption gains are due to kinks in the consumption policy functions, when individuals hit the constraint next period.

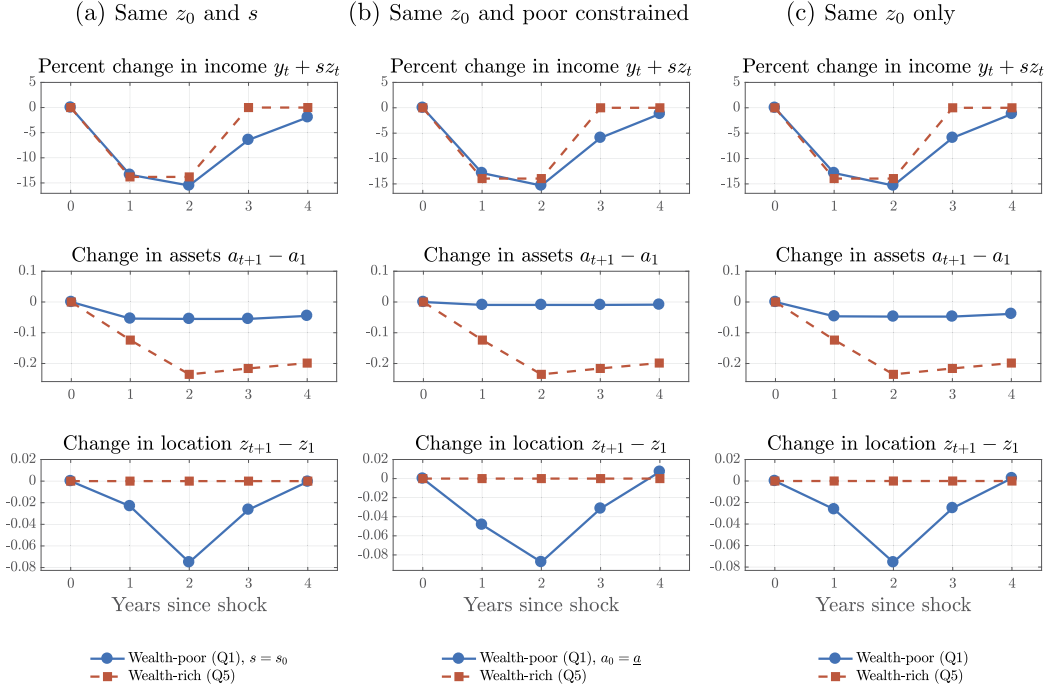(a) Same z_0 and s ... (b) Same z_0 and poor constrained ... (c) Same z_0 only

FIGURE 12.—Disposable income, asset and location response to a front-loaded idiosyncratic income shock, by wealth quintile.

benefit more than agents in the high income state, as they are the most likely to use the "location asset."

Figure 12 shows these individuals' asset and location over time. In column (a), we select only individuals of the same skill $s = s_0 = 1$, where $s_0 = \mathcal{S}^U(z_0)$ is the skill of the unconstrained individuals who reside in location z_0. Selecting individuals of the same skill s_0 in both groups implies that everyone must be unconstrained, and so the low-wealth individuals must hold just enough financial assets to be unconstrained but sufficiently little to be in the bottom quintile. The second row of column (a) shows that, as a response to the negative income shock, wealthy individuals dis-save their financial assets to smooth consumption. By contrast, low-wealth individuals do not dis-save much because they hit the credit constraint rapidly. As shown in the last row, these low-wealth, credit-constrained, individuals smooth consumption using the 'location asset' and downgrading their location. By contrast, wealthy individuals stay in their unconstrained location z_0. After the idiosyncratic component of income reverts to the high state, the initially wealthy individuals start saving again in financial assets. The credit-constrained individuals save in the 'location asset' by upgrading their location. Because we select individuals of the same skill s_0 among both low-wealth and wealthy individuals in column 4, z_0 is the unconstrained location for individuals of both groups. Thus, the low-wealth individuals also revert to z_0 in period 4 when they all accumulate assets and become unconstrained.

In column (b) of Figure 12, we select only wealth-poor individuals who are exactly constrained $a_0 = \underline{a}$ when entering the high income state in period 0. Constrained and unconstrained individuals choose to live in the same location only if the unconstrained wealthy individuals have a lower s and are, therefore, less location-elastic. Hence, we cannot con-

dition on s. The second row of column (b) shows that the asset downgrading of initially low-wealth individuals is minimal, since individuals now hold only the assets they managed to accumulate in period zero, when they had high income. Since low-wealth constrained individual have a higher s than wealthy ones, they are already 'borrowing' with the location asset. As in column (a), however, they downgrade location even more to weather the low income shock. Once the idiosyncratic component of income reverts to the high state, however, they upgrade their location towards an even better location than where they started (this is evident in the last row of column (b) in period 4). In fact, we know that they will start accumulating assets and will stop upgrading their location only when they reach a better location than their wealthy counterparts, since we know they have a higher s.

In column (c), which is identical to Figure 4 in the main text, we consider all individuals who satisfy our initial criteria. As a result, the impulse response of assets and location are a weighted average of those in columns (a) and (b). Overall, all three columns reveal similar patterns.

APPENDIX E: Data Description and Additional Results

E.1. *Data Description and Sample Selection*

Our main data sources are the "Déclaration de Données Sociales" (DADS) Panel, as well as the "Données Sociales et Fiscales" from the "Echantillon Demographique Permanent" (EDP). Both are administrative tax data from the French statistical institute (INSEE).

DADS.   The DADS is a matched employer-employee dataset based on tax returns filed by employers. It has rich information on a representative sample of workers who receive taxable labor income in France. It is a panel of all workers in France born in October of even years (approximately 8%). In this data set, we can track the same individual throughout her employment spells for the period 2002–2015. We start in 2002 to observe workers for a long enough period and estimate long-term returns to mobility.

We extract the following variables from the dataset: (i) anonymized individual identifier, common to DADS and EDP; (ii) total net wage earnings; (iii) age and gender; (iv) municipality of residence and workplace; (v) 2-digit occupation. We extract the highest paying employment spell for each individual and each quarter. We then aggregate wage at the annual level, and select municipality and occupation based on the highest paying spell in the year.

EDP.   The fiscal data in the EDP dataset starts in 2008 and contains income tax return information for French households that are sampled in the DADS or in the baseline EDP sample. The EDP sample contains individuals born in January 2–5, April 1–4, July 1–4, and October 1–4. We link it to the DADS Panel through a common individual identifier.

We extract the following variables from the dataset: (i) anonymized individual identifier, common to DADS and EDP; (ii) income from financial assets: annuities, housing rents, net of expenses (mortgage payments, repairs, etc.), stocks, mutual funds, bonds, taxable bank accounts, excluding capital gains, imputed nontaxable income (life insurance, certain types of bank accounts, etc.). We excluded private equity from the analysis because in many cases it corresponds to ownership of a practice (lawyers, medical doc-

tors, etc.) that it highly illiquid and hard to separate from the worker and sell. We use the residence information from the DADS rather than the fiscal residence information from the EDP due to well-known concern that the fiscal residence is often times different from the actual residence. We indeed find that using the fiscal residence implies a annual migration rate that is an order of magnitude lower than what we find in the DADS, and implausibly low. We must restrict attention to years 2010 to 2015 so that all variables are non-missing.

We compute asset quintiles in every year based on the within-municipality distribution of assets to be fully consistent with our theory. The correlation with unconditional asset quintiles is 0.9. As shown in Figure 5, local wealth quintiles appear to capture substantial variation in monetary terms.

Additional Data. We complement our main sample with additional data from two sources.

Amenities. We use two measures of amenities. First, we use the "Base Permamente des Equipements" in 2007 to construct a measure of amenities. 2007 is the year prior to which the closest year available before our sample with financial income starts. It reports data the number of 136 types of establishments in health services (e.g., hospitals), education services (e.g., preschools), public services (e.g., police stations), and commercial services (e.g., perfumeries). We first compute the number of these establishments per capita in each municipality. Then we extract the first principal components of the corresponding covariance matrix. For each municipality, we obtain the loading on this principal component. We choose the sign of the principal component such that the loadings correlate positively with our measure of z. Finally, we rank these loadings between 0 and 1. This rank is our first measure of amenities. Our second measure of local amenities are the amenities recovered through the structural model in Bilal (2020). We refer to that paper for details.

Commuting Distance. We obtain data on the centroids of each municipality in France from a database publicly available from the French government at https://www.data.gouv. fr/en/datasets/listes-des-communes-geolocalisees-par-regions-departements-circonscript ions-nd/. We then compute the geodesic distance between each residence-workplace municipality pair, and use this distance as our measure of commuting distance.

Background on the French Geography. The French mainland territory is partitioned in about 96 districts ("Départements") and 36,552 municipalities ("Communes"). Départements are fairly large areas (median area is 8763 km² and median population is 531,380 inhabitants), while municipalities are much smaller (median area is slightly above 10 km2, and median population is 432 inhabitants).

Construction of the z Variable. To determine how desirable a municipality is, we compute average annual wage earnings in each municipality in the DADS. We then rank municipalities and compute the corresponding percentile for each municipality.

E.2. *Income Shock*

Figure 13 presents the average wage income shock by wealth quintile that we use for the exercises in Section 4.

(a) In euros   (b) In Percent

Municipality, income, occupation, age and home-ownership controls   Municipality, income, occupation, age and home-ownership controls
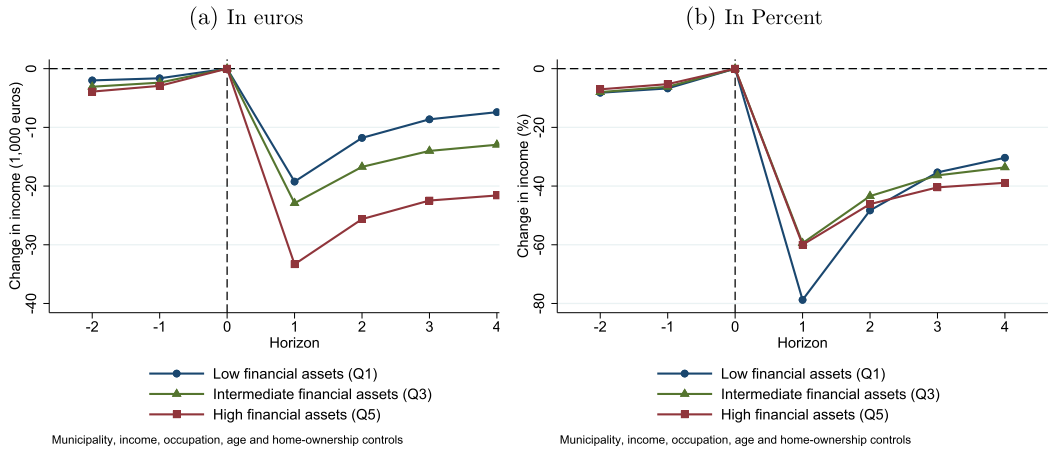
FIGURE 13.—Wage income effect of a negative income shock by financial assets quintile. Note: Difference between wage income of individuals with low financial assets (Q1) and individuals with high financial assets (Q5) $\alpha_{1,1,t} - \alpha_{5,1,t}$ following a negative income shock relative to individuals who do not receive the shock. $t = 0$ is the year before the income shock. Confidence intervals omitted for readability. The set of controls includes: fixed effects for the time-0 municipality, log wage income at period 0, fixed effects for the time-0 2-digit occupation, 5-year age bin fixed effects, and a home-ownership (HO) fixed effect.

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