Econometrica Supplementary Material

SUPPLEMENT TO "AN EMPIRICAL MODEL OF R&D PROCUREMENT CONTESTS: AN ANALYSIS OF THE DOD SBIR PROGRAM"

(*Econometrica*, Vol. 89, No. 5, September 2021, 2189–2224)

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APPENDIX B: TECHNICAL APPENDIX

B.1. *Properties of the Equilibrium*

SUPPOSE THAT THE DENSITY OF VALUES is bounded and atomless on compact support and is continuously differentiable. Suppose that the cost distribution $H(c; t)$ is continuously differentiable in (c, t), and $H(\cdot; t)$ is stochastically decreasing in t. I first prove properties of equilibria in the Phase II game. Suppose that Phase II efforts are bounded above by $\bar{t}$; they are of course bounded below by 0. A player of type v chooses effort t to maximize utility function given by the maximand of (1). Since the distribution of opponents' surplus $G(\cdot)$ is continuous in their efforts (as $H(c, t)$ is continuous in t), this utility function is continuous in all efforts as well.[1] Moreover, this function is supermodular between a firm's type and its effort, if each firm's beliefs about its opponents' values are independent of its own value.

LEMMA B.1—Supermodularity: *If each firm's beliefs about its opponents' values are independent of its own value*, *then the maximand of* (1) *is strictly supermodular.*

PROOF: We can write the first term of the maximand as

$$\eta \int_{\underline{c}}^{v} \left[\int_0^{v-c} (v - c - s)\, dG(s) + (v - c)G(0) \right] dH(c; t)$$

$$= \eta \int_{\underline{c}}^{v} \left[-(v - c)G(0) + \int_0^{v-c} G(s)\, ds + (v - c)G(0) \right] dH(c; t)$$

$$= \left(\int_0^{v-c} G(s)\, ds \right) H(c, t) \Big|_{\underline{c}}^{v} + \int_{\underline{c}}^{v} G(v - c)H(c, t)\, dc = \int_{\underline{c}}^{v} G(v - c)H(c, t)\, dc.$$

The cross-partial with respect to v and t is

$$G(0)\frac{\partial H(v, t)}{\partial t} + \int_{\underline{c}}^{v} g(v - c)\frac{\partial H(c, t)}{\partial t}\, dc,$$

and each term is strictly positive. *Q.E.D.*

An implication of Lemma B.1 is the following existence and monotonicity result.

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[1]Note that, unlike in an auction, there are no discontinuities in payoffs based on the ordering of types or efforts. The randomness in costs smooths this out.

 https://doi.org/10.3982/ECTA16581

PROPOSITION B.1—Monotonicity of Effort: *In a Phase II contest in which each firm's beliefs about its opponents' values are independent of its own value, an equilibrium exists in nondecreasing pure strategies. The equilibrium strategies are strictly increasing if effort is strictly in* $(0, \bar{t})$.

PROOF: Since the distribution of values is bounded and atomless, Condition (A1) in Athey (2001) applies. Lemma B.1 shows that the game satisfies the single crossing condition (SCC). The utility function is continuous in actions, and actions are bounded in $[0, \bar{t}]$. By Corollary 2.1 of Athey (2001), a pure strategy equilibrium exists in nondecreasing strategies.

The proof of Lemma B.1 shows that the partial derivative of the maximand with respect to t exists and is strictly increasing in v. That the equilibrium is strictly increasing follows from Strict Monotonicity Theorem 1 of Edlin and Shannon (1998). *Q.E.D.*

Proposition B.1 shows that an equilibrium exists for any $N_2 \leq \bar{N}_2$ if there is no selection. This covers cases where $\bar{N}_2 = N_1$ or $\bar{N}_2 = 1$. (The latter is a single agent decision problem.) The maximand in (3) is continuous in p^*. By the Theorem of the Maximum, the maximizer $p(p^*)$, that is, the best response if all opponents are playing p^* in Phase I, is upper hemicontinuous and thus has a closed graph. By Kakutani's Fixed Point Theorem, a fixed point exists, and this is a symmetric equilibrium. This proves the following proposition.

PROPOSITION B.2: *If* $\bar{N}_2 = N_1$ *or* $\bar{N}_2 = 1$, *an equilibrium of the contest in Section* 3.1 *exists*.

When there is competition and selection, that is, $1 < N_2 = \bar{N}_2 < N_1$, Lemma B.1 need not apply, as $G(\cdot)$ directly depends on v. Writing this as $G(v - c; v)$ makes it clear that the cross-partial has a term that depends on the partial derivative of this quantity with respect to the second argument. This is negative: higher-v firms expect to face stronger competition, which means they may expect a stronger distribution of the highest surplus generated by rivals. As mentioned in Footnote 16, this may threaten the existence of a monotone equilibrium. I have not found a proof for the existence of such an equilibrium, but I have also not found a counterexample numerically.

It is possible to prove existence (of a possibly nonmonotone) equilibrium in Phase II if $\bar{N}_2 = 2$. I leverage the following result.

LEMMA B.2: *Let* $\mathrm{BR}_i(t(\cdot))$ *denote the best response of player* i *in the Phase II game if its opponent is playing the effort function* $t(\cdot)$. *If* $t(\cdot) \geq \tilde{t}(\cdot)$ *pointwise, then* $\mathrm{BR}_i(t(\cdot)) \leq \mathrm{BR}_i(\tilde{t}(\cdot))$ *pointwise*.

PROOF: Let $G_i(\cdot; v, t(\cdot))$ denote the cdf of the surplus that player i expects to face if it has value v and its opponent is playing the strategy $t(\cdot)$. If $t(\cdot) \geq \tilde{t}(\cdot)$ pointwise, then $G_i(\cdot; v, t(\cdot)) \leq G_i(\cdot; v, \tilde{t}(\cdot))$: the distribution of i's opponent's surplus stochastically increases. Fix a type v, and note that for candidate efforts t_i' and $t_i'' > t_i'$,

$$\int_{\underline{c}}^{v} \big[G_i\big(v-c; v, \tilde{t}(\cdot)\big) - G_i\big(v-c; v, t(\cdot)\big)\big]\big(H\big(c, t_i'\big) - H\big(c, t_i''\big)\big)\, dc \leq 0,$$

using the fact that $H(\cdot; t')$ is stochastically decreasing in t'. By monotone comparative statics arguments, this means that the choice t_i for effort for player i with type v increases if its opponent is playing a strategy with pointwise lower effort. *Q.E.D.*

Lemma B.2 shows strategic substitutability in the 2-player case, which can be used to prove equilibrium existence. I use negative signs to turn this into a game of strategic complements and appeal to the lattice structure of strategies, as in Jia (2008).

PROPOSITION B.3: *Suppose $N_2 = \bar{N}_2 = 2$. Then, a pure strategy Nash equilibrium to the Phase II effort provision game exists.*

PROOF: Let $(t_1(\cdot), \bar{t}_2(\cdot))$ be a vector of functions that denotes that player 1 will play effort $t_1(v)$ if it has value v and player 2 will play effort $-t_2(v)$ if it has type v. The set of all such strategies forms a lattice, with $(t_1(\cdot), \bar{t}_2(\cdot)) \geq (t_1'(\cdot), \bar{t}_2'(\cdot))$ if $t_1(\cdot) \geq t_1'(\cdot)$ and $\bar{t}_2(\cdot) \geq \bar{t}_2'(\cdot)$ (so that $t_2(\cdot) \leq t_2'(\cdot)$), both elementwise. The lattice is complete since effort is always bounded in $[0, \bar{t}]$. Let $\mathrm{BR} : (t_1(\cdot), \bar{t}_2(\cdot)) \to (\mathrm{BR}_1(-t_2(\cdot)), -\mathrm{BR}_2(t_1(\cdot)))$ be the best response map, taking the appropriate negative signs. Lemma B.2 shows that BR is an increasing function. Tarski's Fixed Point Theorem shows that a fixed point exists, and this is a pure strategy Nash equilibrium by construction. *Q.E.D.*

Arguments similar to Proposition B.2 then show that an equilibrium exists in Phase I as well, for $\bar{N}_2 = 2$. Note that Proposition B.3 does not show that a type-symmetric pure strategy equilibrium exists, although in all numerical examples I find a symmetric equilibrium. I have not found an existence proof for $2 < \bar{N}_2 < N_1$, although all computations in this paper for estimation only rely on $\bar{N}_2 \in \{1, 2, N_1\}$.

Finally, none of the results in this section show that the equilibrium, if it does exist, is unique. Identification only relies on necessary conditions of equilibria. Estimation involves solving the model conditional on η; I have tried starting the iterated best response algorithm at many different starting points, but it converges to the same point.

The next result shows an efficiency property of the equilibrium.

PROPOSITION B.4: *Consider a contest that begins in Phase II. The social planner's solution (when the planner is constrained to choose effort schedules that depend only on an individual competitor's value) can be supported by a competitive equilibrium when $\eta = 1$. Moreover, if there is exactly one competitor, the social surplus is monotonically increasing in η.*

PROOF: Suppose the social planner can pick a schedule $t_i(v)$ of effort for each firm i as a function of the firm's realized value v. Define $S_i(v, t_i(v))$ as the random variable representing the surplus of each firm i. Fix a distinguished firm i. Then, the social planner's problem is

$$\max_{t_i, t_{-i}} \left\{ \mathbb{E}\left[\max\left\{ S_i\big(v_i, t_i(v_i)\big), \max_{-i} S_{-i}\big(v_{-i}, t_{-i}(v_{-i})\big) \right\}^+ \right] - \mathbb{E}\big[t_i(v_i)\big] - \sum_{-i} \mathbb{E}\big[t_{-i}(v_{-i})\big] \right\},$$

where the expectations are taken over realization of v. If we denote the social planner's optimum as $t_{-i}^*(\cdot)$, to determine $t_i^*(v)$, the planner will solve

$$\begin{aligned}
&\arg\max_t \left\{ \mathbb{E}\left[\max\left\{ S_i(v, t), \max_{-i} S_{-i}\big(v_{-i}, t_{-i}^*(v_{-i})\big) \right\}^+ \right] - t \right\} \\
&\quad = \arg\max_t \left\{ \mathbb{E}\left[\left\{ S_i(v, t) - \max_{-i} S_{-i}\big(v_{-i}, t_{-i}^*(v_{-i})\big)^+ \right\}^+ + \max_{-i} S_{-i}\big(v_{-i}, t_{-i}^*(v_{-i})\big)^+ \right] - t \right\} \\
&\quad = \arg\max_t \left\{ \mathbb{E}\left[\left\{ S_i(v, t) - \max_{-i} S_{-i}\big(v_{-i}, t_{-i}^*(v_{-i})\big)^+ \right\}^+ \right] - t \right\}, \qquad \text{(B.1)}
\end{aligned}$$

where from the second to the third line, I drop $\max_{-i} S_{-i}(v_{-i}, t_{-i}(v_{-i}))^+$ since it is independent of t. (Note that expectations are taken only over realization of v_{-i} in this sequence.) But, (B.1) is identically the expression for firm i's problem when $\eta = 1$. Thus, the social planner's optimum corresponds to a Nash equilibrium of the game.

To show that the social surplus is monotone in η, consider the problem $\max_t[\eta f(t; v) - t]$ where f is increasing in t. Denote the solution to this problem as $t^*(\eta; v)$. This solution is increasing in η due to the fact that the maximand has increasing differences in η and t. Consider the function $g(v; \eta) \equiv f(t^*(\eta; v); v) - t^*(\eta; v)$. The derivative with respect to η is

$$\frac{dt^*(\eta; v)}{d\eta}\big[f'\big(t^*(\eta; v); v\big) - 1\big].$$

But, $dt^*(\eta; v)/d\eta \geq 0$. Moreover, we know that $\eta f'(t^*(\eta; v); v) = 1$ at an interior solution, so $f'(t^*(\eta; v); v) \geq 1$. Thus, $g(v; \eta)$ is increasing in η for all v. Since the social surplus is $\mathbb{E}[g(v; \eta)]$, where the expectation is taken over η, we have that it is increasing in η. *Q.E.D.*

B.2. *Identification*

This section formalizes the identification sketch in Section 4.1. I present the result under two successively stronger assumptions. The first (Assumption M) is especially general and simply states that the map from values to Phase II research efforts is monotone (conditional on the other primitives). The second (Assumption O) is that the firm chooses the optimal effort, consistent with the model in Section 3.1. Separating these two assumptions clarifies which aspects of the structure are used to identify which parameters. Furthermore, because Assumption M is more general, stating it separately can provide guidance to adapt this framework to other empirical settings to study R&D contests.

Throughout, I assume effort does not depend on opponents' values, and I thus discuss an effort function $\hat{t}(v)$. As noted in Section 3.2, this is consistent with the institutions since the DOD does not provide competitors information about their opponents' values. I denote Phase I efforts $\hat{p}$ to highlight that I do not require it to be set optimally.

ASSUMPTION M: *The Phase II research effort $\hat{t}(v)$ is observed and is an increasing function of the value v. This map may depend on all contest primitives and on the realization of N_2.*

Assumption M(onotonicity) places no restrictions on Phase I efforts $\hat{p}$. The restrictions placed on Phase II efforts are that higher-value firms exert more effort and that effort only depends on one's own value. Assumption M is relatively weak and may be applicable outside the setting of this paper. For instance, in some contests, firms may be given a research award that is an institutionally specified function of a quality score (the "value"), and they may exhaust the award on research for the project.[2] Outside the context of procurement contests, one could imagine that higher-quality startups, which are capital-constrained, also attract more external funding and thus spend more money developing their research projects.

[2]This could be the case when monitoring is especially strong and the monitoring agency can check whether each dollar is spent on the project itself. Alternatively, one can imagine that this is likely when firms are especially small. Such firms may have no other ongoing R&D projects, and as long as the award cannot literally be pocketed and used as profit, they would exhaust the award on research.

PROPOSITION B.5: *Suppose we have data on distributions of Phase III transfers, Phase I and II research efforts, and the realized number of Phase II competitors for a set of contests with a single* $(N_1, \bar{N}_2)$. *Suppose the probability of a Phase III contract is strictly less than* 1 *for any value of the Phase II contract, and* $H(\cdot, t)$ *strictly increasing if it takes on values in* $(0, 1)$, *for all* t. *If Assumption* M *holds and* η *is known, then* (i) V *is nonparametrically identified (and does not depend on* η); (ii) $H(c; t)$ *is nonparametrically identified on* $[0, v(t)]$; *and* (iii) *a single point* $\psi(\hat{p})$ *on* $\psi(\cdot)$ *is identified (where* $\hat{p}$ *is the success probability in Phase I*), *and variation in* $\hat{p}$ *identifies* $\psi(\cdot)$ *entirely.*

This argument follows the discussion in Section 4.1. The following lemma provides the one piece of the argument missing from Section 4.1.

LEMMA B.3: *Let* $\{S_i\}_{i=1}^{N_1}$ *be independent Bernoulli variables with known success probability* p *and* $\{V_i\}_{i=1}^{N_1}$ *be independent draws from a distribution with cdf* $F_V(\cdot)$. *Suppose the distribution of* $V_S \equiv \max\{V_i : S_i = 1\}$ *is known. Then,* $F_V(\cdot)$ *is identified.*

PROOF: Let $F_{V_S}(\cdot)$ denote the cdf of V_S. We can write this as

$$F_{V_S}(v) = \frac{1}{(1-p)^{N_1}} \sum_{N_S=1}^{N_1} \binom{N_1}{N_S} p^{N_S}(1-p)^{N_1-N_S} F_V(v)^{N_S}. \tag{B.2}$$

This is the cdf of the mixture of the top order statistic of between $N_S = 1$ and $N_S = N_1$ draws from $F_V(\cdot)$, where N_S represents the number of successes. The right-hand side of (B.2) is a convex combination of increasing functions of $F_V(v)$. Since $F_{V_S}(v)$ is identified, we can invert (B.2) to identify the cdf $F_V(\cdot)$. *Q.E.D.*

Lemma B.3, a simple extension of the proof of Theorem 1(i) in Athey and Haile (2002), says a known mixture of order statistics of a distribution identifies the distribution itself. This result is useful since the selected distribution of values in Phase II follows (B.2) exactly.

PROOF OF PROPOSITION B.5: First note that the probability of success in Phase I is identified directly from the data: the probability that Phase II does not occur is simply $(1-\hat{p})^{N_1}$.

To prove (i), we follow the first part of Section 4.1. Consider contest with $N_2 = 1$. Fixing t_2, since the probability of a Phase III contract is strictly less than 1 and there are no gaps in the support of costs, the maximum contract observed in the data must be at a cost draw $c = v(t_2)$, where $\hat{t}^{-1}(v(t_2)) = t_2$. Thus, the value is identified for each t_2 as the maximum of the support of the conditional distribution of Phase III transfers. Since the distribution of t_2 is observed, so is the distribution of values conditional on selection into Phase II. Lemma B.3 shows that the unconditional distribution of values is identified.

The proof of (ii) follows from the observation that $c = [T_3 - \eta \cdot v(t_2)]/(1-\eta)$, and the right-hand side is either observed or identified. Since $\hat{p}$ is identified and $\psi(\hat{p})$ is observed as the Phase I effort, (iii) is immediate. Observed variation orthogonal to $\psi(\hat{p})$ (e.g., shifts in the value or cost distributions) lets us estimate other $\hat{p}$ and identify other points on $\psi(\cdot)$. *Q.E.D.*

In this paper, I impose an additional assumption: the contract amount for Phase II coincides with the effort the firm would choose itself, that is, the amounts are firm-optimal.

ASSUMPTION O: *The Phase II effort schedule* $\hat{t}(v)$ *coincides with the type-symmetric Bayesian Nash equilibrium of the model, given by* p^* *and* $\{t^*_{N_2}(\cdot)\}_{N_2 \le \bar{N}_2}$, *which satisfy* (1) *and* (3). *Thus, the firm-optimal Phase II effort is observed given the Phase II contract.*

Proposition B.1 shows that in cases without selection, Assumption O(ptimality) implies Assumption M. With this assumption, I can show that the bargaining parameter is identified.

PROPOSITION B.6: *Suppose we have data on distributions on Phase III transfers, Phase II research efforts, and the realized number of Phase II competitors for a set of contests with a single* $(N_1, \bar{N}_2)$. *Assume the conditions in Proposition* B.5 *and that Assumption* O *holds as well. Then,* (i) η *is identified;* (ii) V *is nonparametrically identified;* (iii) $H(c; t)$ *is nonparametrically identified on* $[0, v(t)]$; *and* (iv) $\psi'(\cdot)$ *is identified within a single-parameter family of functions, and variation that continuously shifts the equilibrium probability of success in Phase I without shifting Phase I costs can identify* $\psi'(\cdot)$ *nonparametrically.*

PROOF: Proposition B.5 shows (ii) directly and that (iii) is identified up to η. To prove (i), recall that the firm sets its research effort in response to the first-order condition (4). Integrating (4) by parts, and letting $H(\cdot)$ denote the cdf associated with $h(\cdot)$, we have

$$\eta \int_{\underline{c}}^{v(t_2)} \frac{dH}{dt}(c; \eta, t_2)\, dc = 1. \tag{B.3}$$

If $C(t_2)$ is a draw from a random variable with cdf $H(\cdot; \eta, t_2)$, note that

$$\begin{aligned} H(c; \eta, t_2) &= \Pr\big(C(t_2) \le c | t_2, \eta\big) = \Pr\big(\eta v(t_2) + (1-\eta)C(t_2) \le \eta v(t_2) + (1-\eta)c\big) \\ &\equiv \hat{F}\big(\eta v(t_2) + (1-\eta)c; t_2\big), \end{aligned}$$

which is the cdf of the transfer evaluated at $\eta v(t_2) + (1-\eta)c$, an observed quantity (as a function of η).[3] Substituting into (B.3), we have

$$\eta \int_{\underline{c}}^{v(t_2)} \left(\frac{d\hat{F}}{dt}\big(\eta v(t_2) + (1-\eta)c; t_2\big) + \eta v'(t_2) \hat{f}\big(\eta v(t_2) + (1-\eta)c; t_2\big) \right) dc = 1.$$

Setting $u = \eta v(t_2) + (1-\eta)c$, we have

$$\frac{\eta}{1-\eta} \int_{\underline{T}}^{v(t_2)} \frac{d\hat{F}}{dt}(u; t_2)\, du + \frac{\eta^2}{1-\eta} \int_{\underline{T}}^{v(t_2)} v'(t_2) \cdot \hat{f}(u; t_2)\, du = 1, \tag{B.4}$$

where $\underline{T}$ is the minimum transfer observed. Given that $v(\cdot)$ and thus $v'(t_2)$ are both identified already from the support of the transfer distribution, the integrals are identified directly from the data. Thus, (B.4) can be rearranged to a quadratic in η and has at most two solutions, only one of which corresponds to the actual optimum (as the other violates the second-order condition). Thus, η is identified, which in turn identifies the cdf $H(c; t_2)$ nonparametrically for all $c \le v(t_2)$, proving (ii) as well.

[3]The transfer distribution $\hat{F}(\cdot; \eta)$ is purely data; I include the dependence on η to highlight that it is generated from a DGP that depends on η. The function $v(t_2)$ is identified from the upper bound argument, η is a candidate parameters, and c is the dummy of integration.

To prove (iii), note that p^* is observed, and all elements but a single parameter would be identified in the first-order condition associated with (3), that is, that $\psi'(p^*) = \pi(p^*)$, where $\pi(\cdot)$ is the profit conditional on success if equilibrium efforts are p^*.[4] *Q.E.D.*

In summary, Assumption O gives us two main benefits. First, the bargaining power is identified. Second, identification of $\psi'(\cdot)$ within a single-parameter family of functions does not require data on Phase I research efforts, which is useful in this paper since it allows me to avoid using the institutionally-set contract amount for most of the estimates. (In the extension with external benefits, I use this observed amount.) Separating the identification argument highlights that the model has empirical content without an equilibrium assumption. It also underscores that alternate models for efforts can inform the bargaining parameter.

The argument in Proposition B.6 only used the first-order condition at one point, and the battery of overidentifying restrictions can add flexibility for more general models that may help researchers extend this paper. For instance, suppose Phase II research provides benefits $b(t_2)$ outside Phase III; with knowledge of $b(\cdot)$ at one point, it can be identified directly from the first-order condition (4). A different extension relevant for the empirical framework is identification of the model with multiplicative contest-level unobserved heterogeneity θ_j. Here, the joint distribution of Phase II transfers (focusing on cases without selection) identifies the distribution of unobserved heterogeneity (Kotlarski (1967), Li and Vuong (1998), Krasnokutskaya (2011)). The failure rate as a function of the observed Phase II effort t_2 and the distribution of Phase III transfers conditional on t_2 are both observed, and they are convolutions of the "base" failure rate and transfers (i.e., with $\theta_j = 1$) and the distribution of θ_j (now identified). Thus, under appropriate technical conditions, deconvolution recovers the base failure rate and Phase III transfer distributions as a function of t_2. This recovers the cost distribution as a function of both η and (unlike in the baseline) $\tilde{v}(t_2)$. The consistency condition that the failure rate equals the probability the cost draw is less than the value identifies $\tilde{v}(\cdot)$. Optimality (as in (4)) identifies η. Formal results corresponding to these observations were in the working paper version and are available upon request.

B.3. *Details of Selection Equations*

With selection in Phase II (i.e., $N_2 = \bar{N}_2 < N_1$), beliefs of the values of opponents' types are correlated and also depend on one's own type. The beliefs a firm with value v who enters Phase II will have that its opponents have values $\mathbf{v}_{-i} = (v_{-i1}, v_{-i2}, \dots, v_{-i,\bar{N}_2-1})$, which I use as notation for the full vector of values and order in decreasing order, are

$$f_v(\mathbf{v}_{-i}; v, \bar{N}_2, p) \propto \sum_{N_S=\bar{N}_2-1}^{N_1-1} \Bigg\{ \overbrace{\frac{(N_1-1)!}{(N_1-\bar{N}_2)!}\left(\prod_{j=1}^{\bar{N}_2-1}\big(p\cdot f(v_{-ij})\big)\right)}^{\text{succeeded, with given values}} \times \underbrace{\binom{N_1-\bar{N}_2}{N_S-(\bar{N}_2-1)}\big[p\cdot F\big(\min\{\mathbf{v}_{-i}, v\}\big)\big]^{N_S-(\bar{N}_2-1)}}_{\text{succeeded but drew lower values}} \times \underbrace{(1-p)^{N_1-N_S-1}}_{\text{did not succeed}} \Bigg\}. \quad \text{(B.5)}$$

[4]Of course, knowing the Phase I research efforts directly would also provide information on $\psi(\cdot)$ without resorting to the first-order condition for optimality of Phase I effort.

Note that N_S indexes the number of successes of other firms in Phase I. Since $\bar{N}_2$ firms enter, at least $\bar{N}_2 - 1$ other firms succeeded. We have $N_1 - 1$ candidates for the firm with value v_{-i1}; the firm must have succeeded (probability p) and drawn a value v_{-i1} (density $f(v_{-i1})$). We repeat until the final firm who enters, for whom there are $N_1 - (\bar{N}_2 - 1)$ candidates. Then, $N_S - (\bar{N}_2 - 1)$ of the remaining $N_1 - \bar{N}_2$ firms succeeded but must have drawn values lower than the minimum value of those who did enter (including the firm in question with value v). The remaining firms must have failed. In (2), I use the notation $f_v(\mathbf{v}_{-i}; v, \bar{N}_2, p)$ generically; without selection, it denotes independent draws from $F_V(\cdot)$.

I now turn to $\lambda(v, N_S, \bar{N}_2)$ in (3), the probability that a successful firm with value v is allowed to enter Phase II if N_S other firms succeed. This expression is

$$\lambda(v, N_S, \bar{N}_2) \equiv \begin{cases} 1 & \text{if } N_S \leq \bar{N}_2 - 1, \\ \displaystyle\sum_{N_b=0}^{\bar{N}_2-1} \binom{N_S}{N_b} F(v)^{N_S-N_b}\bigl(1 - F(v)\bigr)^{N_b} & \text{otherwise.} \end{cases} \tag{B.6}$$

If the number of other firms who succeed is less than $\bar{N}_2 - 1$, then all firms enter. If more than $\bar{N}_2 - 1$ other firms succeed, then the firm with value v enters as long as no more than $\bar{N}_2 - 1$ other firms draw values higher than v. The probability that N_b firms draw values higher than v is given by the summand in the second case.

APPENDIX C: MODEL SOLUTION AND ESTIMATION

C.1. *Solving the Model*

I use a three-step procedure to solve the model in Section 3.1. First, I solve for the effort function in Phase II when $N_2 = 1$ on a fine grid of values; for each value v, I compute the optimal effort using single-variable optimization. Second, for each $1 < N_2 < \bar{N}_2$ (unless $\bar{N}_2 = N_1$, in which case this step applies to $N_2 = \bar{N}_2$ as well), I use iterated best responses to compute the equilibrium Phase II effort function $t^*_{N_2}$. I iterate on (1), with $t^*_{N_2}(\cdot)$ in (1) replaced by the candidate effort function from the previous iteration of the algorithm. Note that for these values of N_2, there is no selection, so the value of p^* is irrelevant. On each step of the iteration, I solve a one-dimensional optimization problem at each grid point. When $1 < N_2 = \bar{N}_2 < N_1$, (1) depends on opponents' Phase I efforts p. I use the same iteration to compute the equilibrium but do this computation for p on a grid from 0.01 to 0.99. Finally, I compute p^* from the first-order condition associated with (3); I compute the profits from arbitrary p by linearly interpolating the effort functions from the previous step.

Solutions for extensions are similar. When values are shocked to $\zeta \cdot v$ in Phase II, I integrate over ζ as needed using quadrature formulas for the beta distribution (Fernandes and Atchley (2006)). The effort function for the case where phases are combined does not depend on N_2, but it does depend on firms' beliefs over the distribution of competitors they face. This distribution is binomial; I use the same strategy as above of computing the function on a grid of p and using interpolation. When α is drawn from a distribution in Phase I, I compute an effort function $p^*(\alpha)$ via iterated best responses in Phase I. This function depends on beliefs over the distribution of competitors in Phase II.[5]

[5]While this distribution is in principle a mixture of Bernoullis for different $p(\alpha)$, I have found that it is well-approximated by a binomial with success probability $\mathbb{E}[p(\alpha)]$, and I use this approximation.

C.2. *Estimation Details*

Step 2. I follow Steps 1–5 of Section 4.1 of Krasnokutskaya (2011) almost exactly, modifying equations to account that "bidders" are homogeneous in my setting. (Using her notation, A_1 and A_2 are i.i.d.) When inverting the characteristic function, I follow suggestions in her appendix and introduce a damping function $d(t; T) = (1 - |t|/T)^+$, with T chosen in a data-driven fashion by matching moments of recovered densities with the data.[6]

Step 3. Fix a particular contest j and guess a θ_j. The map $\tilde{v}$, which matches quantiles of the candidate distribution of values with the quantiles of the empirical distribution of efforts (after deconvolving it from the unobserved heterogeneity) allows one to compute $\tilde{v}_{ij}(\theta_j)$ as $\tilde{v}(\nu_{ij}/\theta_j)$ for all firms i. We can compute the likelihood $L_{\text{values},j}(\theta_j)$ of observing all the values in a contest j with the candidate $f_{\tilde{v}}(\cdot)$ by multiplying the probabilities of seeing each value, fixing the draw of θ_j at the contest.[7] If the contest does not enter Phase III, then it must be that all firms drew costs larger than their values, and we can compute the likelihood $L_{\text{Phase III},j}(\theta_j)$ of this as the product of the candidate counter-cdf of costs at the postulated values. If instead we do observe a Phase III transfer for firm i^*, then we can compute the likelihood $L_{\text{Phase III},j}(\theta_j)$ of observing this transfer directly using the pdf of the candidate cost distribution. With more than one participant in Phase II, we would also have to integrate out over the draw s of the second-highest surplus; we can compute the pdf $f_{\tilde{S},j}(s)$ of the maximum value of the surplus for all firms other than i^* based on the observed values of ν_{ij} and the candidate $\tilde{H}(\cdot;\cdot)$. Note that if, for the particular θ_j, the value of the Phase III transfer exceeds the implied $\tilde{v}_{ij}(\theta_j)$, then the likelihood is zero.[8] Finally, if a Phase III contract is implausibly low (less than \$1 million), I assume that the project succeeded but that the actual value of the Phase III transfer is unobserved (see Appendix A.1). In this case, if i^* is awarded the contract, then it must be that i^* drew a cost less than its value and that the surplus generated by all other competitors is less; the likelihood $L_{\text{Phase III},j}(\theta_j)$ can be computed accordingly. Integrating over θ, the log likelihood of observing this outcome is

$$\log \int L_{\text{values},j}(\theta) \cdot L_{\text{Phase III},j}(\theta) \cdot f_\theta(\theta)\, d\theta.^{9} \tag{C.1}$$

I parameterize $\mu(t)$ to be a decreasing quadratic in $\log t$ on the interval $-2.0 \leq \log t \leq 1.5$, which encompasses almost all the data. I then parameterize this function by three values: (a) $\mu(t)$ at $\log t = -2.0$, (b) $\mu'(t)$ at $\log t = -2.0$, and (c) $\mu'(t)$ at $\log t = 1.5$. I constrain the parameters in (b) and (c) to be negative. To avoid issues related to the quadratic function becoming increasing, I let $\mu(t)$ be linear in $\log t$ outside this range.

[6]I have noticed through visual inspection that this procedure sometimes still leads to distributions that oscillate near the tails, an issue especially relevant using bootstraps. I thus reduce the T estimated for Θ by 10% and the T estimated for $\tilde{t}$ by 50% for some additional smoothing beyond what is given by this procedure.

[7]In the case where $N_1 = 2$ and $\bar{N}_2 = 1$, there is selection into Phase II. I parameterize V as a lognormal in this case as well, but when I compute the likelihood, I note that the distribution of values in Phase II is a mixture between V and the maximum of two draws of V. The mixing probabilities are a function of the probability p^* of success in Phase I, which I can estimate directly.

[8]Since we are integrating out over θ_j, this would not immediately mean that the overall likelihood itself is 0 for that contest. Indeed, this is how the θ_j allows for a degree of smoothing of the upper bound.

[9]For certain parameters, specific observations are computed to have a likelihood of zero. Instead of letting the log likelihood function be $-\infty$ at these parameters, I replace the zeros with a penalty term $\pi_{\text{penalty}} = \exp(-100)$. The optimizer moves away from this parameter region.

I set the semi-elasticity of $\mu(t)$ (i.e., $d\mu(t)/d\log t$) outside this range equal to the value at the closest endpoint.[10]

In the extension in which the values v are shocked to $\zeta \cdot v$ in Phase II, the computation of the likelihood involves one further integral over ζ_{ij} for each firm i in each contest j. The expressions above are modified as appropriate. I compute this integral using quadrature formulas for the beta distribution (Fernandes and Atchley (2006)).

In general, I optimize the likelihood in (C.1) using the global optimizer DIRECT-L (Gablonsky and Kelley (2001)) followed by the local optimizer BOBYQA (Powell (2009)), both implemented by NLopt (Johnson (2010)) and following its guidance. This allows me to explore a large parameter space first before polishing the solution, and it provides the efficiency needed to perform optimization on a fine grid of η.

Step 4. For each contest j, let $\hat{f}_j$ be an indicator for whether the contest failed before entering Phase III. Let $\hat{t}_{3j}$ be the observed Phase III transfer. Since this quantity is undefined for contests that do not enter Phase III, I instead define the moment $\hat{t}'_j$ to be 0 if the contest fails and $\hat{t}_{3j}$ if it does not. I match these to the empirical counterparts, which are the simulated probability of failure $\widetilde{\Pr}(\text{failure}; \eta, \theta^*(\eta))$, where $\theta^*(\eta)$ are the MLE estimates conditional on η from Step 3, and the (partial) expectation of the observed transfer, $(1 - \widetilde{\Pr}(\text{failure}; \eta, \theta^*(\eta))) \cdot \tilde{\mathbb{E}}[t_3; \eta, \theta^*(\eta)]$. I match these moments conditional on (N_1, N_2). Thus, for a contest j, the relevant set of moments is

$$g_j(\eta) = \begin{pmatrix} \hat{f}_j - \widetilde{\Pr}\big(\text{failure}; \eta, \theta^*(\eta)\big) \\ \hat{t}'_j - \big(1 - \widetilde{\Pr}\big(\text{failure}; \eta, \theta^*(\eta)\big)\big) \cdot \tilde{\mathbb{E}}\big[t_3; \eta, \theta^*(\eta)\big] \end{pmatrix} \otimes \mathbb{1}_{(N_{1j}, N_{2j})} \equiv \hat{g}_j - \tilde{g}_j(\eta),$$

where $\mathbb{1}_{(N_1, N_2)}$ is a vector that contains a 1 in the element corresponding to (N_1, N_2) and zeros elsewhere. I estimate η^* by minimizing the Euclidean norm of the average of this vector over all j on a fine grid of η, or (η, β_ζ) in the extension with a shock to v in Phase II.

Step 5. I compute

$$\underset{\{\alpha_{N_1}\}_{N_1=1}^4}{\arg\min} \sum_{(N_1, \bar{N}_2)} w_{(N_1, \bar{N}_2)} \big[\psi'(\hat{p}_{(N_1, \bar{N}_2)}; \alpha_{N_1}) - \hat{\pi}\big(N_1, \bar{N}_2; \eta^*, \theta\big(\eta^*\big)\big)\big]^2,$$

where $\hat{\pi}(N_1, \bar{N}_2; \eta^*, \theta(\eta^*))$ is the expected profit conditional on success and the weight $w_{(N_1, \bar{N}_2)}$ is equal to the number of contests with $(N_1, \bar{N}_2)$.[11] In the modification where firms earn a benefit B of a success in Phase I, I choose $\{\alpha_{N_1}\}_{N_1=1}^4$ and B to minimize

$$\sum_{(N_1, \bar{N}_2)} w_{(N_1, \bar{N}_2)} \big[\psi'(\hat{p}_{(N_1, \bar{N}_2)}; \alpha_{N_1}) - \hat{\pi}\big(N_1, \bar{N}_2; \eta^*, \theta\big(\eta^*\big)\big) - B\big]^2 + w'\left(\sum_j \psi(\hat{p}_{(N_{1j}, \bar{N}_{2j})}; \alpha_{N_{1j}}) - K_j\right)^2,$$

[10] I experimented with specifications in which the cost distribution is a mixture of a lognormal and a mass point at ∞, which has probability $\gamma(t)$, to rationalize a failure rate without necessarily resorting to a large standard deviation of the cost distribution. Somewhat surprisingly, these specifications in practice tend to place low mass on this "outright" failure rate and not change the estimated cost distribution appreciably.

[11] Restricting to contests without selection into Phase II is unnecessary (monotonicity is not leveraged) and nonsensical: it is unknown at the start of Phase I which contests would have selection into Phase II.

where the second sum is over contracts i, K_j is the Phase I contract amount, and w' is a weight so that the two elements are on the same scale.[12]

APPENDIX D: ADDITIONAL DISCUSSION OF THE STRUCTURAL MODEL

D.1. *Further Analysis of Forces*

This appendix provides further decompositions of the total effects in Section 6.1 to quantify various forces and inefficiencies.

Quantifying the Business-Stealing Effect. I start with a different decomposition of the total inefficiency than the one presented in Section 6.1. I first set $\eta = 1$ to alleviate holdup. In doing so, I let Phase I effort readjust to the new equilibrium, rather than holding it constant as in Section 6.1: this illustrates the sentences in Section 6.2 that increasing η exacerbates the other inefficiencies. I then eliminate reimbursement, still keeping $\eta = 1$. I finally compute the optimum social surplus, so the remaining change is due to business-stealing. This lets me decompose the latter two effects.

Table D.I shows these results. For $N_1 = 1$, eliminating holdup and computing the new equilibrium increases surplus, consistent with Section 6.1. For larger N_1, simply increasing η significantly lowers surplus, as firms overprovide effort. One way to reduce overprovision is to eliminate reimbursement. Doing so has no effect when $N_1 \leq 2$: here, Phase I effort is already at a corner solution of high p^*, and eliminating one source of inefficiency is not enough to move it away from the corner. For larger N_1, eliminating reimbursement substantially improves surplus by 45–55% of the baseline. Finally, the remaining inefficiency is business-stealing. This is mechanically not present when $N_1 = 1$, but in all other cases it improves surplus by about 80% of the baseline. Overall, the business-stealing effect is slightly larger than reimbursement, but they are comparable in magnitude.

While the previous decomposition quantifies business-stealing fixing $(N_1, \bar{N}_2)$, one way to think about changing competition is to decompose the total effect into business-stealing and the "change in expected rewards." Adding competitors changes the gains from succeeding in Phase I. Changes in the surplus fixing the rewards the firms expect to earn from success is similar to "business-stealing." (Another term may be a "change in competition" effect.) The remainder is due to the change in the expected rewards from success.[13]

TABLE D.I

DECOMPOSITION OF INEFFICIENCIES IN THE BASELINE DESIGN[a]

			Change in Social Surplus		
N_1	$\bar{N}_2$	Baseline Surplus ($M)	$\eta = 1$	+ No Reimbursement	+ No Business Stealing
1	1	0.250	38.7%	38.7%	38.7%
2	1	0.104	−44.6%	−44.6%	43.9%
3	2	0.117	−76.9%	−20.3%	61.0%
4	2	0.189	−84.8%	−40.1%	42.9%

[a]The final three columns consider first setting $\eta = 1$ to eliminate holdup, then also eliminating all reimbursement, and then adding necessary fees to get to the optimum. The table reports the percent change from the baseline surplus. In all steps, the equilibrium is recomputed to determine the effects on surplus.

[12]I use the square of the average value of $\hat{\pi}(\cdot)$ divided by the square of the average value of K_i.

[13]I am grateful to an anonymous referee for suggesting this decomposition.

TABLE D.II

BUSINESS-STEALING AND "CHANGE IN EXPECTED REWARDS" EFFECTS ON SOCIAL SURPLUS[a]

(a) Business-stealing

$\bar{N}_2$	1	2	3	4
$N_1 = 2$	23.2%	95.0%		
$N_1 = 3$	9.2%	154.4%	185.8%	
$N_1 = 4$	−26.4%	177.6%	254.7%	267.6%

(b) Change in expected rewards

$\bar{N}_2$	1	2	3	4
$N_1 = 2$	10.1%	−0.5%		
$N_1 = 3$	42.8%	−1.9%	−2.2%	
$N_1 = 4$	91.6%	13.2%	−3.3%	−3.1%

[a]This table reports percent changes relative to social surplus with $N_1 = \bar{N}_2 = 1$ competitor, for parameters with $N_1 = 4$.

Table D.II shows this decomposition. In Panel (a), firms exert effort assuming success in Phase I gives them the same expected rewards as when $N_1 = \bar{N}_2 = 1$. Panel (b) shows the change in surplus when firms readjust to the new expected rewards, noting that more competition means lower rewards. The change in expected rewards effect is generally smaller, although the opposite is true for design changes when $N_1 = 1$. For intuition, note the expected reward from a success in Phase I comes from (i) the prospect of entering Phase II and (ii) the surplus from the R&D and procurement contracts. Since failure rates are high in Phase II, the expected reward does not depend strongly on competition. Thus, when entry is not too restricted ($\bar{N}_2$ is large), the change in the expected rewards is minimal. When entry is restricted ($\bar{N}_2 = 1$), the change in expected rewards from a Phase I success are high.

This decomposition is tied closely to the direct and the incentive effects below.

Quantifying the Direct and Incentive Effects. As discussed in Section 6.2, adding contestants has the direct effect of more chances of success, governed by the uncertainty in R&D. (Boudreau, Lacetera, and Lakhani (2011) called this the parallel paths effect.) In general, taking advantage of this direct effect of competition comes hand-in-hand with an incentive effect that competitors adjust their efforts endogenously. This incentive effect is a fundamental economic force behind the rule of thumb that entry should be restricted (Taylor (1995), Che and Gale (2003)). I separately quantify these effects here.

Consider a contest with $(N_1, \bar{N}_2)$ and any outcome $S(N_1, \bar{N}_2, p, \{t_{N_2}(\cdot)\}_{N_2 \le \bar{N}_2})$, defined as a function of the number N_1 of Phase I participants, the limit $\bar{N}_2$ of Phase II participants, effort p in Phase I, and the effort functions $t_{N_2}(\cdot)$. In equilibrium, the firms would exert the effort level $p^*_{(N_1,\bar{N}_2)}$ and the effort functions $t^*_{N_2}(\cdot; p^*_{(N_1,\bar{N}_2)})$. The total effect of moving from a contest with one contestant to one with $(N_1, \bar{N}_2)$ is

$$\underbrace{S\big(N_1, \bar{N}_2, p^*_{(N_1,\bar{N}_2)}, \big\{t^*_{N_2}\big(\cdot; p^*_{(N_1,\bar{N}_2)}\big)\big\}_{N_2 \le \bar{N}_2}\big) - S\big(1, 1, p^*_{(1,1)}, \big\{t^*_1(\cdot)\big\}\big)}_{\text{total effect}}$$

$$= \underbrace{S\big(N_1, 1, p^*_{(1,1)}, \big\{t^*_1(\cdot)\big\}\big) - S\big(1, 1, p^*_{(1,1)}, \big\{t^*_1(\cdot)\big\}\big)}_{\text{direct effect of Phase I competition}}$$

$$+ \underbrace{S\big(N_1, \bar{N}_2, p^*_{(1,1)}, \{t^*_1(\cdot)\}\big) - S\big(N_1, 1, p^*_{(1,1)}, \{t^*_1(\cdot)\}\big)}_{\text{direct effect of Phase II competition}}$$

$$+ \underbrace{S\big(N_1, \bar{N}_2, p^*_{(N_1,\bar{N}_2)}, \{t^*_1(\cdot)\}\big) - S\big(N_1, \bar{N}_2, p^*_{(1,1)}, \{t^*_1(\cdot)\}\big)}_{\text{incentive effect from Phase I competition}}$$

$$+ \underbrace{S\big(N_1, \bar{N}_2, p^*_{(N_1,\bar{N}_2)}, \{t^*_{N_2}(\cdot; p^*_{(N_1,\bar{N}_2)})\}_{N_2 \le \bar{N}_2}\big) - S\big(N_1, \bar{N}_2, p^*_{(N_1,\bar{N}_2)}, \{t^*_1(\cdot)\}\big)}_{\text{incentive effect from Phase II competition}}.$$

The direct effect of Phase I adds Phase I competitors without changing efforts. The direct effect of Phase II subsequently increases $\bar{N}_2$, again without any change in efforts. The incentive effect from Phase I allows firms to adjust their research efforts in Phase I to the equilibrium effort given by the new competitive structure. The incentive effect from Phase II allows firms to adjust their Phase II efforts and arrives at the new equilibrium.

Table D.III quantifies these four effects. Panel (a) shows the direct effect of adding Phase I competitors, which is definitionally independent of $\bar{N}_2$. Increasing N_1 without in-

TABLE D.III

DIRECT AND INCENTIVE EFFECTS ON SOCIAL SURPLUS[a]

(a) Direct (Phase I)

$\bar{N}_2$	1	2	3	4
$N_1 = 2$	23.2%	23.2%		
$N_1 = 3$	9.2%	9.2%	9.2%	
$N_1 = 4$	−26.4%	−26.4%	−26.4%	−26.4%

(b) Direct (Phase II)

$\bar{N}_2$	1	2	3	4
$N_1 = 2$	0.0%	74.4%		
$N_1 = 3$	0.0%	150.3%	184.2%	
$N_1 = 4$	0.0%	211.0%	294.6%	309.0%

(c) Incentive (Phase I)

$\bar{N}_2$	1	2	3	4
$N_1 = 2$	10.1%	−0.6%		
$N_1 = 3$	42.8%	−2.5%	−2.8%	
$N_1 = 4$	91.6%	12.0%	−5.1%	−4.9%

(d) Incentive (Phase II)

$\bar{N}_2$	1	2	3	4
$N_1 = 2$	0.0%	−2.5%		
$N_1 = 3$	0.0%	−4.5%	−6.9%	
$N_1 = 4$	0.0%	−5.8%	−11.7%	−13.2%

[a] This table reports percent changes relative to social surplus with $N_1 = \bar{N}_2 = 1$ competitor, for parameters with $N_1 = 4$.

creasing $\bar{N}_2$ increases total Phase I expenditures and the value of the Phase II competitor slightly, but it does not improve the probability of success in Phase II appreciably—other than by decreasing the probability that the contest fails in Phase I. This latter effect is important up to $N_1 = 3$, and the direct effect increases surplus. However, the added cost of a fourth Phase I competitor is not worth the lower probability of failure in Phase I, and the direct effect is negative for $N_1 = 4$. Panel (b) shows the direct effect of increasing entry into Phase II, which is larger and positive (but definitionally 0 for $\bar{N}_2 = 1$). Once again, the low chance of Phase II success means that firms are not close substitutes in Phase II; thus, the benefit of an additional draw is not dampened by substitutability, and each additional draw outweighs the cost (ignoring all effects on effort). The total direct effect is thus positive as long as $\bar{N}_2 > 1$.

Panel (c) shows the incentive effect for Phase I. Phase I effort decreases with N_1 and increases with $\bar{N}_2$. Firms' Phase I efforts at $N_1 = \bar{N}_2 = 1$ are socially excessive with more competitors, so there are in general social gains from letting firms adjust their efforts down when only one successful competitor will be admitted to Phase II. (When more competitors are admitted to Phase II, allowing firms to adjust efforts in equilibrium can harm surplus slightly.) Finally, the incentive effect for Phase II trades off savings in the effort cost with higher cost draws. This effect is unsurprisingly small across the board, especially in comparison to the direct effect of Phase II. A firm factors in competition when determining its effort to the extent that it expects to influence its marginal surplus; because the probability that one's opponent succeeds is low, this event does not influence incentives much.

Overall, the reason the DOD would want to invite more contestants into both phases is primarily the direct effect (and in particular the direct effect of Phase II), which ties back to the level of uncertainty estimated in Phase II. The incentive effect can be large, but mostly when entry is limited in Phase II, and the incentive effect in Phase II is small.

D.2. *Models of Changing Contest Structure*

Here, I provide further details about the counterfactuals related to contest structure.

Combining Phases. If the DOD combines Phases I and II, then any firm generating a successful innovation with value v in early-stage research (what used to be Phase I) can then choose to exert effort t to draw a delivery cost $c \sim H(\cdot; t)$. When choosing t, the number of Phase II participants (i.e., the number of Phase I successes) is not revealed. Thus, the equilibrium consists of a Phase I effort p^* and a single Phase II effort function $t^*(\cdot)$. The equilibrium satisfies an analogue of (1), with the beliefs over the distribution of the highest opponent surplus ((2) in the baseline) replaced with a mixture of the distributions with between 0 and $N_1 - 1$ other Phase II competitors. The mixture probabilities are given by the equilibrium Phase I effort p^*, which satisfies the analogue of (3), with $\lambda(\cdot, \cdot, \cdot)$ replaced by 1 since all successful firms are allowed to enter "Phase II."

IP Sharing. The model of IP sharing has the same timeline as in Section 3.1. To it, I add the fact that the firms receive a prize $K(v)$ for any successful innovation with value v at the end of Phase I, and they must make the plans of the project public. If no one generates a successful project, then the contest fails. Otherwise, the DOD shares the highest-v plan with $\bar{N}_2$ firms. For concreteness, I take the stance that the DOD shares the plans with successful firms first, and then the unsuccessful firms; this also maximizes effort in Phase I since it provides an added benefit to developing a successful project.

Overall, this means that if the contest enters Phase II, then exactly $\bar{N}_2$ firms enter. As before, they conduct research in Phase II independently and each draw costs $c_i \sim H(\cdot; t_i)$. In Phase III, the DOD chooses the firm with the lowest cost draw (as long as it is smaller than v) and pays it an amount equal to its delivery cost c_i, plus a fraction η of the incremental surplus it generates above the next-best firm. Firms who enter Phase II thus solve

$$t^*(v) = \arg\max_t \left\{ \eta \int_{\underline{c}}^{v} \int_{c}^{\bar{c}} \left(\min\{v, c'\} - c\right) dH_{\bar{N}_2-1:\bar{N}_2-1}\left(c'; t^*(v)\right) dH(c; t) - t \right\}, \qquad \text{(D.1)}$$

where $H_{\bar{N}_2-1:\bar{N}_2-1}(\cdot; t)$ is the minimum of $\bar{N}_2 - 1$ draws from $H(\cdot; t)$. To understand (D.1), note that the incremental surplus a firm generates is $v - c$ if the lowest cost draw of its opponents is larger than v, and $c' - c$ if the lowest rival cost draw c' is less than v. Let $\pi_s(v)$ denote the maximized value of (D.1). From $\pi_s(v)$, firms compute the profit conditional on success $\mathbb{E}[\pi_{\text{success}}(v; p^*, K)]$, which is the sum of the Phase I prize $K(v)$ and the Phase II profits (taking into account that one can enter Phase II and then get a better draw of values from one's opponents). In this setup, firms also have the potential of earning profits even when they do not generate a successful innovation, if they are selected to enter Phase II and use plans generated by a different firm; this happens with positive probability if fewer than $\bar{N}_2$ of their opponents generate successful innovations. Denote the profits from this avenue by $\pi_{\text{failure}}(p^*)$, as they depend on opponents' equilibrium efforts.

An equilibrium of the R&D contest with IP sharing is a pair $(p^*, t^*(\cdot))$ such that $t^*(\cdot)$ satisfies (D.1) and p^* satisfies (D.2) below:

$$p^* = \arg\max_p \left\{ p \cdot \mathbb{E}\left[\pi_{\text{success}}\left(v; p^*, K(\cdot)\right)\right] + (1 - p) \cdot \pi_{\text{failure}}\left(p^*\right) - \psi(p) \right\}. \qquad \text{(D.2)}$$

When I consider mandatory IP sharing in Sections 6.2 and 6.3, I set $K(v) = 0$. When I consider an alternative where $K(v)$ is chosen to make IP sharing incentive-compatible, I say that all successful firms can still enter Phase II, subject to the limit $\bar{N}_2$, but they forgo the payment $K(v)$ if they keep their breakthroughs private. To compute this incentive-compatible schedule $K(v)$, I need to specify the profits under a deviation in which a firm with value v refuses to share information. I consider the following setting: at the end of Phase I, the DOD offers the prize $K(v)$ to all firms with successful innovations. However, unlike before, the DOD allows any firm to forgo the prize $K(v)$ in exchange for keeping the invention secret; the firm is still allowed to enter Phase II if its draw of v is high enough to merit entry into Phase II. The DOD does not reveal whether firms shared their information, and it still shares the plans of the highest-value project from the other firms with the holdout. Moreover, it does not reveal whether or not each firm accepted the prize.[14]

The deviation I consider consists of the following steps. A firm with value v gives up the prize $K(v)$ but enters Phase II if v is in the top $\bar{N}_2$ of the draws. It must decide whether

[14]One could imagine other mechanisms. For instance, the DOD could refuse to share other firms' plans with a firm that does not accept the prize $K(v)$. One justification for the willingness to share plans involves social surplus: while the DOD could improve its welfare by committing to not share plans (and reduce the prize it has to pay), the planner would want a firm to have access to a project that could be potentially better, ignoring incentive effects. The DOD could also announce which firms were willing to share their plans. However, I avoid this possibility out of convenience: if deviations were public, I would have to be explicit about off-path beliefs, which in turn would affect the incentives to deviate.

to accept the Phase I prize before learning how many other firms succeeded. In Phase II, it gets access to the highest draw v' of all other firms and develops that project if and only if $v' > v$. If no other firm succeeded in Phase I, the deviator is the only firm in Phase II and exerts effort according to the equilibrium in Section 3.1, with $N_2 = 1$. Beliefs of all other firms are passive, so all other firms in Phase II (if any) exert the equilibrium effort $t^*(v')$ on the project v'. These criteria together let us derive the equilibrium effort exerted by a firm with value v that deviates, if all other firms are using the technology with value v'. Denote this profit by $\hat{\pi}_{\text{success}}(v, v'; p^*, K(\cdot))$. The incentive compatibility condition is that $K(v) \geq \mathbb{E}[\hat{\pi}_{\text{success}}(v, v'; p^*, K(\cdot))] - \pi_{\text{success}}(v; p^*, K(\cdot))$, where the first expectation is taken over the realization of successes as well as the best value of the opponents.

I do not ask whether this is the optimal schedule for the DOD in the setting in which IP sharing is not mandatory; the DOD may well choose a prize schedule that induces sufficiently high-value firms to keep their IP private. It could also provide only the highest-value firm a prize. This incentive-compatible schedule simply serves as a natural benchmark.

D.3. *Model Fit*

Section 5.1 shows one measure of model fit: the Phase I contract amounts, which are not used in estimation, are comparable to the true ones. That the estimated amount is less than the modal institutionally-set amount could indicate a desire on the part of the DOD SBIR program to cover costs for all contracts (including those with high θ_j) or could represent the existence of other incentives that increase. Section 5.2 investigated this second possibility.

Figure D.1 plots two further measures of fit. Panel (a) plots the observed failure rate (from Phase II to Phase III) for bins of (N_1, N_2) against the model-implied one. Error bars indicate one standard error in the data moments: some bins are rare, and the probability of failure is imprecise here. The model overpredicts failure somewhat, although the differences between predicted and observed failure rates are largest when the observed rates are especially imprecise. The figure also plots the fit for the bins $(N_1, N_2) = (4, 2)$ and $(3, 2)$ (in blue circles), which were held out in Phase II estimation due to the possibility of a nonmonotone strategy; the fit here is comparable to that in other bins. Panel (b) plots the observed contract amounts against the model-implied values. The fit is fairly close, with the predictions usually within one standard error of the observed means. The model does slightly overpredict the transfers for the held-out contests, but they are still within 1.5 standard errors of the observed values. Overall, the parsimonious specification fits the data reasonably.

Panel (c) shows the fit of entry into Phase II. I solve the full model for all contests and predict the distribution of the number of Phase II competitors N_2, taking as given the distribution of $(N_1, \bar{N}_2)$ in the data. The model slightly overpredicts the number of contests with 0 or 2 Phase II entrants relative to 1, but the fit is close. Overall, the model fits surplus-relevant quantities reasonably.

APPENDIX E: INSTITUTIONAL DETAILS ABOUT THE DOD SBIR PROGRAM

I discuss the institutional details of DOD SBIR, expanding on Section 2 and focusing on differences with other agencies and on the discussion in Section 3.2. SBIR is coordinated by the Small Business Administration but administered by individual federal agencies. Accordingly, while the general structure of the program is similar across agencies, the focus and implementation can be different. This is especially true for the DOD, which runs by far the largest SBIR program and is responsible for more than half the total award funding.

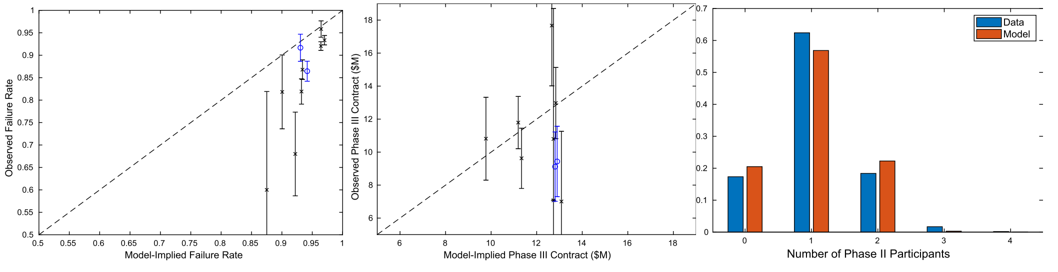
As mentioned in Section 2, all federal agencies with an extramural R&D budget of more than $100 million must allocate approximately 3% of it competitively through SBIR. SBIR awards can only be offered to for-profit small businesses (with fewer than 500 employees) that operate within the United States and are majority-owned by a citizen or permanent resident of the U.S.[15] All federal agencies follow a three-phase SBIR process. The first two phases are part of the core program and funded by the set-aside dollars that must amount to at least 3% of the agency's R&D budget. Phase I involves a smaller award: this was about $80,000 during my sample period, but more recently the SBA has offered guidance that the award amount can be larger. Phase II involves larger awards, with guidance of up to $750,000. However, the SBA does not mandate the award amounts. For instance, as discussed in Section 2, the Phase II award amounts for the Navy are quite variable. Indeed, the SBA does not even mandate the structure of the contest itself: for example, different services of the DOD allow firms to compete for Phase II awards without winning a Phase I award. In sum, despite the general guidance from the SBA, the agencies have considerable latitude to modify the program to meet their own goals.

Because of this guidance, the DOD has focused its program on procurement of products that would be part of acquisition programs. This is reflected (especially in the Navy) by its treatment of Phase III. The SBA defines Phase III in general as the following:

> The objective of Phase III, where appropriate, is for the small business to pursue commercialization objectives resulting from the Phase I/II R/R&D activities.[16]

Importantly, Phase III is not funded by dollars set aside for SBIR. Most agencies do not even offer Phase III awards themselves. The DOD, however, treats Phase III as a formal part of the SBIR program and generally has the intention of offering delivery contracts to firms to "commercialize" the product as part of the program, funding these contracts from the budget of the acquisition programs themselves.[17]

As described by the National Research Council (2009, henceforth NRC), this focus on procurement has developed over time and is in line with Congressional mandate:



(a) Failure Rate (b) Phase III Contract Amounts (c) Phase II Contestants

FIGURE D.1.—Observed and model-implied (a) failure rates in Phase II, (b) Phase III contract amounts, and (c) number of entrants from Phase I to Phase II. The error bars indicate one standard error.

[15]This limit on the number of employees does not apply to Phase III contracts given by the DOD.

[16]See https://www.sbir.gov/about.

[17]The National Research Council (2009, p. 3) notes, "Unlike some major agency participants in the program (e.g., NIH & NSF), DoD seeks to acquire and use many of the technologies and products developed through the program. For many DoD officials, this is the primary objective of the program."

> In the early years of the SBIR program, Phase III was not a very high priority. SBIR topics were defined and awards were made largely in line with the interests and activities of the wider R&D programs—for example, the Army Research Labs. During the 1990s, following the renewal of the program, growing pressure from Congress, and changes in priorities of the leadership in the Pentagon, gradually shifted the SBIR program's emphasis toward serving the warfighter more directly, and specifically to the issue of Phase III. [...] Congress has repeatedly directed SBIR programs generally, and DoD in particular, to emphasize commercialization and to promote the use of SBIR-sponsored technologies in acquisition programs (p. 159).

The DOD Interim Defense Acquisition Guidebook from 2004 (Section C.2.9.15, quoted by NRC (p. 165)) also highlights this emphasis on acquisition:

> The Program Manager shall develop an acquisition strategy that plans for the use of technologies developed under the SBIR program, and gives favorable consideration for funding of successful SBIR technologies. At milestone and appropriate program reviews for ACAT I programs [major defense acquisition programs], the PM shall address the program's plans for funding the further development and insertion into the program of SBIR-developed technologies.

The Navy has a particular emphasis on designing solicitations centered around procurement.[18] The vast majority of solicitations in the Navy SBIR program are designed by the Program Execution Office of the acquisition programs in conjunction with the prime contractor in charge of the program.[19] Finally, NRC writes

> The Navy considers the Phase III reported in the DD350 [a contracting form] its primary tool for such purposes, emphasizing once again the stress laid on active insertion into weapons programs by the services as the core metric of success from the agency perspective (p. 118).

The emphasis on procurement motivates Navy SBIR as a case study for an R&D procurement contest and justifies studying surplus generated by procurement as the baseline metric.

As discussed in Section 2, Phase I is described as a "feasibility study to determine the scientific or technical merit of an idea or technology that may provide a solution to DON needs or requirements." Phase II is "typically a demonstration phase in which prototypes are built and tested" and "is expected to produce a well-defined deliverable prototype";[20] accordingly, the R&D contract amounts for Phase II are considerably larger as well.[21]

[18]Chapter 5 of NRC discusses the unique role of the Navy in the evolution of the DOD SBIR program into one that focused on procurement, citing John Williams, then the director of the Navy SBIR program. I am grateful for conversations with John Williams near the start of this project.

[19]See https://www.navysbir.com/natconf14f/presentations/3-09-Navy-Comm-Williams.pdf for a presentation by John Williams (in his capacity of the director of the Navy SBIR program) with information about the process to design solicitations. Another presentation, also by John Williams, at https://ndiastorage.blob.core.usgovcloudapi.net/ndia/2005/small_business/williams.pdf as well as National Research Council (2009, p. 176) provides a discussion of the Navy Primes Program, which helped integrate the prime contractors into solicitation design. The Navy SBIR Guidebook for Program Managers, Contracting Officers, and Small Business Professionals provides extensive discussion of how to integrate SBIR programs into an acquisition program (https://bit.ly/2HSKlCi).

[20]These quotes come from the Navy SBIR Phase III Guidebook (https://bit.ly/2HSKlCi) and https://www.navysbir.com/phaseii.htm.

[21]A natural question is whether firms use internal funds to supplement what they are awarded in Phase II from the DOD. I do not have data on such funds at the contract level, and the DOD award is in principle supposed to cover all reasonable costs. A survey by NRC shows that 34% of respondents do claim to use internal funds in research stemming from Phase II (Table 4-10 of National Research Council (2009)), but this additional internal investment is about $300,000 (Table 4-11 of National Research Council (2009)) in the early part of the program. Moreover, this survey evidence is possibly an overestimate, as firms may have included research funding for any project even loosely connected to the specific solicitation. (Given this response includes other follow-on funding, it is difficult to interpret this response precisely.) Regardless, this provides sugges-

Phase III is a delivery contract. As described in the Navy SBIR Phase III Guidebook (p. 8), a benefit of offering a procurement contract through Phase III SBIR is that Phases I and II automatically satisfy the full and open competition requirements from the Defense Federal Acquisition Regulations. Thus, Phase III contracts are only open to contestants who participated in Phases I and II; while firms have the option of dropping out of the contest even if they generate successful research in Phase I, doing so precludes them from the possibility of a major source of revenues through Phase III contracts. While I do not have data on how many successful firms in Phase I choose not to apply to Phase II, this opportunity cost supports anecdotes that such attrition is not common in DOD SBIR.

Similarly, I maintained in the body that the DOD chooses how many firms to admit into the contest, although this assumption does not affect estimation. In reality, firms choose to apply to Phase I. However, the number of applications for each contest generally exceeds the number of Phase I awards, and thus the DOD does limit entry into contests.[22] Together with the fact that many firms participate in this market, this suggests that the DOD's reluctance to expand entry does not stem from lack of interest on the firms' side or lack of availability of firms. Of course, without data on the technical scores of Phase I applicants, I cannot say whether the firms the DOD chose not to invite would have been as competitive as the entrants in that specific contest; the extension to heterogeneous firms captures this possibility.

As mentioned above, given the DOD's and in particular the Navy's focus on procurement, this paper primarily considers the surplus from procurement. However, as with any setting with innovation, there is the possibility of additional benefits from research, for example, via commercialization outside the Navy (either through government or non-government contracts) or indirect improvements of the quality of future projects lead to higher quality of research in future projects. This is especially plausible given a feature of SBIR contracts (inherited from the Congressional mandate given to the SBA) is that firms retain "data rights" to their technologies and in principle can use them outside the specific contest.

As discussed in Section 5.2, I estimate external benefits are up to half the surplus generated in the program, and I benchmark this number here. Survey evidence reported in Section 4.2.3.1 of NRC, for firms participating in the early part of my sample, provide a sense of the magnitude of such additional benefits. Respondents among a survey sent to firms in all DOD SBIR programs report that only 10% of Phase II awards result in commercialization revenues (including a Phase III award) of at least \$5 million; this is remarkably in line with the Phase III award rate in the data set. Almost two-thirds of respondents reported little to no revenue at all.[23] A more recent analysis of Company Commercialization Reports[24] held by the Air Force SBIR Program from 2015–2018 documents that only 7.6% of projects are commercialized in any way (Rask (2019)), consistent with the Phase III contract being the primary source of commercialization. Thus, while there certainly is the

tive evidence that the assumption of $\gamma = 1.75$ is likely an overestimate: not all firms supplement Phase II with internal funds, and those that do still spend on average less than the Phase II contract.

[22]This is confirmed by conversations with officials and by a field in the FPDS that records the "number of offers received" for each contest, which has a median of 7 and mean of 9 for Phase I contracts.

[23]About one-quarter of respondents reported \$1–\$5 million of commercialization: there may be revenues that are at least in part connected to the Phase II research not accounted for by Phase III contracts. However, the expected revenue from such possibilities seem to be less significant than the ones from Phase III.

[24]Company Commercialization Reports are self-reports of any revenues the recipient believes were related to Phases I and II, including a Phase III delivery contract. Given this is self-reported, the direction of the bias is difficult to ascertain. Rask (2019) discussed these issues, so the number should be treated with caution.

possibility of commercialization beyond Phase III, there is not much evidence in adjacent settings that this is very large. Of course, there may be sources of surplus beyond simply commercialization, or participation in SBIR could lead to commercialization that is not reported in these surveys or reports. Overall, the estimates in the extension likely provide a reasonable upper bound.

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Co-editor Aviv Nevo handled this manuscript.

Manuscript received 9 August, 2018; final version accepted 11 February, 2021; available online 2 March, 2021.