

SUPPLEMENT TO “PRICE SETTING WITH MENU COST FOR MULTIPRODUCT FIRMS”: ONLINE APPENDICES  
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APPENDIX B: GENERAL EQUILIBRIUM SETUP

THIS APPENDIX OUTLINES THE GENERAL EQUILIBRIUM SETUP that underlies our approximation. The preferences of the representative agents are given by

$$(31) \quad \int_0^\infty e^{-rt} \left[ U(c(t)) - \alpha \ell(t) + \log\left(\frac{M(t)}{P(t)}\right) \right] dt,$$

where  $c(t)$  is an aggregate of the goods produced by all firms,  $\ell(t)$  is the labor supply,  $M(t)$  is the nominal quantity of money, and  $P(t)$  is the nominal price of one unit of consumption, formally defined below (all variables at time  $t$ ). We use  $U(c) = (c^{1-\varepsilon} - 1)/(1 - \varepsilon)$ , where  $\varepsilon > 1$ . There is a unit mass of firms, indexed by  $k \in [0, 1]$ , and each of them produces  $n$  goods, indexed by  $i = 1, \dots, n$ . There is a preference shock  $A_{k,i}(t)$  associated with good  $i$  produced by firm  $k$  at time  $t$ , which acts as a multiplicative shifter of the demand of each good  $i$ . Let  $c_{k,i}(t)$  be the consumption of the product  $i$  produced by firm  $k$  at time  $t$ . The composite Dixit–Stiglitz consumption good  $c$  is

$$(32) \quad c(t) = \left[ \int_0^1 \left( \sum_{i=1}^n A_{k,i}(t)^{1/\eta} c_{k,i}(t)^{(\eta-1)/\eta} \right) dk \right]^{\eta/(\eta-1)}.$$

For firm  $k$  to produce  $y_{k,i}(t)$  of the  $i$  good at time  $t$  requires  $\ell_{k,i}(t) = y_{k,i}(t)Z_{k,i}(t)$  units of labor, so that  $W(t)Z_{k,i}(t)$  is the marginal cost of production. We assume that  $A_{k,i}(t) = Z_{k,i}(t)^{\eta-1}$  so the (log of) marginal cost and the demand shock are perfectly correlated. We assume that  $Z_{k,i}(t) = \exp(\sigma \mathcal{W}_{k,i}(t))$ , where  $\mathcal{W}_{k,i}$  are standard BM's, independent across all  $i, k$ .

The budget constraint of the representative agent is

$$M(0) + \int_0^\infty Q(t) \left[ \bar{\Pi}(t) + \tau(t) + (1 + \tau_\ell)W(t)\ell(t) - R(t)M(t) - \int_0^1 \sum_{i=1}^n P_{k,i}(t)c_{k,i}(t) dk \right] dt = 0,$$

where  $R(t)$  is the nominal interest rate,  $Q(t) = \exp(-\int_0^t R(s) ds)$  is the price of a nominal bond,  $W(t)$  is the nominal wage,  $\tau(t)$  is the lump sum nominal transfers,  $\tau_\ell$  is a constant labor subsidy rate, and  $\bar{\Pi}(t)$  is the aggregate (net) nominal profits of firms.

The first order conditions (f.o.c.'s) for the household problem are (with respect to  $\ell, m, c, c_{k,i}$ )

$$\begin{aligned} 0 &= e^{-rt} \alpha - \lambda_0 (1 + \tau_\ell) Q(t) W(t), \\ 0 &= e^{-rt} \frac{1}{M(t)} - \lambda_0 Q(t) R(t), \\ 0 &= e^{-rt} c(t)^{-\varepsilon} - \lambda_0 Q(t) P(t), \\ 0 &= e^{-rt} c(t)^{-\varepsilon} c(t)^{1/\eta} c_{k,i}(t)^{-1/\eta} A_{k,i}(t)^{1/\eta} - \lambda_0 Q(t) P_{k,i}(t), \end{aligned}$$

where  $\lambda_0$  is the Lagrange multiplier of the agent budget constraint. If the money supply follows  $M(t) = M(0) \exp(\mu t)$ , then in equilibrium

$$(33) \quad \lambda_0 = \frac{1}{(\mu + r)M(0)} \quad \text{and}$$

$$\text{for all } t: R(t) = r + \mu, W(t) = \frac{\alpha}{1 + \tau_\ell} (r + \mu) M(t).$$

Moreover, the f.o.c.'s for  $\ell$  and for  $c$  give the output equation

$$(34) \quad c(t)^{-\varepsilon} = \frac{\alpha}{1 + \tau_\ell} \frac{P(t)}{W(t)}.$$

From the household's f.o.c.'s of  $c_{k,i}(t)$  and  $\ell(t)$ , we can derive the demand for product  $i$  of firm  $k$ , given by

$$(35) \quad c_{k,i}(t) = c(t)^{1-\varepsilon\eta} A_{k,i}(t) \left( \frac{\alpha}{1 + \tau_\ell} \frac{P_{k,i}(t)}{W(t)} \right)^{-\eta}.$$

In the impulse response analysis of Section 5, we assume  $\mu = 0$ ,  $\tau_\ell = 0$ , and that the initial value of  $M(0)$  is such that  $M(0)/P(0)$ , computed using the invariant distribution of prices charged by firms, is different from its steady state value.

The nominal profit of a firm  $k$  from selling product  $i$  at price  $P_{k,i}$ , given the demand shock is  $A_{k,i}$ , marginal cost is  $Z_{k,i}$ , nominal wages are  $W$ , and aggregate consumption  $c$ , is (we omit the time index)

$$c^{1-\varepsilon\eta} A_{k,i} \left( \frac{\alpha}{1 + \tau_\ell} \frac{P_{k,i}}{W} \right)^{-\eta} [P_{k,i} - W Z_{k,i}].$$

Alternatively, collecting  $W Z_{k,i}$  and using that  $A_{k,i} Z_{k,i}^{1-\eta} = 1$  gives

$$W c^{1-\varepsilon\eta} \left( \frac{\alpha}{1 + \tau_\ell} \frac{P_{k,i}}{W Z_{k,i}} \right)^{-\eta} \left[ \frac{P_{k,i}}{W Z_{k,i}} - 1 \right]$$

so that the nominal profit of firm  $k$  from selling product  $i$  with a price gap  $p_{k,i}$  is

$$(36) \quad W(t)c(t)^{1-\varepsilon\eta}\Pi(p_{k,i}(t)) \quad \text{where}$$

$$\Pi(p_{k,i}) \equiv \left( \frac{\alpha}{1+\tau_\ell} \frac{\eta}{\eta-1} \right)^{-\eta} e^{-\eta p_{k,i}} \left[ e^{p_{k,i}} \frac{\eta}{\eta-1} - 1 \right],$$

where we rewrite the actual markup in terms of the price gap  $p_{k,i}$ , defined in equation (14), that is,  $\frac{P_{k,i}}{WZ_{k,i}} = e^{p_{k,i}} \frac{\eta}{\eta-1}$ . This shows that the price gap  $p_{k,i}$  is sufficient to summarize the value of profits for product  $i$ . Note also that by simple algebra,  $\Pi(p_{k,i})/\Pi(0) = e^{-\eta p_{k,i}} [1 + \eta e^{p_{k,i}} - \eta]$ , which we use below.

Next we show that the ideal price index  $P(t)$ , that is, the price of one unit of the composite good, can be fully characterized in terms of the price gaps. Using the definition of total expenditure (omitting the time index)  $Pc = \int_0^1 \sum_{i=1}^n (P_{k,i} c_{k,i}) dk$ , replacing  $c_{k,i}$  from equation (35), and using the first order condition with respect to  $c$  to substitute for the  $c^{-\varepsilon}$  term gives

$$(37) \quad P = W \left( \int_0^1 \sum_{i=1}^n \left( \frac{P_{k,i}}{WZ_{k,i}} \right)^{1-\eta} dk \right)^{1/(1-\eta)},$$

which is the usual expression for the ideal price index and can be written in terms of the price gaps using  $\frac{P_{k,i}}{WZ_{k,i}} = e^{p_{k,i}} \frac{\eta}{\eta-1}$ .

### B.1. The Firm Problem

We assume that if firm  $k$  adjusts any of its  $n$  nominal prices at time  $t$ , it must pay a fixed cost equal to  $\psi_\ell$  units of labor. We express these units of labor as a fraction  $\psi$  of the steady state frictionless profits from selling one of the  $n$  products, that is, the dollar amount that has to be paid in the event of a price adjustment at  $t$  is  $\psi_\ell W(t) = \psi W(t) \bar{c}^{1-\eta\varepsilon} \Pi(0)$ . To simplify notation, we omit the firm index  $k$  in what follows, and denote by  $p$  the vector of price gaps and by  $p_i$  its  $i$ th component.

The time 0 problem of a firm selling  $n$  products that starts with a price gap vector  $p$  is to choose  $\{\tau, \Delta p\} \equiv \{\tau_j, \Delta p_i(\tau_j)\}_{j=1}^\infty$  to minimize the negative of the expected discounted (nominal) profits net of the menu cost. The signs are chosen so that the value function is comparable to the loss function in equation (1):

$$-\mathbb{E} \left[ \int_0^\infty e^{-rt} \left( \sum_{i=1}^n W(t)c(t)^{1-\varepsilon\eta} \Pi(p_i(t)) \right) dt \right. \\ \left. - \sum_{j=1}^\infty e^{-r\tau_j} W(t) \psi_\ell \Big| p(0) = p \right].$$

Letting  $\hat{\Pi}(p_i) \equiv \Pi(p_i)/\Pi(0)$ , using that equilibrium wages are constant  $W(t)/\bar{W} = e^\delta$ , and using the parameterization of fixed cost in terms of steady state profits  $\psi_\ell = \psi \bar{c}^{1-\eta\epsilon} \Pi(0)$  gives (where the overbars denote steady state values)

$$(38) \quad \mathcal{V}(\boldsymbol{\tau}, \Delta \mathbf{p}, \mathbf{c}; p) \\ \equiv -\bar{W} e^\delta \bar{c}^{1-\epsilon\eta} \Pi(0) \\ \times \mathbb{E} \left[ \int_0^\infty e^{-rt} \sum_{i=1}^n \mathcal{S}(c(t), p_i(t)) dt - \sum_{j=1}^\infty e^{-r\tau_j} \psi \Big| p(0) = p \right]$$

subject to equation (2),  $\Delta p_i(\tau_j) \equiv \lim_{t \downarrow \tau_j} p_i(t) - \lim_{t \uparrow \tau_j} p_i(t)$  for all  $i \leq n$  and  $j \geq 0$ , where  $\mathbf{c} = (c(t))_{t \geq 0}$ , and where the function  $\mathcal{S}: \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}$  gives the normalized per-product profits as a function of aggregate consumption  $c$  and the price gap of the  $i$ th product  $p_i$  as

$$\mathcal{S}(c, p_i) \equiv \left(\frac{c}{\bar{c}}\right)^{1-\eta\epsilon} \hat{\Pi}(p_i) = \left(\frac{c}{\bar{c}}\right)^{1-\eta\epsilon} e^{-\eta p_i} [1 + \eta e^{p_i} - \eta].$$

Expanding  $\mathcal{S}(c, p_i)$  around  $c = \bar{c}$ ,  $p_i = 0$  and using that

$$\left. \frac{\partial \mathcal{S}(c, p_i)}{\partial p_i} \right|_{p_i=0} = \left. \frac{\partial^2 \mathcal{S}(c, p_i)}{\partial p_i \partial c} \right|_{p_i=0} = 0, \quad \left. \frac{\partial^2 \mathcal{S}(c, p_i)}{\partial p_i \partial p_i} \right|_{p_i=0, c=\bar{c}} = \eta(1 - \eta)$$

in equation (38), we obtain

$$\mathcal{V}(\boldsymbol{\tau}, \Delta \mathbf{p}, \mathbf{c}; p) \\ = \bar{W} \Pi(0) \bar{c}^{1-\eta\epsilon} e^\delta \left\{ \mathbf{V}(\boldsymbol{\tau}, \Delta \mathbf{p}; p) - \frac{1}{r} \right. \\ \left. - (1 - \epsilon\eta) \int_0^\infty e^{-rt} \left( \frac{c(t) - \bar{c}}{\bar{c}} + \frac{1}{2} \eta\epsilon \left( \frac{c(t) - \bar{c}}{\bar{c}} \right)^2 \right. \right. \\ \left. \left. - \frac{1}{6} \eta\epsilon(1 + \eta\epsilon) \left( \frac{c(t) - \bar{c}}{\bar{c}} \right)^3 \right) dt \right. \\ \left. - \mathbb{E} \left[ \int_0^\infty e^{-rt} \frac{(2\eta - 1)\eta(\eta - 1)}{6} \left( \sum_{i=1}^n p_i(t)^3 \right) dt \Big| p(0) = p \right] \right. \\ \left. + \mathbb{E} \left[ \int_0^\infty e^{-rt} (1 - \epsilon\eta) \frac{\eta(\eta - 1)}{2} \right. \right.$$

$$\begin{aligned} & \times \left( \frac{c(t) - \bar{c}}{\bar{c}} \sum_{i=1}^n p_i(t)^2 \right) dt \Big|_{p(0) = p} \Big] \\ & + \mathbb{E} \left[ \int_0^\infty e^{-rt} o(\|(p(t), c(t) - \bar{c})\|^3) dt \Big|_{p(0) = p} \right] \Big\}, \end{aligned}$$

where  $\mathbf{V}(\boldsymbol{\tau}, \Delta \mathbf{p}; p)$  is given by equation (1) with  $B = (1/2)\eta(\eta - 1)$ . We can then write

$$\begin{aligned} & \mathcal{V}(\boldsymbol{\tau}, \Delta \mathbf{p}, \mathbf{c}; p) \\ & = Y e^{\delta} \mathbf{V}(\boldsymbol{\tau}, \Delta \mathbf{p}; p) \\ & + \mathbb{E} \left[ \int_0^\infty e^{-rt} o(\|(p(t), c(t) - \bar{c})\|^2) dt \Big|_{p(0) = p} \right] + \iota(\delta, \mathbf{c}), \end{aligned}$$

where the constant  $Y = \bar{W}\Pi(0)\bar{c}^{1-\eta\epsilon}$  is the per-product maximum (frictionless) nominal profits in steady state and where the function  $\iota$  does not depend on  $(\boldsymbol{\tau}, \Delta \mathbf{p})$ .

#### APPENDIX C: NUMERICAL ACCURACY OF THE APPROXIMATIONS

This appendix documents the precision of our analytical results in comparison to the exact numerical solution of a model that uses no approximations. In particular, recall that our solution used a second order approximation of the profit function, no drift in the price gaps, and the impulse response functions were computed using the steady state decision rules, that is, ignoring the general equilibrium feedback effect, which, as stated in Proposition 7, were shown to be second order. This section explores the robustness of our approximations compared to a model that features an asymmetric profit function, the presence of drift, and takes into account the general equilibrium feedback on decision rules following the aggregate shock. To this end, we solve numerically two models that can be computed: one model for the case of  $n = 1$  and one model for the case of  $n = \infty$ . We compare the results with the results produced by our approximations in Section 5. We show that the approximate results are very close to the exact results. The reason, explained in Proposition 7, is that the general equilibrium feedback effect on the decision rules is second order.

##### C.1. *On the Accuracy of the Impulse Responses*

First we describe the case of  $n = 1$ . We solve for the optimal policy of a firm in steady state. This is done using the nonquadratic objective function from the implied CES preferences described in the general equilibrium setup in Online Appendix B. The optimal policy is of the *sS* nature, but given the lack of symmetry on the objective function, the thresholds are not symmetric (i.e., the

distance between the optimal return point and the lowest adjustment threshold, which gives the size of the price increases, is not equal to the distance between the highest thresholds and the optimal return point, which gives the size of price decreases). Another difference with the model in the main body of the paper is that we reported results assuming the price gap had no drift, due to zero inflation and no drift in the real marginal costs. In this section, we assume that marginal cost has a negative drift, due to productivity growth  $Z_t$ , which is equal to 2%. Similar results are obtained by assuming a small inflation rate. Because of these differences, the optimal return point (that is, the optimal price upon resetting) does not need to be equal to zero, that is, the frictionless optimal.<sup>22</sup> A positive drift in the price gap will give the firm a motive to set a positive price gap to hedge against the anticipated depreciation of the sale price. Another motive for the nonzero price gap is that the profit function associated with the CES demand is asymmetric, so that prices below the optimum are more costly (in terms of foregone revenues) than prices above the optimum. Both forces will give the firm a motive for setting a positive price gap upon resetting.

We solve numerically a discrete time model with a very small time period (half a day), where the shock to the (log of the) firm marginal cost follows a discrete time analogue to the Brownian motion (used in the main model) with drift equal to the trend growth of productivity, so that the price gap will have a small drift. The parameterization of the nonlinear model is chosen to be the same as that of the quadratic model (with the noted exception of the small drift in the price gap).

Solving for the impulse response involves the following steps:

- We compute the steady state (invariant) distribution of the price gaps. Since the thresholds are not symmetric, the distribution is not necessarily symmetric either.
- We draw a large number of firms ( $N = 500,000$ ) with price gaps distributed according to the invariant distribution. In the cases of  $n = 1$  and  $n = \infty$ , such an invariant distribution can be derived analytically by solving the ODE of the associated Kolmogorov forward equation.
- We shock the nominal value of each firm's price gap by the same proportion at time  $t = 0$ . This uses the fact that, as in Golosov and Lucas (2007), the equilibrium path of nominal wages and nominal interest rates can be solved independently of the aggregate output and the distribution of prices of the final good (see Online Appendix B).
- (i) We simulate the shocks for each of the  $N$  firms until  $T$  years, keeping track of the price gap of each of the  $j$  firms in each period. We use the decision rules obtained for a given assumed path of future aggregate consumption  $\{c_t\}$ .
- (ii) For each time period between  $t = 0$  and  $t = T$ , we use the cross section of the  $N$  firms's price gap to compute the ideal price index and the associated

<sup>22</sup>Of course, these differences vanish as the adjustment cost  $\psi$  goes to zero.

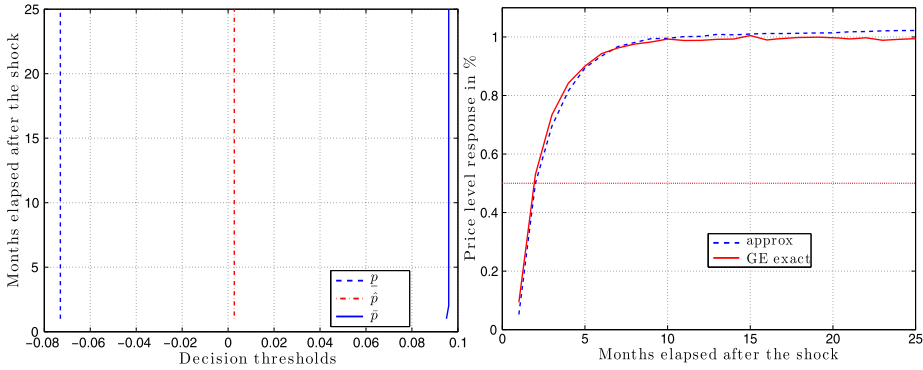


FIGURE 9.—Approximate vs. exact solution after a 1% shock to money supply for  $n = 1$ . The parameter values are  $\eta = 6.8$  (so that  $B = 20$ ),  $\sigma = 0.10$ ,  $\psi = 0.035$ , and  $\rho = 0.02$ ; the productivity drift is 2%; in the approximate model, these produce  $N_a = 1$ ,  $\text{Std}(\Delta p) = 0.10$ . Very similar values are produced by the exact model. The simulation to compute the impulse response function uses a cross section of 500,000 firms.

aggregate consumption. At the end of this procedure, we have a path for the aggregate consumption and a path for the price level:  $\{c'_i, P_i\}_{i=0}^T$ .

- If the assumed aggregate consumption path  $\{c_i\}$  equals the new path  $\{c'_i\}$ , up to numerical tolerance (namely that one changes in the IRF values are smaller than 0.03 percent), we stop the algorithm. If it does not, we let  $\{c_i\} = \{c'_i\}$ , return to (i), and iterate again until convergence.

The left panel of Figure 9 plots the decision rules produced by this procedure for the model with  $n = 1$  for a shock  $\delta = 1\%$ , as considered in the main body of the paper: the threshold levels for the price gaps  $\underline{p}$ ,  $\bar{p}$ , delimiting the inaction range, and the optimal return point  $\hat{p}$ . The vertical axis measures the time elapsed since the shock occurred. These lines are virtually vertical, indicating that the optimal decision rules are virtually overlapping with the steady state ones. The only visible effect appears for  $\bar{p}$  in the periods immediately following the shocks. Much larger shocks are needed, in the order of  $\delta = 10\%$ , to see more actions (still rather small) in the decision thresholds. The reason was given in Proposition 7, where it was shown that the general equilibrium feedback effect on the decision rules is second order.

The right panel of Figure 9 plots the “exact” impulse response function that takes into account the general equilibrium effect, the drift, and the non-quadratic profit function as well as the impulse response produced by our model for the analogue parameterization, which was presented in Figure 4 of Section 5. The two curves appear almost on top of each other and their half-lives are virtually identical. The figures shows that our model provides a very good approximation for a 1% monetary shock (which is not a small shock historically).

C.1.1. *The Decision Rule Along a Transition for the  $n = 1$  Model*

Using equation (38), we write the profit function relative to the steady state frictionless profit. We do this for the  $n = 1$  case. Let  $T$  be the time when consumption reverts to the steady state. For each  $t \in (0, T)$ , there is a triplet, two inaction bands, and an optimal return point that satisfy value matching and smooth pasting for the value functions

$$\bar{v}(c_t, p_t) = \mathcal{S}(c_t, p_t)\Delta + \frac{1}{1 + \Delta r} \mathbb{E}v(c_{t+\Delta}, p_{t+\Delta})$$

and

$$\hat{v}(c_t, p_t) = \max_{\hat{p}} \left( \mathcal{S}(c_t, p_t)\Delta - \psi + \frac{1}{1 + \Delta r} \mathbb{E}v(c_{t+\Delta}, p_{t+\Delta}) \Big| p_t = \hat{p} \right),$$

where

$$\mathcal{S}(c, p) = \left( \frac{c}{\bar{c}} \right)^{1-\eta\varepsilon} e^{-\eta p} [1 + \eta e^p - \eta].$$

C.1.2. *On the Exact Solution of the  $n = \infty$  Case*

Consider a value function defined in the augmented state  $\tilde{V}(p_1, \dots, p_n; \tau)/n$ , where  $(p_1, \dots, p_n)$  is the vector of price gaps and  $\tau$  is the time since last adjustment. The period return for this Bellman equation is  $\sum_{i=1}^n c(t)^a \Pi(p_i(t))/n$ , where  $a = 1 - \eta\varepsilon$  and  $\Pi(p_i)$  is the function in equation (36) deflated by nominal wages. As  $n \rightarrow \infty$ , the law of large numbers allows us to write the period return as

$$\sum_{i=1}^n c(\tau)^a \Pi(p_i(\tau))/n \rightarrow c(\tau)^a \mathbb{E}[\Pi(p(\tau)) | p(0)] = c(\tau)^a F(\tau, p(0)),$$

where  $F(\tau, p(0))$  is a function that gives the expected value of  $\Pi(p(\tau))$  after  $\tau$  periods since resetting each price gap at  $p(0)$ . We can write the steady state Bellman equation as

$$\hat{V} = \max_{p, T} \int_0^T e^{-rt} c^a F(t, p) dt + e^{-rT} [\hat{V} - \psi_\ell],$$

where, abusing notation, we use  $\hat{V}$  to denote the value of the value function right after an adjustment of prices, that is,

$$\hat{V} = \max_{p_1, \dots, p_n} \tilde{V}(p_1, \dots, p_n, 0)/n$$



and  $\psi_\ell = \Pi(0)c^a\psi$ , as assumed in Section B.1. Recall from equation (36) that

$$\Pi(p) \equiv \left( \frac{\alpha}{1 + \tau_\ell} \frac{\eta}{\eta - 1} \right)^{-\eta} e^{-\eta p} \left[ e^p \frac{\eta}{\eta - 1} - 1 \right].$$

Recall that the price gap  $p$  is given by equation (14), so it has the diffusion

$$dp = (\gamma - \mu) dt + \sigma dB.$$

Next define the function  $f(\tau, p)$  as the ratio between the expected profits  $\tau$  periods after resetting a price gap  $p$  and the frictionless profit term  $\Pi(0)$ :

$$\begin{aligned} (39) \quad f(\tau, p) &\equiv \mathbb{E} \left[ \frac{\Pi(p_\tau)}{\Pi(0)} \mid p_0 = p \right] = \frac{F(\tau, p)}{F(0, 0)} \\ &= \eta e^{(1-\eta)p} e^{((\eta-1)(\mu-\gamma) + (\sigma^2/2)(\eta-1)^2)\tau} \\ &\quad - (\eta - 1) e^{-\eta p} e^{(\eta(\mu-\gamma) + (\sigma^2/2)\eta^2)\tau}. \end{aligned}$$

Then the firm's value function, scaled by the frictionless profits  $\Pi(0)c^a$ , solves the Bellman equation

$$(40) \quad \hat{v} = \max_{p, T} \int_0^T e^{-rt} f(t, p) dt + e^{-rT} (\hat{v} - \psi).$$

To match the model moments to the observables, note that to keep the number of adjustments finite, let  $\psi = n\psi_1$ , so that as  $n$  increases, the cost per good stays constant at  $\psi_1$ . Thus as  $n \rightarrow \infty$ , we have that  $N_a = \sqrt{\frac{B\sigma^2}{2\psi_1}}$  and  $\text{Std}(\Delta p) = \sqrt{\sigma^2/N_a}$ . Under the invariant, the distribution of  $y/\bar{y}$  is uniform in  $(0, 1)$  as in Section 5.2. After the shock hits, the distribution is shifted.

### C.1.3. The Decision Rule Along a Transition for the $n = \infty$ Model

Let  $T$  be the time when consumption reverts to the steady state. For each  $t \in (0, T)$ , there is a value function  $v_t$ , an optimal return point  $\hat{p}_t$ , and a time until the next review  $\tau_t$  that solve the Bellman equation (scaled by the steady state frictionless profits  $\Pi(0)c^a$ )

$$(41) \quad \hat{v}_t = \max_{p_t, \tau_t} \int_0^{\tau_t} e^{-rs} \left( \frac{c_{t+s}}{\bar{c}} \right)^{1-\eta\epsilon} f(s, p_t) ds + e^{-r\tau_t} (\hat{v}_{t+\tau_t} - \psi)$$

and use  $\hat{v}_T = \hat{v}$ , that is, the steady state value function. For a given guess of the aggregate consumption profile  $c_t$ , the value functions can be solved backwards.

We first determine which firms will adjust prices immediately as the shock arrives. Let  $\hat{p}$  be the price gap chosen by firms in the steady state and let  $\hat{T}$  be

the time until the next adjustment in the steady state. After a monetary shock, all firms find their price gaps reduced by  $\delta$ , so their value function corresponds to a function on which the last price gap was reset at  $\hat{p} - \delta$ . This determines a new planned date for adjusting prices:  $\tau_0$ , which by the first order condition with respect to  $\tau_t$  in equation (41) solves

$$(42) \quad f(\tau_0, \hat{p} - \delta) - r(\hat{v}_{\tau_0} - \psi) = 0.$$

After computing the value functions  $v_t$ , one can thus determine the new times until adjustment  $\tau_0$ . All firms that adjusted prices  $t$  periods ago with  $t \in (\tau_0, \hat{T})$  will immediately adjust prices. Thus the fraction of firms that will jump on impact after the monetary shock is given by  $\frac{\hat{T} - \tau_0}{\hat{T}}$ . All other firms will adjust when the age of their price reaches  $\tau_0$  and, at that point, use the decision rules  $\{\tau_t, p_t\}$  prescribed by equation (41). Numerically, as occurred for the  $n = 1$  case, the left panel of Figure 10 shows that for a shock  $\delta = 1\%$ , such as those considered in the main text, these rules are virtually identical to the rules of the steady state. The main difference compared to the approximate rule derived in the main text of the paper concerns the size of the impact effect, which the model slightly underestimates due to the fact that the rule in the paper uses  $\hat{T}$  as the optimal adjustment date whereas the exact model prescribes  $\tau_0$ . The right panel of Figure 10 shows that the difference between the approximate and the exact impulse response is tiny.<sup>23</sup>

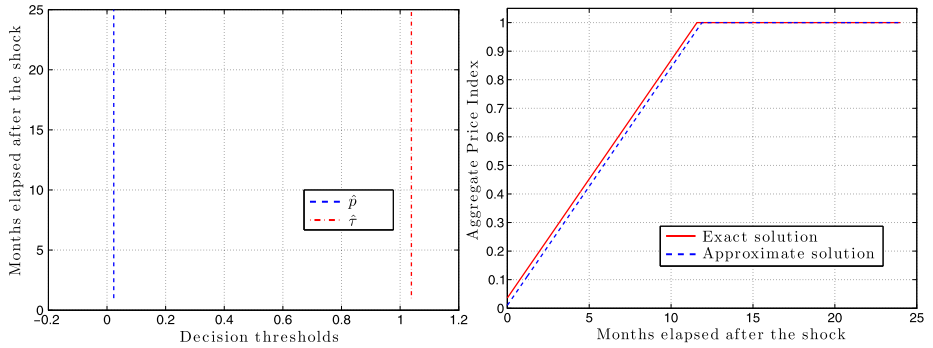


FIGURE 10.—Approximate vs. exact solution after a 1% shock to money supply for  $n = \infty$ . The parameter values are  $\eta = 6.8$  (so that  $B = 20$ ),  $\sigma = 0.10$ ,  $\psi = 0.035$ , and  $\rho = 0.02$ ; the productivity drift is 2%; in the approximate model, these produce  $N_a = 1$  and  $\text{Std}(\Delta p) = 0.10$ . Very similar values are produced by the exact model. The simulation to compute the impulse response function uses a cross section of 500,000 firms.

<sup>23</sup>Likewise, Figure 7 in Golosov and Lucas (2007) compares an IRF that includes the general equilibrium feedback effect with an IRF computed ignoring this effect, that is, keeping the firms decision rules constant. The authors conclude that “[e]vidently, the approximation works very

## APPENDIX D: EVIDENCE ON SYNCHRONIZATION OF PRICE CHANGES

We now turn to a detailed discussion of the evidence. [Lach and Tsiddon \(1996\)](#) documented more synchronization of price changes of different products within a store than of products of the same type across stores:

Our analysis leads us to conclude that Figure 1 is the result of staggering across price-setters, while price changes of different products are synchronized (non-staggered) within the store. That is, the data exhibit across-stores staggering and within-store synchronization in the timing of price changes.

This is the first piece of evidence that motivated interest in the hypothesis of a common fixed cost, which the authors proposed.<sup>24</sup>

[Levy, Bergen, Dutta, and Venable \(1997\)](#) and [Dutta, Bergen, Levy, and Venable \(1999\)](#) studied the pricing behavior of four large U.S. supermarket chains and a large U.S. drugstore company. They found evidence of large synchronization of price changes of a given type of good in both types of stores (for example, "...96.4 percent of the weekly price changes in the supermarket chains are done during a two-day (Sunday and Monday) period, on a regular basis"), as well as documenting that a large component of the menu cost is common to the pricing of all products of a given type (for example "[m]any components of menu costs we document in this paper are indeed store-specific, rather than product-specific"). See Section 5 and Table 4 in [Dutta et al. \(1999\)](#) for a summary of their findings on synchronization across the two papers. [Fisher and Konieczny \(2000\)](#) studied daily the prices of Canadian newspapers. They proposed an index of synchronization of price changes and found that price changes of papers owned by the same firm are synchronized. On the other hand, independently owned newspapers do not appear to change their prices together. [Chakrabarti and Scholnick \(2007\)](#) used price data from online book retailers (Amazon.com and Barnes and Noble.com) to compute the index proposed by [Fisher and Konieczny \(2000\)](#) and documented that the degree of price synchronization is at least as high if not higher than that found for Canadian newspapers.

[Midrigan \(2009\)](#) (see Section 2.C and Appendix 2) used scanner data from Dominick's to document that price changes in narrow product categories within a store are synchronized (see also page 1160 of [Midrigan \(2011\)](#)). He

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well for the effects of a one-time shock, even a large one." Likewise, small general equilibrium feedback effects are found in [Alvarez, Lippi, and Paciello \(2013\)](#) and in the Online Appendix C, where we solve numerically a model with the general equilibrium structure of [Golosov and Lucas](#) using values for the idiosyncratic shocks and adjustment cost values that are close to the values used in this paper for  $n = 1$ .

<sup>24</sup>The authors analyzed monthly price changes over 18 months for 21 product groups in 55 stores that sell wine products and 25 that sell meats products. In Section IV (Within-Store Synchronization), the authors rejected the hypothesis that the proportion of price changes within a store is the result of price changes binomially independently distributed across products. In particular, they showed a relatively large fraction of months where the stores either changed no prices or changed the prices of all of these products; see Tables 5 and 7.

also showed that the hazard of a good's price change depends not only on that good's desired price change (proxied by changes in costs and the deviation of the good's price from that of its competitors), but also on the desired price change of other goods within a store. [Bhattarai and Schoenle \(2011\)](#) studied U.S. firm's producer price index (PPI) prices from the BLS. They found evidence for substantial synchronization of individual price adjustment decisions within the firm by estimating logit regressions on price changes of a firm's product, which included other firms' product changes as well as other firms' price changes of the same types of products. They documented that synchronization within the firm is much stronger than within the industry.<sup>25</sup>

[Cavallo \(2010\)](#) used scraped daily data from online grocery retailers stores in four developing countries. He found that there is daily synchronization in the timing of price changes among closely competing goods within each store. He labeled this "synchronization within the aisle." His measure compares the actual distribution of price changes of closely related products with the distribution that would arise if price changes across products within the aisle were independent (and were to have the same frequency of price changes as in the data). In words, he found that there is a higher frequency of many products changing their prices in a day or very few products changing their prices in a day relative to what is expected from price changes that are independently distributed across products.

A related, but different, phenomenon is the one of "uniform pricing rules," where a retailer decides to charge the same price for all variants of a product, regardless of demand and possible cost changes. Obviously, a firm that follows this practice will have perfect synchronized price changes in all the variants of the product under uniform pricing rules. [Anderson, Jaimovich, and Simester \(2012\)](#) documented this practice and reported a variety of statistics for a very large U.S. retail store.

#### APPENDIX E: CORRELATION, DRIFT, AND CROSS-PRODUCTS

Our paper studies the problem of a firm that controls an  $n$ -dimensional vector of price gaps  $p \in \mathbb{R}^n$  subject to a common menu cost  $\psi$ . Assuming that the individual price gaps  $p_i$  had no drift and were mutually uncorrelated, and that the objective function was to minimize the square of the price gaps, we showed that the  $n$ -dimensional state of the problem could be collapsed into a single state variable  $y = \sum_{i=1}^n p_i^2$ , measuring the squared norm of the price gaps. This delivered a lot of analytical tractability.

This appendix extends the model to the possibility that the price gaps  $p_i$  are mutually correlated and/or that the gaps have a common drift and that the

<sup>25</sup>We note that the results are obtained despite the fact that their measure of the number of product that a firm produces is noisy: they used the number of product that are sampled by the BLS for a given firm.

objective function has nonzero cross-partial terms (all equal to a common constant). These extensions impair the symmetry of the problem so that one might fear losing the tractability that was obtained before. Surprisingly (at least to us), we show that despite the apparent complexity of these extensions, the modified problem remains tractable: instead of the single state variable  $y$  defined above, the state of the problem with either drift, correlation, cross-product, or any combination of them now includes only one additional variable that measures the sum of the coordinates of the vector, namely  $z = \sum_{i=1}^n p_i$ . Importantly, this not only allows us to solve numerically the steady state problem of the firm for any  $n \geq 1$ , but also to compute the impulse response, since the effect on the aggregate price level can be obtained by keeping track of  $z$  for each firm.

For ease of exposition and because its implications are more important to judge the robustness of the benchmark case, the next section shows how to solve the firm problem when the price gaps are correlated but there is no drift. The value function and decision rules for the problem are presented in Section E.1. Section E.2 illustrates the cross-section implications of an economy where firms follow these decision rules, presenting the implications for the cross-section distribution of price changes—a statistic that is central to the empirical analyses of the price-setting problem. Section E.3 moves on to characterize how the aggregate economy will respond to monetary shocks. We show how the response of the economy to a monetary shock varies as we change (i) the number of goods  $n$  sold by each firm and (ii) the correlation  $\rho$  between the shocks of the price gaps of the firm. Finally, Section E.4 shows how to further extend the firm problem to include a common drift in all price gaps, for example, inflation, and Section E.5 shows how to include nonzero cross-partial derivatives (between the price gaps of the different goods) in the instantaneous return function.

### E.1. *The Case of Correlated Price Gap*

We assume that the price gaps are diffusions that satisfy

$$(43) \quad \mathbb{E}[dp_i(t)] = 0 dt, \quad \mathbb{E}[dp_i^2(t)] = \hat{\sigma}^2 dt, \quad \text{and} \\ \mathbb{E}[dp_i(t) dp_j(t)] = \rho \hat{\sigma}^2 dt$$

for all  $i = 1, \dots, n$  and  $j \neq i$ , and for two positive constants  $\hat{\sigma}^2$  and  $\rho$ . Then we can write that each price gap follows

$$(44) \quad dp_i(t) = \bar{\sigma} d\bar{W}(t) + \sigma d\mathcal{W}_i(t) \quad \text{for all } i = 1, \dots, n,$$

where  $\bar{\mathcal{W}}$ ,  $\mathcal{W}_i(t)$  are independent standard BM's, so that  $\hat{\sigma}^2 = \bar{\sigma}^2 + \sigma^2$  and the correlation parameter is  $\rho = \frac{\bar{\sigma}^2}{\bar{\sigma}^2 + \sigma^2}$ . Define

$$(45) \quad y(t) = \sum_{i=1}^n p_i^2(t) \quad \text{and} \quad z(t) = \sum_{i=1}^n p_i(t).$$

Using Ito's lemma,

$$\begin{aligned} dy(t) &= [n\sigma^2 + n\bar{\sigma}^2] dt \\ &\quad + 2\sigma \sum_{i=1}^n p_i(t) d\mathcal{W}_i(t) + 2\bar{\sigma} \left[ \sum_{i=1}^n p_i(t) \right] d\bar{\mathcal{W}}(t) \end{aligned}$$

and

$$dz(t) = n\bar{\sigma} d\bar{\mathcal{W}}(t) + \sigma \sum_{i=1}^n d\mathcal{W}_i(t).$$

This implies that

$$(46) \quad \begin{aligned} \mathbb{E}[dy(t)]^2 &= 4\sigma^2 \left( \sum_{i=1}^n p_i^2(t) \right) dt + 4\bar{\sigma}^2 \left( \sum_{i=1}^n p_i(t) \right)^2 dt \\ &= 4\sigma^2 y(t) dt + 4\bar{\sigma}^2 z(t)^2 dt, \end{aligned}$$

$$(47) \quad \mathbb{E}[dz(t)]^2 = \sigma^2 n dt + \bar{\sigma}^2 n^2 dt,$$

$$(48) \quad \begin{aligned} \mathbb{E}[dy(t) dz(t)] &= 2\sigma^2 \left( \sum_{i=1}^n p_i(t) \right) dt + 2n\bar{\sigma}^2 \left( \sum_{i=1}^n p_i(t) \right) dt \\ &= 2(\sigma^2 + n\bar{\sigma}^2) z(t) dt. \end{aligned}$$

Thus define the diffusions

$$(49) \quad dy(t) = n[\sigma^2 + \bar{\sigma}^2] dt + 2\sigma\sqrt{y(t)} d\mathcal{W}^a(t) + 2\bar{\sigma}z(t) d\mathcal{W}^c(t),$$

$$(50) \quad dz(t) = n\bar{\sigma} d\mathcal{W}^c(t)$$

$$+ \sqrt{n}\sigma \left[ \frac{z(t)}{\sqrt{ny(t)}} d\mathcal{W}^a(t) + \sqrt{1 - \left( \frac{z(t)}{\sqrt{ny(t)}} \right)^2} d\mathcal{W}^b(t) \right],$$

where  $(\mathcal{W}^a, \mathcal{W}^b, \mathcal{W}^c)$  are three standard independent BM's.

Note that if  $\bar{\sigma} = 0$ , then  $z$  does not affect  $y$  and, hence, the state of the problem is  $y$ . Also note that if  $\bar{\sigma} > \sigma = 0$ , then the specification coincides with

a 1-good model ( $n = 1$  and no correlation), so the state can be taken to be  $y$  too. In the case where  $\sigma$  and  $\bar{\sigma}$  are positive, the state of this problem will be the pair  $(y, z)$ . We offer some preliminary characterization of the problem.

(i) We require that

$$(51) \quad z(t)^2 \leq ny(t).$$

If  $z(0)^2 \leq ny(0)$  and  $\{y(t), z(t)\}_{t>0}$  generated by equation (49) and equation (50) satisfy this inequality. To see why, let us consider the case where  $z = \sqrt{yn}$  and use Ito's lemma to compute  $d(yn)$  and  $d(z^2)$ . At this point, the two process have the same drift and diffusion. A similar argument follows at  $z = -\sqrt{yn}$ , where the diffusions' coefficients differ only in their sign. Thus  $z^2/(yn)$  stays in  $[0, 1]$ .

(ii) The diffusions defined by equation (49) and equation (50) satisfy equation (46), equation (47), and equation (48).

(iii) The value function has arguments  $(y, z)$ , denoted by  $v(y, z)$ . Alternatively,  $V(p_1, p_2, \dots, p_n) = v(\sum_{i=1}^n p_i^2, \sum_{i=1}^n p_i)$ .

(iv) The value function is symmetric in  $z$  around zero, so  $v(y, z) = v(y, -z)$  for all  $(y, z) \in \mathbb{R}_+^2$ . This follows because clearly  $V(p) = V(-p)$ .

(v) The optimal policy is to have an inaction region  $\mathcal{I} = \{(y, z) : 0 \leq \bar{y}(z)\}$  for some function  $\bar{y}(z)$ .

(vi) At the threshold, we have value matching and if the function is  $C^1$  in the entire domain, we have smooth pasting:

$$(52) \quad V_i(p) = 0 \quad \text{and} \quad V(p) = V(0) + \psi \quad \text{if} \quad \bar{y}\left(\sum_{i=1}^n p_i\right) = \sum_{i=1}^n p_i^2 \quad \text{or}$$

$$(53) \quad v(\bar{y}(z), z) = v(0, 0) + \psi \quad \text{and} \quad v_1(\bar{y}(z), z)2z + nv_2(\bar{y}(z), z) = 0.$$

Differentiating value matching with respect to (w.r.t.)  $z$  and comparing with smooth pasting, we have

$$(54) \quad v_1(\bar{y}(z), z)\bar{y}'(z) + v_2(\bar{y}(z), z) = 0 \quad \text{all} \quad z \\ \implies v_1(\bar{y}(z), z)\left[\bar{y}'(z) - \frac{2z}{n}\right] = 0.$$

We conjecture that for all  $z$ , we have  $v_1(\bar{y}(z), z) = 0$  and, hence,  $v_2(\bar{y}(z), z) = 0$  too. These are required if  $v$  is  $C^1$  in the entire domain.

(vii) The threshold  $\bar{y}(z)$  satisfies  $\bar{y}(z) = \bar{y}(-z) > 0$  for all  $z > 0$  and  $\bar{y}'(z) = -\bar{y}'(-z)$  for all  $z > 0$ .

There are two special cases of interest for which we can solve for  $\bar{y}(z)$ . One is when  $\bar{\sigma} = 0 < \sigma$  so the correlation is zero, which is our benchmark case for which we have an analytical solution of the problem. In this case,  $\bar{y}(z)$  does not depend on  $z$  and  $\bar{y}'(z) = 0$  for all  $z$ . The second case corresponds to perfect

correlation, that is, when  $\sigma = 0 < \bar{\sigma}$ . This case corresponds to the case with only one product, since for any history where  $p_i(0) = 0$  for all  $i = 1, \dots, n$ , we have  $p_i(t) = p(t)$  and  $y(t) = np(t)^2 = (1/n)[np(t)]^2 = z(t)^2/n$ . In this case, only the values of  $\bar{y}(z)$  at the edges of the state space can be achieved. The two diffusions give

$$(55) \quad dy(t) = n\bar{\sigma}^2 dt + 2z(t)\bar{\sigma} d\mathcal{W}^c(t),$$

$$(56) \quad dz(t) = n\bar{\sigma} d\mathcal{W}^c(t),$$

where  $\mathcal{W}^c$  is standard BM. If  $y = z^2/n$ , we can write this also as

$$(57) \quad dy(t) = (n\bar{\sigma}^2) dt + 2\sqrt{y(t)(n\bar{\sigma}^2)} d\mathcal{W}^c(t),$$

which coincides with the law of motion of the case of one product with an innovation variance of  $n\bar{\sigma}^2$ . The common optimal value for  $\bar{y}$  at these two points can be found by solving the problem with no correlation with the same  $r, B$ , and  $\psi$ , but with  $(\sigma', n') = (\sqrt{n}\bar{\sigma}, 1)$ .

### E.1.1. The Case of a Large Number of Products

In this section, we analyze the limit case as  $n \rightarrow \infty$  in the presence of correlation. The process for the average square gap is the sum of two processes obtained in the benchmark case of no correlation. One process corresponds to the case of  $n = 1$  with instantaneous variance  $\bar{\sigma}^2$ ; the other process is the deterministic process that corresponds to the case of  $n = \infty$  with drift  $\sigma^2$ . Let us define

$$\tilde{y}(t) \equiv \frac{y(t)}{n} = \frac{1}{n} \left[ \sum_{i=1}^n \sigma^2 W_i(t)^2 + \bar{\sigma}^2 \bar{W}(t)^2 + 2\sigma\bar{\sigma} W_i(t)\bar{W}(t) \right]$$

with  $\tilde{y}(0) = 0$ . Thus

$$\begin{aligned} \text{Var}[\tilde{y}(t)|\tilde{y}(0) = 0] &= \frac{1}{n^2} \left[ \sum_{i=1}^n \sigma^4 E[W_i(t)^4] + \bar{\sigma}^4 E[\bar{W}(t)^4] \right. \\ &\quad \left. + 2\sigma^2\bar{\sigma}^2 E[W_i(t)^2] E[\bar{W}(t)^2] \right] \\ &\quad + \frac{n(n-1)}{n^2} \bar{\sigma}^4 E[\bar{W}(t)]^4 \\ &= \frac{t^2}{n} [\sigma^4 + \bar{\sigma}^4 + 2\sigma^2\bar{\sigma}^2] + \frac{t^2(n-1)}{n} \bar{\sigma}^4, \end{aligned}$$



so we can write

$$(58) \quad d\tilde{y}(t) = [\sigma^2 + \bar{\sigma}^2] dt + 2\bar{\sigma}\tilde{z}(t) d\mathcal{W}(t) \quad \text{and} \quad dz(t) = \bar{\sigma} d\mathcal{W}(t)$$

or

$$(59) \quad \tilde{y}(t) = \sigma^2 t + y_1(t) \quad \text{where} \quad dy_1 = \bar{\sigma}^2 dt + 2\bar{\sigma}\sqrt{y_1} d\mathcal{W}.$$

Thus we can consider a problem where the objective function depends on  $y_1$  and time since last adjustment:  $B(y_1 + \sigma^2 t)$  and the law of motion of  $y_1$  is given by equation (59).

### E.1.2. Discrete Time Approximation

Let  $\Delta > 0$  be the length of the time period. We approximate the pair of diffusions as

$$(60) \quad y' = \mathcal{Y}(y, z, e) \\ \equiv \max \{ y + n[\sigma^2 + \bar{\sigma}^2]\Delta + 2\sqrt{\Delta}\sigma\sqrt{y}e^a + 2\sqrt{\Delta}\bar{\sigma}ze^c, 0 \},$$

$$(61) \quad z' = \mathcal{Z}(y', y, z, e) \\ \equiv \max \left[ -\sqrt{y'n}, \min \left\{ z + n\sqrt{\Delta}\bar{\sigma}e^c \right. \right. \\ \left. \left. + \sqrt{n\Delta}\sigma \left[ \frac{z}{\sqrt{ny}}e^a + \sqrt{1 - \left( \frac{z}{\sqrt{ny}} \right)^2} e^b \right], \sqrt{ny'} \right\} \right],$$

where  $e = \{e^a, e^b, e^c\}$  is a vector of three independent random variables, with zero mean and unit variance. An example is three binomials, each taking the values  $\pm 1$  with probability  $1/2$ . The set of binomial shocks is denoted by  $E = \{e^i \in \{-1, 1\} \text{ for } i = a, b, c\}$ . We let  $E$  denote the set of innovations and for notational purposes, we use  $F$  for its c.d.f. The max and min operators in the previous definitions ensure that  $y$  stays positive and that  $z^2 \leq ny$ . Let the state space be  $\mathbb{S} = \{(y, z) : y \geq 0, -\sqrt{yn} \leq z \leq \sqrt{yn}\} \subset \mathbb{R}_+ \times \mathbb{R}$ .

To simplify the notation, we let  $\mathcal{S} : \mathbb{S} \times E \rightarrow \mathbb{S}$  map  $(y, z, e)$  into  $(y', z')$  via

$$(62) \quad (y', z') = \mathcal{S}(y, z, e) \equiv (\mathcal{Y}(y, z, e), \mathcal{Z}(\mathcal{Y}(y, z, e), y, z, e)).$$

The discrete time Bellman equation becomes, for all  $(y, z) \in \mathbb{S}$ ,

$$(63) \quad v(y, z) = \min \left\{ \psi + v(0, 0), \Delta B y + e^{-\Delta r} \int_{e \in E} v(\mathcal{S}(y, z, e)) dF(e) \right\}.$$

We solve  $v(y, z)$  by repeated iterations in a grid included in  $\mathbb{S}$ . We use the value function for the uncorrelated case (with the same volatility for each price

gap, that is,  $\sqrt{\sigma^2 + \bar{\sigma}^2}$ ) as the initial function. To compute the expected value of the value function in each iteration, we need to be able to evaluate the value function outside the grid points. Let us denote a set of  $N$  grid points in  $\mathbb{S}$  by  $\mathbb{G}$ . To do so, in each iteration, we fit a polynomial in  $(y, z)$  to the grid points that are in the inaction region. We use the polynomial

$$(64) \quad v(y, z) = \beta_0 + \beta_1 y + \beta_2 y^2 + \beta_3 y^3 + \beta_4 z^2 + \beta_5 z^4 \\ + \beta_6 y z^2 + \beta_7 y^2 z^2 + \beta_8 y^3 z^2$$

for  $(y, z) \in \mathbb{G}$ . Note that this polynomial imposes symmetry w.r.t.  $z$  and includes the third order approximation for the case of no correlation. For the case with no correlation, we have found that this functional form gives very accurate results. The coefficients of this polynomial are fitted to the grid points  $(y_i, z_i)$  for which  $v(y_i, z_i) < \max_{j \in \mathbb{G}} v(y_j, z_j)$ , that is, they are fitted to the inaction set.

We display a numerical example of the value function and policies for the following parameters. We measure time in years and let the real discount rate be 5% or  $r = 0.05$ , use a markup of about 15%, which implies  $B = 20$ , and use a volatility of each price gap of 10% with a pairwise correlation of  $1/2$ , so  $\sigma = \bar{\sigma} = 0.05$ . The menu cost is 4% of frictionless profits per good, so  $\psi/n = 0.04$ . We solve the model for daily periods, so  $\Delta = 1/365$ . We display results for the case of  $n = 10$  products per firm in Figure 7 in the main body of the paper: the left panel plots the value function as a function of  $y$  and  $z$ ; the right panel plots the decision rule of the firm. This figure plots the level of the value function in all the grid points we have used to compute it. The values of the value function for which control (i.e., price adjustment) is optimal are marked with bold dots. The feasible state space for the firm is given by the region in  $y, z$  inside the parabola. For each  $z$ , the value function has a similar shape as that for the case of no correlation. Fixing  $y$ , the value function is decreasing in  $|z|$ . This is because higher  $|z|$  implies higher conditional variance of  $y$  and, hence, higher option value. As anticipated, the function  $\bar{y}(z)$  is symmetric around  $z = 0$  and increasing for larger values of  $|z|$ . The fact that  $\bar{y}$  is increasing in  $|z|$  reflects the option value effect of  $z$  just described. For comparison, we plot a horizontal line with the value  $\bar{y}$  for the case of uncorrelated price gaps but with the same innovation variance per unit of time of each of the price gaps, that is, with  $\sigma^2 + \bar{\sigma}^2$  as well as the case with perfectly correlated price gaps. While the inaction set can be summarized in an  $\mathbb{R}^2$  space, we emphasize that the state of the problem is  $n$ , which can be much higher, for instance, it is  $n = 10$  for this example.

## E.2. Cross-Section Implications With Correlated Shocks

We use the decision rules described above to produce the invariant distribution of a cross section of firms using simulations. The model parameter-

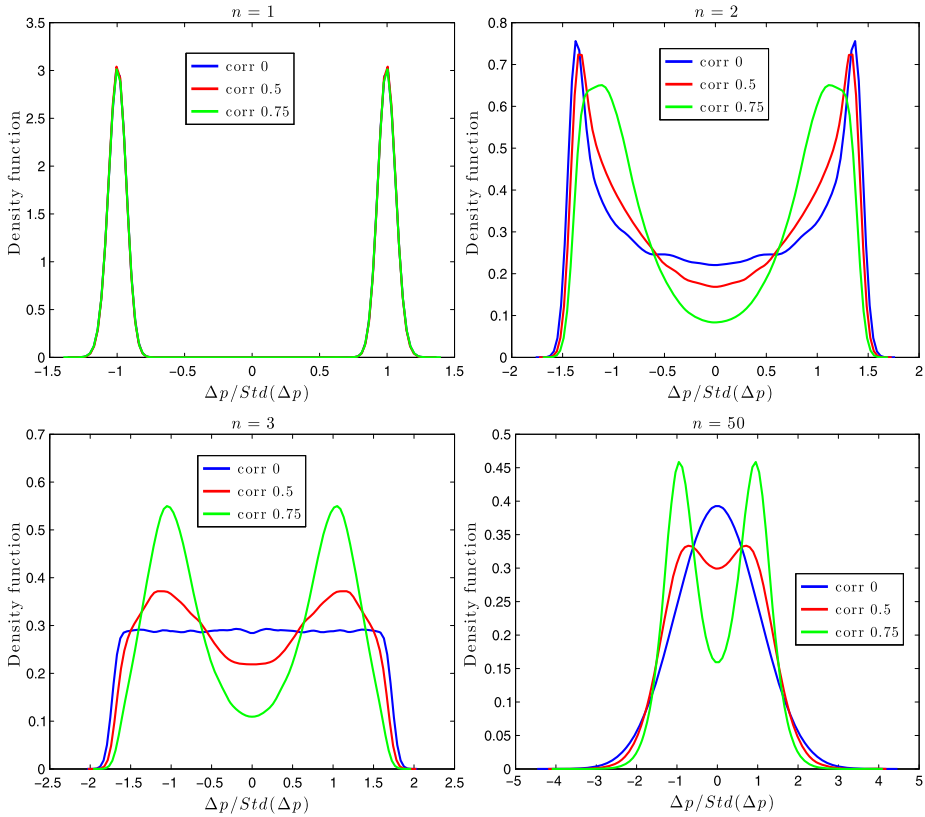


FIGURE 11.—Distribution of price changes:  $\Delta p_i$ . All distributions have been standardized to have  $\text{Std}(\Delta p) = 0.1$ .

ization is close to that used in the main body of the paper, that is, it produces a frequency of adjustments per year  $N_a = 1.3$  and a standard deviation  $\text{Std}(\Delta p) = 0.11$ . Figure 11 plots the standardized distribution of price changes  $w(\Delta p)$  for different values of  $n = 1, 2, 3, 50$  and different levels of correlation between the shocks:  $\rho = 0$ , our baseline case, as well as  $\rho = 0.5$  and  $\rho = 0.75$ .

The marginal distribution of price changes is obtained as follows. First we solve for the optimal decision rules, which gives us the function  $\bar{y}(\cdot)$ . Then we simulate a discrete time version of the  $n$ -dimensional process  $\{p_t\}$  described equation (44), and use the optimal policy to stop it the first time it reaches the adjustment region, upon which the  $n$  price gaps are set to zero. In particular, each draw of the joint  $n$ -dimensional distribution is obtained by starting  $p_{0,i} = 0$  for all  $i = 1, \dots, n$ , simulating  $\{p_t\}$  and the associated  $y_t, z_t$  defined by equation (66). Letting  $\tau$  be the first time that  $y_\tau \geq \bar{y}(z_\tau)$ , we obtain each

of the  $n$  price changes as  $\Delta p_i = -p_{\tau,i}$ . We set the length of the time period  $\Delta = 1/(2 \times 365)$ , that is, half a day, and simulate 50,000 price changes of the  $n$  products.<sup>26</sup> We represent the outcome of the simulations by fitting a smooth kernel density to the simulated data.

The first panel contains the distribution of price changes for the case of one product, that is,  $n = 1$ . In this case,  $\bar{y}$  is flat and the correlation should make no difference. The distribution of price changes should be degenerate, but given that we simulate a discrete time process, albeit with a small time period, the price changes are distributed tightly, but not degenerately, around two values. This is included as a check of the procedure and to control the difference that is due to the discretization of the model. The case of  $n = 2$  shows that as the correlation increases, the distribution has more mass for small price changes. Not surprisingly, adding correlation to the shocks makes the  $n = 2$  case closer to the  $n = 1$  case, a feature that is important for both its empirical plausibility (i.e., the comparison with empirical distribution of price changes) and for the predicted effect of monetary shocks. The case of  $n = 3$  is particularly revealing, since, for zero correlation, the distribution is uniform, but as the correlation is positive, the density decreases to have a minimum at zero and two maxima at a high values of the absolute value, as in the case of  $n = 1$ . The case of  $n = 50$  is also informative because with zero correlation, the marginal distribution of price changes is essentially normal. Nevertheless, with positive correlation the distribution of price changes remains bimodal, with a minimum of its density at zero.

Interestingly, the simultaneous near normality and bimodality (or the dip on the density of the distribution on a central value of price changes) that is displayed in the figure for  $n = 50$  is apparent in several data sets—such as [Midrigan \(2009\)](#), who used scanner AC Nielsen data for the United States (see Figure 1, bottom two panels) and [Wulfsberg \(2010\)](#), who used Norway’s consumer price index (CPI) data (see Figure 4)—and has been explicitly tested and estimated by [Cavallo and Rigobon \(2010\)](#) using online supermarket data for 23 countries.

### E.3. *Impulse Responses With Correlated Shocks*

In this section, we compute the impulse response function of the price level to a once and for all shock to the money supply. We investigate the effect of correlation on the results. In particular, for a shock of the same size and for several values of  $n$ , we compare the IRF of prices for correlations  $\rho = 0$ ,  $\rho = 0.5$ , and  $\rho = 0.75$ . We stress that to solve for the IRF for any  $n$ , we only need to

<sup>26</sup>We simulate half as many, and then we use symmetry to reflect it and obtain a sample twice as large, a standard importance sampling procedure.

keep track of a two-dimensional object, which makes the procedure computationally feasible.

We obtain the IRF as follows. We start with the optimal steady state decision rules, summarizing them by the function  $\bar{y}(\cdot)$ .

- We simulate a discrete time version of the process for  $\{y_t^j, z_t^j\}$  for a large number of firms, say  $j = 1, \dots, M$ . We use  $M = 500,000$ .

- We let  $t = 0$  denote the first period,  $t = T$  denote the period where the aggregate monetary shock of size  $\delta$  occurs, and  $t = T + T'$  denote the last period of the simulation.

- The first  $T$  periods discrete time versions of the firms' state are simulated so that at  $t = T$ , the distribution of  $(y_t^j, z_t^j)$  across  $j = 1, \dots, M$  gives an accurate representation of the invariant distribution without aggregate shocks. During the first  $T$  periods, whenever a firm's state reaches  $y_t^j \geq \bar{y}(z_t^j)$ , we set  $y_t^j = z_t^j = 0$ , corresponding to a price adjustment on the  $n$  products, and we keep simulating the process for  $y_{t+1}^j, z_{t+1}^j$  according to its law of motion.

- At time  $t = T$ , we shock the values of  $(z_t^j, y_t^j)$  of each of the  $j$  firms by decreasing the price gap in each of the  $n$  components by  $\delta > 0$ .

- The value of the state for each firm right after the shock but right before the adjustment can be characterized as a the following two-dimensional shift. We denote with  $Y'$  and  $Z'$  the post-monetary-shock (but pre-adjustment) value of the state for a firm with state  $y, z$ :

$$Y'(\delta, y, z) \equiv \sum_{i=1}^n (p_i - \delta)^2 = y - 2\delta z + n\delta^2 \quad \text{and}$$

$$Z'(\delta, y, z) \equiv \sum_{i=1}^n (p_i - \delta) = z - n\delta.$$

- At time  $t = T$  before the adjusting decision takes place, we replace  $y_t^j$  by  $Y'(\delta, y_t^j, z_t^j)$  and  $z_t^j$  by  $Z'(\delta, y_t^j, z_t^j)$  for all  $j = 1, \dots, J$ .

- We simulate the state process for each firm  $j$  up to the first time  $t = \tau_j \geq T$  in which it adjusts its prices. In particular, the first time  $\tau_j$  were  $y_{\tau_j}^j \geq \bar{y}(z_{\tau_j}^j)$ . Note that at time  $\tau_j$ , firm  $j$ 's sum of the price changes across the  $n$  goods equals the negative of  $z_{\tau_j}^j$ . If at time  $t = T + T'$ , firm  $j$  has not adjusted its price, we force it to change it.

- For each time  $t = T, \dots, T + T'$ , we compute the contribution of each firm to the change in equal-weighted aggregate price level,

$$\theta_t = -\frac{1}{M} \sum_{j=1}^M z_{\tau_j}^j \times \mathcal{I}_{t=\tau_j} \quad \text{for } t = T, T + 1, \dots, T + T',$$

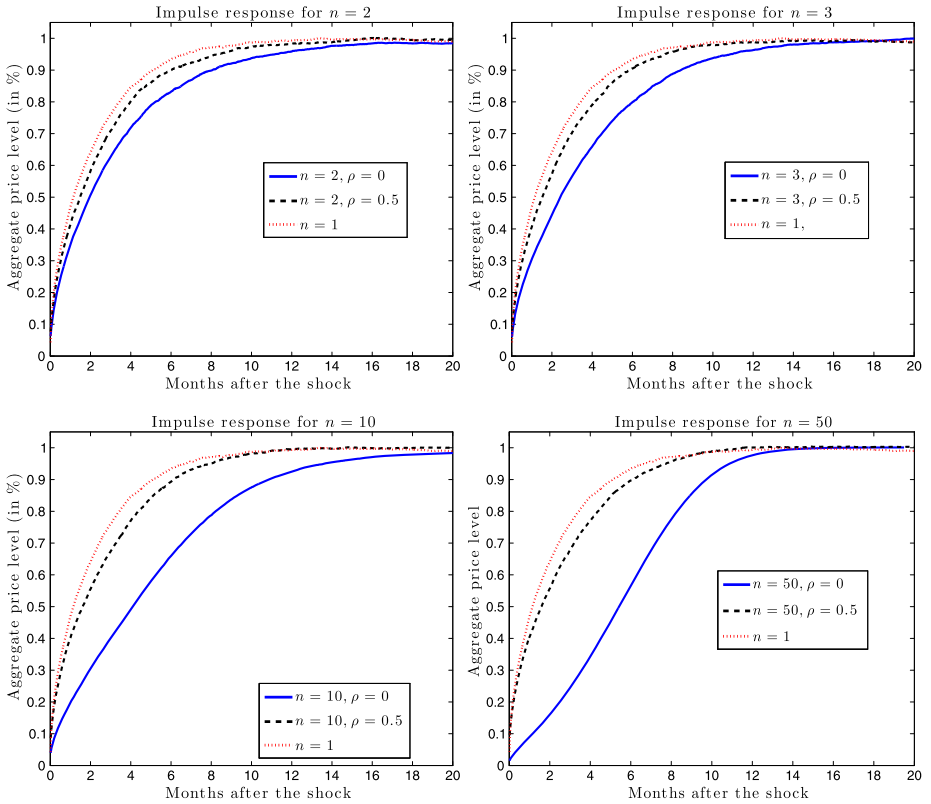


FIGURE 12.—Impulse response to a monetary shock:  $\delta/\text{Std}(\Delta p) = 0.1$ .

where  $\mathcal{I}_{t=\tau_j}$  is the indicator function that takes the value of 1 if firm  $j$  adjusts the price at time  $t$  and 0 otherwise.

- The effect on the equal-weighted price level at time  $t$  is

$$\mathcal{P}(\delta, t) = \sum_{s=T}^t \theta_s \quad \text{for } t = T, T + 1, \dots, T + T'.$$

Figure 12 displays the result of IRF of the price level with respect to a monetary shock. Each panel of this figure considers different values of the number of products ( $n = 2, 3, 10$  and  $n = 50$ ), and for each  $n$ , we plot three cases: the case of  $n = 1$  (which is the same as the case with perfect correlation), correlation equal to zero ( $\rho = 0$ ), and correlation equal to one-half ( $\rho = 1/2$ ). Motivated by the scaling and stretching results we have shown for the case of zero correlation, we normalize the parameters so that the expected number of price changes per year is 1 ( $N_a = 1$ ) and consider a shock of 10% of the size of the steady state standard deviation of price changes (i.e., say  $\delta = 0.01$  and

$\text{Std}(\Delta p) = 0.1$ , i.e., 1% change in money supply and 10% steady state standard deviation of price changes). Thus, each figure corresponds to an economy with the same steady state. The case of  $n = 2$  shows that going from zero correlation to one-half reduces by more than half the distance between the  $n = 2$  and  $n = 1$  cases, that is, it significantly increases the price flexibility at all horizons. The other cases are even more extreme, that is, the vertical distance between the IRF with correlations  $\rho = 1/2$  and  $\rho = 1$  is very small compared with the distance between the IRF's with  $\rho = 1/2$  and  $\rho = 0$ . Recall that the effect on output is proportional to the vertical distance between the level of the IRF and a constant at  $\delta$ , so a correlation of one-half reduces the effect of output significantly toward the case of  $n = 1$ , that is, toward the Golosov and Lucas case.

#### E.4. *The Case With Drift and Correlation*

This section further extends the problem to the case of the joint presence of drift and correlation. Let each price gap follow

$$(65) \quad dp_i(t) = -\mu dt + \bar{\sigma} d\bar{W}(t) + \sigma d\mathcal{W}_i(t) \quad \text{for all } i = 1, \dots, n,$$

where  $\bar{W}$ ,  $\mathcal{W}_i(t)$  are independent standard BM's. Define

$$(66) \quad y(t) = \sum_{i=1}^n p_i^2(t) \quad \text{and} \quad z(t) = \sum_{i=1}^n p_i(t).$$

Using Ito's lemma,

$$\begin{aligned} dy(t) &= [n\sigma^2 + n\bar{\sigma}^2 - 2\mu z(t)] dt + 2\sigma \sum_{i=1}^n p_i(t) d\mathcal{W}_i(t) \\ &\quad + 2\bar{\sigma} \left[ \sum_{i=1}^n p_i(t) \right] d\bar{W}(t) \end{aligned}$$

and

$$dz(t) = -n\mu dt + n\bar{\sigma} d\bar{W}(t) + \sigma \sum_{i=1}^n d\mathcal{W}_i(t).$$

This implies that

$$\begin{aligned} (67) \quad \mathbb{E}[dy(t)]^2 &= 4\sigma^2 \left( \sum_{i=1}^n p_i^2(t) \right) dt + 4\bar{\sigma}^2 \left( \sum_{i=1}^n p_i(t) \right)^2 dt \\ &= 4\sigma^2 y(t) dt + 4\bar{\sigma}^2 z(t)^2 dt, \end{aligned}$$

$$(68) \quad \mathbb{E}[dz(t)]^2 = \sigma^2 n dt + \bar{\sigma}^2 n^2 dt,$$

$$(69) \quad \mathbb{E}[dy(t) dz(t)] = 2\sigma^2 \left( \sum_{i=1}^n p_i(t) \right) dt + 2n\bar{\sigma}^2 \left( \sum_{i=1}^n p_i(t) \right) dt \\ = 2(\sigma^2 + n\bar{\sigma}^2)z(t) dt.$$

Thus define the diffusions

$$(70) \quad dy(t) = [n\sigma^2 + n\bar{\sigma}^2 - 2\mu z(t)] dt + 2\sigma\sqrt{y(t)} d\mathcal{W}^a(t) \\ + 2\bar{\sigma}z(t) d\mathcal{W}^c(t),$$

$$(71) \quad dz(t) = -n\mu dt + n\bar{\sigma} d\mathcal{W}^c(t) \\ + \sqrt{n}\sigma \left[ \frac{z(t)}{\sqrt{ny(t)}} d\mathcal{W}^a(t) + \sqrt{1 - \left( \frac{z(t)}{\sqrt{ny(t)}} \right)^2} d\mathcal{W}^b(t) \right],$$

where  $(\mathcal{W}^a, \mathcal{W}^b, \mathcal{W}^c)$  are three standard independent BM's.

### E.5. Cross-Partials and Different Elasticities Within and Across Firms

In this section, we show the following results.

(i) A quadratic approximation to a cost function that is symmetric across the  $n$  price gaps but with nonzero cross-partial derivative can be accommodated by adding the term  $Dz$  for a constant  $D$  to the flow cost function, which becomes  $By + Dz^2$ .

(ii) The approximation with a nonzero cross-partial derivative can be used to consider a nested CES case, where the aggregate of products produced by a firm have elasticity of substitution  $\eta$  between firms and the products produced by a firm have elasticity  $\varrho$  between them. This yields the expressions for the cost function,  $B$ , and  $D$ ,

$$(72) \quad By + Dz^2 \equiv \frac{1}{n} \left( \frac{\varrho(\eta - 1)}{2} y - \frac{(\varrho - \eta)(\eta - 1)}{2n} z^2 \right),$$

which becomes  $By = \eta(\eta - 1)/(2n)y$  in the benchmark case.

(iii) The effect of the different elasticities described in item (ii) in the constant  $D$  of the cross-product is proportional to  $1/n$ , so it vanishes for moderately high  $n$ , as can be seen in equation (72).

(iv) The effect on  $B$  of incorporating different elasticities is that the value of  $B$  can be greater than that implied by the elasticity  $\eta$  and its optimal markup in the frictionless case or, equivalently, the model produces the same behavior with larger fixed cost  $\psi$ . Equation (72) shows that  $B$  is essentially the product of the two elasticities,  $\eta$  and  $\varrho$ .



(v) From the previous analysis, one can conclude that as  $n \rightarrow \infty$ , the dynamics in the model with different elasticities is identical to the dynamics in the model with the same elasticities.

(vi) The effect of introducing two different elasticities is quantitatively very small in both the shape of the distribution of price changes and the IRF to monetary shocks, especially for moderately large values of  $n$ .

The rest of this section develops the ideas presented below in detail. Before getting into that we offer a few remarks on the results listed above.

To see that introducing symmetric cross-partials yields the expression in item (i), just develop the squares in the relevant expressions. For completeness, we include the relevant algebra below.

To understand the expressions for  $B$  and  $D$  as a function of the elasticities in equation (72), and the effect in item (iv), where if the products sold by the same firm are better substitutes than the aggregate across firms, then  $B$  is larger, we consider two simple examples. Assume that  $\varrho$  is almost  $\infty$  and, therefore, products sold by the same firm are almost perfect substitutes. Furthermore, just to simplify, assume that there are only two products,  $n = 2$ . We consider two examples, where  $y$  is the same but  $z^2$  differs, as a way to understand the expressions in equation (72). In the first example, the price gaps across the two goods are equal in absolute value and are of opposite sign, so  $z = 0$ ; in the second example, the price gaps are equal in absolute value and sign, so  $z^2 > 0$ . In the first case, the firm only sells the good with the lower price. In the second case, profits for the firm are higher (cost is smaller) since the relative prices are same.

The reason why item (vi) holds is that is that, contrary to the case with correlation, there is no change in the law of motion on  $y$  and  $z$ , just a different optimal function  $\bar{y}(\cdot)$ . But given the previous result of the expressions for  $D$  and  $B$ , the function  $\bar{y}$  is almost flat for moderate  $n$ .

### *Cross-Products in the Approximation of the Profit Function*

Consider a profit function of the firm as a function  $\Pi(p)$  of the  $n$  price gaps  $p = (p_1, \dots, p_n)$  and assume that the price gaps are interchangeable, so that profits are the same for any permutation of the price gaps such that, for example,  $\Pi(a, b, \dots) = \Pi(b, a, \dots)$ . Evaluating this function around the maximizing choice  $p_i = 0$  for all  $i$ , we have

$$\bar{b} \equiv -\frac{1}{\Pi(0, 0, \dots, 0)} \frac{\partial \Pi^2(0, \dots, 0)}{\partial p_i \partial p_i} \quad \text{and}$$

$$\bar{d} \equiv -\frac{1}{\Pi(0, 0, \dots, 0)} \frac{\partial \Pi^2(0, \dots, 0)}{\partial p_j \partial p_i} \quad \text{if } i \neq j,$$

where the negative sign is included to define the cost problem. We can write

$$\begin{aligned} & \frac{\Pi(0, 0, \dots, 0) - \Pi(p_1, p_2, \dots, p_n)}{\Pi(0, 0, \dots, 0)} \\ &= \frac{\bar{b}}{2} \left( \sum_{i=1}^n p_i^2 \right) + \bar{d} \left( \sum_{1 \leq i \neq j \leq n} p_i p_j \right) + o(\|p\|^2) \\ &= \frac{\bar{b} - \bar{d}}{2} y + \frac{\bar{d}}{2} z^2 + o(\|p\|^2) \equiv By + Dz^2 + o(\|p\|^2). \end{aligned}$$

Thus we can define the second order approximation of  $\Pi(\cdot)$  in terms of  $y$  and  $z$  as defined above. For  $\partial^2 \Pi / (\partial p \partial p)$  to be negative semidefinite around  $p = 0$  (or, equivalently, for the cost problem to be convex), we require:  $\bar{b} - \bar{d} > 0$  and  $\bar{b} + (n-1)\bar{d} > 0$ , since  $0 \leq z^2 \leq ny$  and  $y \geq 0$ . Note that if  $\bar{d} = 0$ , we recover our benchmark case setting  $\bar{b}/2 = B$ .

#### *Different Elasticities Between Firms and Within Firms' Products*

Now we consider the particular case where the cross-product comes from a different elasticity of substitution between products produced by the firm (denoted by  $\varrho$ ) and between the composite goods produced by different firms (denoted by  $\eta$ ). Let the period  $t$  utility be

$$\begin{aligned} & \frac{c(t)^{1-\varepsilon}}{1-\varepsilon} \quad \text{with} \quad c(t) = \left[ \int_0^1 c_k(t)^{1-1/\eta} dk \right]^{\eta/(\eta-1)} \quad \text{and} \\ & c_k(t) = \left[ \sum_{i=1}^n (Z_{ki}(t) c_{ki}(t))^{1-1/\varrho} \right]^{\varrho/(\varrho-1)}. \end{aligned}$$

Using the CES structure of preference, we can write the demand from product  $i$  of firm  $k$  at time  $t$  as

$$c_{ki}(t) = \left( \frac{P_{ki}(t)}{P_k(t)} \right)^{-\varrho} Z_{ki}(t)^{\varrho-1} \left( \frac{P_k(t)}{P(t)} \right)^{-\eta} c(t),$$

where  $P_k(t)$  is the ideal price index of the products produced by firm  $k$  and  $P(t)$  is the ideal price index of all the goods produced in the economy:

$$\begin{aligned} & P(t) = \left[ \int_0^1 P_k(t)^{1-\eta} dk \right]^{1/(1-\eta)} \quad \text{and} \\ & \frac{P_k(t)}{W(t)} = \left[ \sum_{i=1}^n \left( \frac{P_{ki}(t)}{W(t) Z_{ki}(t)} \right)^{1-\varrho} \right]^{1/(1-\varrho)}. \end{aligned}$$

The time  $t$  nominal profits of the firm  $k$  are

$$\begin{aligned}
& \sum_{i=1}^n (P_{ki}(t) - Z_{ki}(t)W(t))c_{ki}(t) \\
&= W(t) \left( \frac{P_k(t)}{P(t)} \right)^{-\eta} c(t) \sum_{i=1}^n Z_{ki}(t)^{\varrho-1} \left( \frac{P_{ki}(t)}{P_k(t)} \right)^{-\varrho} \left( \frac{P_{ki}(t)}{W(t)} - Z_{ki}(t) \right) \\
&= W(t) \left( \frac{W(t)}{P(t)} \right)^{-\eta} \left( \frac{P_k(t)}{W(t)} \right)^{\varrho-\eta} c(t) \\
&\quad \times \sum_{i=1}^n \left( \frac{P_{ki}(t)}{W(t)Z_{ki}(t)} \right)^{-\varrho} \left( \frac{P_{ki}(t)}{W(t)Z_{ki}(t)} - 1 \right).
\end{aligned}$$

Using the f.o.c. for  $\ell(t)$  and  $c(t)$ ,

$$\frac{W(t)(1 + \tau_\ell)}{P(t)} = \alpha c(t)^\varepsilon,$$

we can write the nominal profit of the firm  $k$  at time  $t$  as

$$\begin{aligned}
& \sum_{i=1}^n (P_{ki}(t) - Z_{ki}(t)W(t))c_{ki}(t) \\
&= W(t) \left( \frac{\alpha}{1 + \tau_\ell} \right)^{-\eta} c(t)^{1-\varepsilon\eta} \left( \frac{P_k(t)}{W(t)} \right)^{\varrho-\eta} \\
&\quad \times \sum_{i=1}^n \left( \frac{P_{ki}(t)}{W(t)Z_{ki}(t)} \right)^{-\varrho} \left( \frac{P_{ki}(t)}{W(t)Z_{ki}(t)} - 1 \right).
\end{aligned}$$

Alternatively, omitting time indices and using  $p_i$  for the price gap of the firm  $k$  defined as  $\exp(p_i) = \frac{\eta}{\eta-1} P_{ki}/(WZ_{ki})$ , we get

$$\begin{aligned}
\Pi(p_1, \dots, p_n) &= \left( \frac{\eta}{\eta-1} \right)^{-\eta} \frac{1}{\eta-1} \left[ \sum_{i=1}^n e^{p_i(1-\varrho)} \right]^{(\varrho-\eta)/(1-\varrho)} \\
&\quad \times \sum_{i=1}^n e^{-p_i\varrho} (\eta e^{p_i} - (\eta-1)),
\end{aligned}$$

where the scaled profit satisfies

$$\begin{aligned} & \sum_{i=1}^n (P_{ki}(t) - Z_{ki}(t)W(t))c_{ki}(t) \\ &= W(t) \left( \frac{\alpha}{1 + \tau_\ell} \right)^{-\eta} c(t)^{1-\varepsilon\eta} \Pi(p_{1k}(t), \dots, p_{kn}(t)) \end{aligned}$$

so that  $\Pi(0, \dots, 0) = \left(\frac{\eta}{\eta-1}\right)^{-\eta} 1/(\eta-1)n^{1+(\varrho-\eta)/(1-\varrho)}$ . We have

$$\begin{aligned} & \frac{\Pi_j(p_1, \dots, p_n)}{\Pi(0, \dots, 0)} \\ &= \frac{1}{n} \left[ \frac{1}{n} \sum_{i=1}^n e^{p_i(1-\varrho)} \right]^{(\varrho-\eta)/(1-\varrho)} \\ & \quad \times \left\{ e^{(1-\varrho)p_j} \left[ (\varrho-\eta)\eta - (\varrho-\eta)(\eta-1) \frac{\sum_{i=1}^n e^{-\varrho p_i}}{\sum_{i=1}^n e^{p_i(1-\varrho)}} \right] \right. \\ & \quad \left. + [(1-\varrho)\eta e^{p_j(1-\varrho)} + \varrho(\eta-1)e^{-\varrho p_j}] \right\}. \end{aligned}$$

Thus

$$\begin{aligned} 0 &= \frac{\Pi_j(0, \dots, 0)}{\Pi(0, \dots, 0)} \quad \text{for all } j = 1, \dots, n, \\ \bar{b} &\equiv \frac{\Pi_{jj}(0, \dots, 0)}{\Pi(0, \dots, 0)} \\ &= -\frac{1}{n} \left\{ (\varrho-\eta)(1-\varrho) + \frac{(\varrho-\eta)(\eta-1)}{n} \right. \\ & \quad \left. + (1-\varrho)^2\eta + \varrho^2(1-\eta) \right\}, \\ \bar{d} &= \frac{\Pi_{ji}(0, \dots, 0)}{\Pi(0, \dots, 0)} = -\frac{1}{n} \frac{(\varrho-\eta)(\eta-1)}{n} \quad \text{for } j \neq i. \end{aligned}$$

The conditions for concavity of the profit function (or convexity of the cost function) are

$$\begin{aligned}\bar{b} - \bar{d} &= -(\varrho - \eta)(1 - \varrho) - (1 - \varrho)^2\eta - \varrho^2(1 - \eta) \\ &= \varrho(\eta - 1) > 0 \quad \text{and} \\ \bar{b} + (n - 1)\bar{d} &= -(\varrho - \eta)(1 - \varrho) + (\varrho - \eta)(\eta - 1) \\ &\quad - (1 - \varrho)^2\eta - \varrho^2(1 - \eta) \\ &= \eta(\eta - 1) > 0,\end{aligned}$$

which are satisfied provided that  $\eta > 1$ .

### *Effect of Different Elasticities for Large $n$*

We finish this section with an asymptotic result: as  $n$  get large, the presence of cross-products can be ignored. The form of the coefficient for the cross-products derived above means that we can write the period return as

$$\frac{\varrho(\eta - 1)}{2} \frac{y}{n} - \frac{(\varrho - \eta)(\eta - 1)}{2} \left(\frac{z}{n}\right)^2.$$

As we let  $n \rightarrow \infty$ , by the law of large numbers,  $z/n$  converges with probability 1 to its expected value, namely 0. In this case, the objective function, and thus the decision rules, converge to the same expressions derived for the case with no cross-products; thus  $\bar{y}(z)$  is flat, that is, independent of  $z$ . As  $n \rightarrow \infty$ , the process for  $y/n$  becomes the same deterministic process as in the benchmark case with one common elasticity. Thus all the analysis for the case of no cross-product applies as  $n \rightarrow \infty$ . The only difference in this case is the interpretation of  $B$ .

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