Econometrica Supplementary Material

SUPPLEMENT TO "FROM BOTTOM OF THE BARREL TO CREAM OF THE CROP: SEQUENTIAL SCREENING WITH POSITIVE SELECTION": PARTNERSHIPS

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CONSIDER NOW A PARTNERSHIP (whether professional or in private life) composed of $n \geq 2$ agents. This partnership is dissolved if any of its members exits. Let $\phi_i(\theta_i, s_t)$ denote agent i's date-t surplus, where θ_i is his type, distributed according to c.d.f. $F_i(\theta_i)$ and density $f_i(\theta_i)$, with support $[\underline{\theta}_i, \overline{\theta}_i]$ and s_t, as earlier, the date-t state. The distributions F_i are independent. The principal, as we explain below, is a coordinating and taxing entity, and so we assume that $\psi = 0$.

In their celebrated contribution, Myerson and Satterthwaite (1983) looked at the problem of efficiently *forming* a partnership. They derived the mechanism that delivers the highest expected social surplus subject to individual rationality, incentive compatibility, and budget balance. Assuming that virtual valuations $\phi_i(\theta_i, s) - \mu[\frac{1-F_i(\theta_i)}{f_i(\theta_i)}]\frac{\partial \phi_i}{\partial \theta_i}$ are strictly increasing in θ_i for $\mu \in [0, 1]$ and, for simplicity, that the environment is stationary ($s_t = s$ for all t), they showed that a partnership is optimally formed if and only if, for some μ in $(0, 1)$ reflecting the intensity of the budget-balance constraint, the sum of the virtual valuations is positive:

$$\sum_i \left[\phi_i(\theta_i, s) - \mu \left[\frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \right] \frac{\partial \phi_i}{\partial \theta_i} \right] \geq 0. \tag{S.1}$$

Myerson and Satterthwaite's (1983) efficient bargaining corresponds to the case of a *benevolent social planner* eager to maximize expected total surplus but unable to put money on the table. Alternatively, one can look at a *profit-maximizing multi-sided platform*, in which a platform enables agents to interact and thereby enjoy partnership surplus. This latter case admits the same characterization (S.1), except that the coefficient μ is now equal to 1, and thus is larger for a profit-maximizing platform than for a social planner. Finally, a social planner with a positive shadow cost of public funds would also deliver condition (S.1), again with a μ in $(0, 1)$. We will therefore call the allocation defined by condition (S.1), which defines the contours of the partnership, the "Myerson–Satterthwaite allocation," regardless of the identity of the principal (social planner with or without cost of public funds, for-profit platform).

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This supplement, by contrast, looks at the possibility of *dissolving*[1] a partnership, and does so in a dynamic rather than static context. For notational simplicity only, we assume a stationary state ($s_t = s$ for all t) and a finite horizon T.

Let us assume that a mechanism is designed, in which the agents truthfully reveal their type; the allocation specified by the mechanism (partnership/no partnership, transfers) is then implemented. Under commitment, $X^t = 1$ (for all t) if and only if (S.1) is satisfied.

Suppose, by contrast, that in each period t and conditional on $X^{t-1} = 1$ (the partnership has not been dissolved), the principal designs a mechanism for that period. A mechanism defines an allocation x_t for period t, as well as payments p_{it} conditional on the agents' reports.

Suppose that the date-0 outcome delivers the Myerson–Satterthwaite outcome. Because from (S.1), $X^{t-1} = 1$ implies that $\sum_i \phi_i(\theta_i, s) > 0$, and so ex post there are always gains from trade. If the principal is a social planner preoccupied solely by the efficiency of trading, the principal is indifferent as to the vector of transfers and so the utilities from date 1 on (which condition truthtelling at date 0) are indeterminate. To avoid this indifference, we rather study the case of a profit-maximizing principal or that of a social planner that puts at least a bit of weight on his budget (say, due to a shadow cost of public funds) and not only on efficiency. Proposition 1 below applies also if the principal has lexicographic preferences, maximizing first social surplus and, if indifferent, maximizing its revenue. Summing up, we let $X^t[\sum_i \phi_i(\theta_i, s) + \lambda p_{it}]$ denote the principal's flow payoff at date t; the polar cases are that of a for-profit platform ($\lambda \to \infty$) and of a lexicographic social planner ($\lambda \to 0$).

Multi-sided platform: Without loss of generality,[2] one can assume that announcements at date t are made to a machine, which reveals whether the partnership continues, and that the date-t transfers are made only at the end of date T. The question is whether information beyond the minimal information transmission on the continuation of the relationship should be forwarded to the principal or to the agents. As for the principal, we will shortly show that, provided that the date-0 allocation is efficient, the principal can costlessly learn types at date 1 even if he has no information; this is a fortiori the case if he receives information at the end of date 0. On the agents' side, and even if the

[1]The material presented here differs from Cramton, Gibbons, and Klemperer (1987) in two essential aspects (see Segal and Whinston (2014) and the references therein for more recent contributions concerning the impact of status quo outcomes on the efficiency of bargaining). First, we study dynamics and time consistency, while Cramton et al., like Myerson–Satterthwaite, focused on a one-shot trade. Second, Cramton et al. studied a situation in which the agents in the partnerships have initial shares r_i (with $\sum_i r_i = 1$) in the partnership and therefore status quo utility (in our notation) $r_i \phi_i(\theta_i, s)$. The goal is to reshuffle ownership rights toward the agent with the highest surplus $\max_i\{\phi_i(\theta_i, s)\}$. Their striking result is that there exists an efficient mechanism provided that the initial shares are "not too different." Here each agent can destroy the other agents' status quo utility by quitting the relationship.

[2]See Myerson (1982).

mechanism conveys no other information than the continuation decision, each agent i knows from date 1 on information about the other agents that is not available to the principal; as (S.1) indicates, conditional on $X^{t-1} = 1$, a higher θ_i makes agent i more pessimistic about the others' types: in the dyad case, for instance, agent 1 has posterior distribution $f_2(\theta_2|\theta_1) \equiv f_2(\theta_2)/[1 - F_2(\theta_2^*(\theta_1))]$ with support $[\theta_2^*(\theta_1), \overline{\theta}_2]$, where $\theta_2^*(\cdot)$ is a strictly decreasing function of θ_1. We call this information structure the *minimal* (or coarsest) *information structure*.

Proposition 1 (whose proof can be found in Appendix B) considers only minimal information transmission and shows that if the date-0 allocation corresponds to the Myerson–Satterthwaite allocation, the principal at date $t \geq 1$ can design a mechanism that allows him to extract all agents' information at no cost and appropriate the total surplus $\sum_i \phi_i(\theta_i, s)$ forever.[3]

PROPOSITION 1—Time Inconsistency With Multiple Agents: *Consider a multi-period n-agent partnership, which is dissolved whenever an agent quits. Suppose that Assumption* 4 *holds and that the principal's utility is either strictly increasing in money or lexicographic in efficiency and then money. There is no efficient and time-consistent allocation such that the agents learn at the end of each period only whether the relationship continues or not.*

Thus, even though agents consume zero or one unit of the partnership, they need to reveal more than just whether they want to stay in the partnership at the current price. And because each agent reveals fine information about himself, he necessarily learns, from the partnership not being dissolved, information about the other agents' types, even if he does not observe their reports. This impossibility result raises interesting research questions; in particular, we leave for future research the characterization of the time-consistent solution.

PROOF OF PROPOSITION 1: Suppose that the agents tell the truth at date 0 (otherwise the optimal allocation cannot be implemented at date 0) and assume minimal information transmission. Let

$$\xi_i \equiv \sum_{j \neq i} \left[\phi(\theta_j, s) - \mu \left[\frac{1 - F_j(\theta_j)}{f_j(\theta_j)} \right] \frac{\partial \phi_j}{\partial \theta_j} \right] \tag{S.2}$$

denote a random variable, with distribution $H_i(\xi_i)$ on $\mathbb{R}$. Suppose that the Myerson–Satterthwaite allocation defined by (S.1) is time consistent. For any

[3]Like most of the static literature on the elicitation of correlated informations, we assume unlimited transfers and no collusion among the agents. Little is known outside this framework. Robert (1991) showed that unlimited transfers are needed if informations are nearly independent. Crémer (1996) provided results on coalitions in auctions with correlated values when both the auction and the coalition formation must be in dominant strategies.

announcement $\widehat{\theta}_i^0$ at date 0 (not necessarily θ_i, as we allow for a unilateral deviation), the partnership is not dissolved at that date if and only if

$$\xi_i \geq K_i(\widehat{\theta}_i^0),$$

where

$$K_i(\theta_i) \equiv -\left[\phi(\theta_i, s) - \mu\left[\frac{1 - F_i(\theta_i)}{f_i(\theta_i)}\right]\frac{\partial \phi_i}{\partial \theta_i}\right]$$

is a decreasing function of θ_i. Assume that at date 1, the principal and the agents know only that the relationship has continued at date 0. Choose an arbitrary function $w_i(\xi_i, \widehat{\theta}_i^1)$ that is strictly decreasing in ξ_i and strictly decreasing in $\widehat{\theta}_i^1$ and such that agent i breaks even provided that the announcement corresponds to the true conditional distribution of the ξ_i:

$$\int_{K_i(\widehat{\theta}_i^1)}^{\infty} w_i(\xi_i, \widehat{\theta}_i^1)\, dH_i(\xi_i) = 0.$$

Let the principal at date 1 offer the following side-bet mechanism, in which all agents reveal their types and agent i receives a side-bet payment $\widehat{p}_i(\widehat{\theta}_i^1, \xi_i)$, where ξ_i is computed as in (S.2) from the date-1 other reports and $\widehat{\theta}_i^1$ is agent i's report of his own type. Let $\widehat{p}_i(\widehat{\theta}_i^1, \xi_i) = -\infty$ if $\xi_i < K_i(\widehat{\theta}_i^1)$ (this rules out under-reports $\widehat{\theta}_i^1 < \widehat{\theta}_i^0$) and for some $k > 0$,

$$\widehat{p}_i(\widehat{\theta}_i^1, \xi_i) = k w_i(\xi_i, \widehat{\theta}_i^1) \quad \text{if} \quad \xi_i \geq K_i(\widehat{\theta}_i^1).$$

The fact that w_i is strictly decreasing in $\widehat{\theta}_i^1$ rules out over-reports ($\widehat{\theta}_i^1 > \widehat{\theta}_i^0$).

Note that with this mechanism, $\widehat{\theta}_i^1 = \widehat{\theta}_i^0$ is optimal on a stand-alone basis. But $\widehat{\theta}_i^0 = \theta_i$ if the Myerson–Satterthwaite allocation is to be implemented already at date 0. So necessarily $\widehat{\theta}_i^1 = \theta_i$. Furthermore, the loss from lying for any $|\widehat{\theta}_i - \theta_i| > \varepsilon$ (for ε small) goes to infinity as k goes to infinity. Last, link the mechanism with the demand of a payment for staying in the partnership

$$p_i(\widehat{\theta}_i^1) = \phi(\widehat{\theta}_i^1, s) - \varepsilon'$$

for some small ε'; p_i is thus increasing in $\widehat{\theta}_i^1$. The total date-1 payment is then $p_i(\widehat{\theta}_i^1) - \widehat{p}_i(\widehat{\theta}_i^1, \xi_i)$. By taking k to infinity, all agents report their date-0 report ($\widehat{\theta}_j^1 = \widehat{\theta}_j^0$ for all j).

Thus, if the agents tell the truth at date 0 and the Myerson–Satterthwaite allocation is implemented at that date, the principal perfectly extracts the agents' rent from date 1 on. Therefore, if $U_i^0(\theta_i)$ denotes the intertemporal payoff of type θ_i and $x_i^{\mathrm{MS}}(\theta_i)$ denotes her ex ante probability of trade in the Myerson–Satterthwaite allocation (given by (S.1)), then

$$\frac{dU_i^0(\theta_i)}{d\theta_i} = x_i^{\mathrm{MS}}(\theta_i)\frac{\partial \phi_i}{\partial \theta_i}.$$

However, the consideration of repeated small under-announcements $\widehat{\theta}_i^t = \widehat{\theta}_i^0 = \theta_i - \varepsilon$, for ε small, yields

$$\frac{dU_i^0(\theta_i)}{d\theta_i} = x_i^{\mathrm{MS}}(\theta_i)\big[1 + \delta + \cdots + \delta^T\big]\frac{\partial \phi_i}{\partial \theta_i},$$

a contradiction. *Q.E.D.*

REFERENCES

CRAMTON, P., R. GIBBONS, AND P. KLEMPERER (1987): "Dissolving a Partnership Efficiently," *Econometrica*, 55, 615–632. [2]

CRÉMER, J. (1996): "Manipulation by Coalition Under Asymmetric Information: The Case of Groves, Mechanisms," *Games and Economic Behavior*, 13, 39–73. [3]

MYERSON, R. (1982): "Optimal Coordination Mechanism in Generalized Principal–Agent Problems," *Journal of Mathematical Economics*, 10, 67–81. [2]

MYERSON, R., AND M. A. SATTERTHWAITE (1983): "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory*, 29, 265–281. [1]

ROBERT, J. (1991): "Continuity in Auction Design," *Journal of Economic Theory*, 55, 169–179. [3]

SEGAL, I., AND M. WHINSTON (2014): "Property Rights and the Efficiency of Bargaining," Report. [2]

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