Econometrica Supplementary Material

SUPPLEMENT TO "INEQUALITY AND UNEMPLOYMENT IN A GLOBAL ECONOMY": TECHNICAL APPENDIX[1]

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S1. INTRODUCTION

THIS TECHNICAL APPENDIX contains the full derivations of the expressions in the paper and additional supplementary derivations. Sections S2–S6 of the appendix correspond to Sections 2–6 of the paper. Section S7 contains additional supplementary derivations. Section S7.1 derives the production technology from human capital complementarities or a managerial time constraint. Section S7.2 examines the relationship between the screening ability threshold and the marginal product of labor. Section S7.3 contains the derivation of the Stole–Zwiebel solution to the bargaining game. Section S7.4 contains the derivation of the search cost (b) from a constant returns to scale matching technology and a cost of posting vacancies.

S2. SECTORAL EQUILIBRIUM

S2.1. *Model Setup*

The real consumption index for the sector (Q) is

$$Q = \left[\int_{j\in J} q(j)^{\beta}\,dj\right]^{1/\beta}, \quad 0<\beta<1, \tag{S1}$$

where j indexes varieties, J is the set of varieties within the sector, and $q(j)$ denotes output of variety j. The price index dual to Q is denoted by

$$P = \left[\int_{j\in J} p(j)^{-\beta/(1-\beta)}\,dj\right]^{-(1-\beta)/\beta},$$

where $p(j)$ denotes the price of variety j. As is well known, given this specification of sectoral demand, the inverse CES demand curve for a firm is

$$p(j) = E^{1-\beta}P^{\beta}q(j)^{-(1-\beta)},$$

[1]This technical appendix combines and extends results previously reported separately in Helpman, Itskhoki, and Redding (2008a, 2008b).

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where $E = PQ$ denotes sectoral expenditure. Substituting for price in firm revenue, we obtain

$$\text{(S2)} \qquad \begin{aligned} r(j) &= p(j)q(j) = Aq(j)^{\beta}, \\ A &\equiv E^{1-\beta}P^{\beta} = Q^{1-\beta}P. \end{aligned}$$

Output of each variety (y) depends on the productivity of the firm (θ), the measure of workers hired (h), and the average ability of these workers ($\bar{a}$),

$$\text{(S3)} \qquad y = \theta h^{\gamma}\bar{a}, \quad 0 < \gamma < 1,$$

where firm productivity is drawn from a Pareto distribution $G_{\theta}(\theta) = 1 - (\theta_{\min}/\theta)^{z}$.

S2.2. *Problem of the Firm*

Given a Pareto distribution of worker ability $G_a(a) = 1 - (a_{\min}/a)^k$, a firm that chooses a screening threshold a_c hires a measure $h = n(a_{\min}/a_c)^k$ of workers with average ability $\bar{a} = ka_c/(k-1)$. Therefore, the production technology (S3) can be written as

$$\text{(S4)} \qquad y = \kappa_y \theta n^{\gamma} a_c^{1-\gamma k}, \quad \kappa_y \equiv \frac{k}{k-1} a_{\min}^{\gamma k}.$$

Since in equilibrium all firms with the same productivity behave symmetrically, we index firms by θ from now onward. Exporters choose output to supply to the domestic market ($y_d(\theta)$) and to the export market ($y_x(\theta)$) to equate marginal revenues in the two markets, which from (S2) implies

$$\text{(S5)} \qquad [y_d(\theta)/y_x(\theta)]^{\beta-1} = \tau^{-\beta}(A^*/A).$$

Total firm output is

$$\text{(S6)} \qquad y(\theta) = y_d(\theta) + y_x(\theta).$$

Together (S5) and (S6) imply

$$\text{(S7)} \qquad y_d(\theta) = y(\theta)/Y(\theta), \quad y_x(\theta) = y(\theta)[Y(\theta) - 1]/Y(\theta),$$

where

$$\text{(S8)} \qquad Y(\theta) \equiv 1 + I_x(\theta)\tau^{-\beta/(1-\beta)}\left(\frac{A^*}{A}\right)^{1/(1-\beta)}$$

and $I_x(\theta)$ is an indicator variable that equals 1 if the firm exports and 0 otherwise. Note that for nonexporters, we have $y_d(\theta) = y(\theta)$ and $Y(\theta) = 1$, which implies that the allocation rule in (S7) also holds for nonexporters. Total firm revenue is

$$r(\theta) = r_d(\theta) + r_x(\theta). \tag{S9}$$

Together (S2), (S5), and (S9) imply

$$r_d(\theta) = r(\theta)/\Upsilon(\theta), \qquad r_x(\theta) = r(\theta)[\Upsilon(\theta) - 1]/\Upsilon(\theta),$$

where again this allocation rule also holds for nonexporters. Additionally, (S2), (S5), and (S7) imply

$$r(\theta) \equiv r_d(\theta) + r_x(\theta) = \Upsilon(\theta)^{1-\beta} A y(\theta)^{\beta}. \tag{S10}$$

Therefore, the firm's problem can be written as

$$\pi(\theta) \equiv \max_{\substack{n \geq 0, \\ a_c \geq a_{\min}, \\ I_x \in \{0,1\}}} \left\{ \frac{1}{1+\beta\gamma} \left[1 + I_x \tau^{-\beta/(1-\beta)} \left(\frac{A^*}{A} \right)^{1/(1-\beta)} \right]^{1-\beta} \right.$$
$$\left. \times A(\kappa_y \theta n^{\gamma} a_c^{1-\gamma k})^{\beta} - bn - \frac{c}{\delta} a_c^{\delta} - f_d - I_x f_x \right\},$$

where $1/(1+\beta\gamma)$ is the equilibrium share of revenue received by the firm as the outcome of the bargaining game with workers, which is modelled as in Stole and Zweibel (1996a, 1996b). We discuss the bargaining game and derive its solution in Section S7.3.

S2.3. *Producer Equilibrium*

The firm's first-order conditions for the measure of workers sampled (n) and the screening ability threshold (a_c) are

$$\frac{\beta\gamma}{1+\beta\gamma} r(\theta) = bn(\theta), \tag{S11}$$

$$\frac{\beta(1-\gamma k)}{1+\beta\gamma} r(\theta) = c a_c(\theta)^{\delta}. \tag{S12}$$

Combining the two first-order conditions (S11) and (S12), we obtain the following relationship between $n(\theta)$ and $a_c(\theta)$:

$$(1-\gamma k) bn(\theta) = \gamma c a_c(\theta)^{\delta}.$$

Using the expression for $r(\theta)$ in (S10), we can solve explicitly for

$$n(\theta) = \phi_1 \phi_2^{\beta(1-\gamma k)} c^{-\beta(1-\gamma k)/(\delta\Gamma)} b^{-(\beta\gamma+\Gamma)/\Gamma} Y(\theta)^{(1-\beta)/\Gamma} Q^{-(\beta-\zeta)/\Gamma} \theta^{\beta/\Gamma},$$

$$a_c(\theta) = \phi_1^{1/\delta} \phi_2^{1-\beta\gamma} c^{-(1-\beta\gamma)/(\delta\Gamma)} b^{-\beta\gamma/(\delta\Gamma)} \times Y(\theta)^{(1-\beta)/(\delta\Gamma)} Q^{-(\beta-\zeta)/(\delta\Gamma)} \theta^{\beta/(\delta\Gamma)},$$

where $\Gamma = 1 - \beta\gamma - \beta(1-\gamma k)/\delta$ and we have introduced two constants

$$\phi_1 \equiv \left[\frac{\beta\gamma}{1+\beta\gamma}\left(\frac{k a_{\min}^{\gamma k}}{k-1}\right)^{\beta}\right]^{1/\Gamma} \quad \text{and} \quad \phi_2 \equiv \left(\frac{1-\gamma k}{\gamma}\right)^{1/(\delta\Gamma)},$$

and, as in the text of the paper, $Y(\theta) = Y_d = 1$ for $\theta \in [\theta_d, \theta_x)$ and $Y(\theta) = Y_x = 1 + \tau^{-\beta/(1-\beta)}(Q^*/Q)^{-(\beta-\zeta)/(1-\beta)}$ for $\theta \geq \theta_x$. Also we solve for

$$\text{(S13)} \quad \frac{\beta\gamma}{1+\beta\gamma} r(\theta) = bn(\theta) = \phi_1 \phi_2^{\beta(1-\gamma k)} c^{-\beta(1-\gamma k)/(\delta\Gamma)} b^{-\beta\gamma/\Gamma} \times Y(\theta)^{(1-\beta)/\Gamma} Q^{-(\beta-\zeta)/\Gamma} \theta^{\beta/\Gamma},$$

$$\text{(S14)} \quad \pi(\theta) + f_d + I_x(\theta) f_x = \frac{\Gamma}{1+\beta\gamma} r(\theta) = \frac{\Gamma}{\beta\gamma} \phi_1 \phi_2^{\beta(1-\gamma k)} c^{-\beta(1-\gamma k)/(\delta\Gamma)} b^{-\beta\gamma/\Gamma} \times Y(\theta)^{(1-\beta)/\Gamma} Q^{-(\beta-\zeta)/\Gamma} \theta^{\beta/\Gamma},$$

$$h(\theta) = n(\theta)\left(\frac{a_{\min}}{a_c(\theta)}\right)^k = a_{\min}^k \phi_1^{(1-k/\delta)} \phi_2^{-(k-\beta)} c^{(k-\beta)/(\delta\Gamma)} b^{-(1-\beta/\delta)/\Gamma} \times Y(\theta)^{(1-\beta)(1-k/\delta)/\Gamma} Q^{-(\beta-\zeta)(1-k/\delta)/\Gamma} \theta^{\beta(1-k/\delta)/\Gamma},$$

so that $\kappa_r \equiv \phi_1 \phi_2^{\beta(1-\gamma k)}$. Finally, we solve for the wage rate:

$$\text{(S15)} \quad w(\theta) = \frac{\beta\gamma}{1+\beta\gamma} \frac{r(\theta)}{h(\theta)} = b\frac{n(\theta)}{h(\theta)} = b\left(\frac{a_c(\theta)}{a_{\min}}\right)^k = a_{\min}^{-k} \phi_1^{k/\delta} \phi_2^{(1-\beta\gamma)k} c^{-(1-\beta\gamma)k/(\delta\Gamma)} b^{(1-\beta\gamma-\beta/\delta)/\Gamma} \times Y(\theta)^{(1-\beta)k/(\delta\Gamma)} Q^{-(\beta-\zeta)k/(\delta\Gamma)} \theta^{\beta k/(\delta\Gamma)}.$$

Note that we have the relationship

$$w(\theta)h(\theta) = bn(\theta) = \frac{\beta\gamma}{1+\beta\gamma} r(\theta),$$

which proves useful in further derivations.

S2.4. *Firm-Specific Variables*

Now, using the zero-profit cutoff condition

$$\begin{aligned}\pi(\theta_d) &= \frac{\Gamma}{1+\beta\gamma} r(\theta) - f_d \\ &= \frac{\Gamma}{\beta\gamma} \phi_1 \phi_2^{\beta(1-\gamma k)} c^{-\beta(1-\gamma k)/(\delta\Gamma)} b^{-\beta\gamma/\Gamma} Q^{-(\beta-\zeta)/\Gamma} \theta_d^{\beta/\Gamma} - f_d = 0,\end{aligned}$$

we can express all firm-level variables solely as the following functions of θ/θ_d, b, and $Y(\theta)$ reported in the paper:

$$\begin{aligned}
&r(\theta) = Y(\theta)^{(1-\beta)/\Gamma} \cdot r_d \cdot \left(\frac{\theta}{\theta_d}\right)^{\beta/\Gamma}, \quad r_d \equiv \frac{1+\beta\gamma}{\Gamma} f_d, \\
&n(\theta) = Y(\theta)^{(1-\beta)/\Gamma} \cdot n_d \cdot \left(\frac{\theta}{\theta_d}\right)^{\beta/\Gamma}, \quad n_d \equiv \frac{\beta\gamma}{\Gamma}\frac{f_d}{b}, \\
&a_c(\theta) = Y(\theta)^{(1-\beta)/(\delta\Gamma)} \cdot a_d \cdot \left(\frac{\theta}{\theta_d}\right)^{\beta/(\delta\Gamma)}, \quad a_d \equiv \left[\frac{\beta(1-\gamma k)}{\Gamma}\frac{f_d}{c}\right]^{1/\delta}, \\
&h(\theta) = Y(\theta)^{(1-\beta)(1-k/\delta)/\Gamma} \cdot h_d \cdot \left(\frac{\theta}{\theta_d}\right)^{\beta(1-k/\delta)/\Gamma}, \\
&h_d \equiv \frac{\beta\gamma}{\Gamma}\frac{f_d}{b}\left[\frac{\beta(1-\gamma k)}{\Gamma}\frac{f_d}{ca_{\min}^{\delta}}\right]^{-k/\delta}, \\
&w(\theta) = Y(\theta)^{k(1-\beta)/(\delta\Gamma)} \cdot w_d \cdot \left(\frac{\theta}{\theta_d}\right)^{\beta k/(\delta\Gamma)}, \\
&w_d \equiv b\left[\frac{\beta(1-\gamma k)}{\Gamma}\frac{f_d}{ca_{\min}^{\delta}}\right]^{k/\delta}.
\end{aligned} \tag{S16}$$

S2.5. *Sectoral Variables*

S2.5.1. *Labor Market Tightness and Hiring Costs*

Search costs (b) depend on the tightness of the labor market (x) as

$$b = \alpha_0 x^{\alpha_1}, \quad \alpha_0 > 1, \alpha_1 > 0. \tag{S17}$$

We show in Section S7.4 that this specification of search costs can be derived from a constant returns to scale matching technology and a cost of posting vacancies following Blanchard and Gali (2010).

Expected income in the differentiated sector is

$$\omega = xb. \tag{S18}$$

Together (S17) and (S18) imply

$$b = \alpha_0^{1/(1+\alpha_1)} \omega^{\alpha_1/(1+\alpha_1)} \quad \text{and} \quad x = \left(\frac{\omega}{\alpha_0}\right)^{1/(1+\alpha_1)}, \tag{S19}$$

and we consider parameter values for which $\alpha_0 > \omega$ so that $0 < x < 1$.

S2.5.2. *Productivity Cutoffs and Demand*

From (S14), the zero-profit productivity is determined by

$$\frac{\Gamma}{1+\beta\gamma} \kappa_r \left[c^{-\beta(1-\gamma k)/\delta} b^{-\beta\gamma} A \theta_d^{\beta} \right]^{1/\Gamma} = f_d. \tag{S20}$$

Similarly, from (S14), the exporting productivity cutoff is determined by

$$\frac{\Gamma}{1+\beta\gamma} \kappa_r \left[c^{-\beta(1-\gamma k)/\delta} b^{-\beta\gamma} A \theta_x^{\beta} \right]^{1/\Gamma} \left[\Upsilon_x^{(1-\beta)/\Gamma} - 1 \right] = f_x. \tag{S21}$$

These two conditions imply the relationship between the productivity cutoffs,

$$\left[\Upsilon_x^{(1-\beta)/\Gamma} - 1 \right] \left(\frac{\theta_x}{\theta_d} \right)^{\beta/\Gamma} = \frac{f_x}{f_d}, \tag{S22}$$

where, in a symmetric equilibrium, $A = A^*$ and, hence, $\Upsilon_x = 1 + \tau^{-\beta/(1-\beta)}$. Therefore, the ratio of the two productivity cutoffs in a symmetric equilibrium is pinned down by (S22) alone.

The free entry condition that equates the expected value of entry to the sunk entry cost is

$$\int_{\theta_d}^{\infty} \pi(\theta)\, dG(\theta) = f_e,$$

where, from (S14) and (S20)–(S21), we have

$$\pi(\theta) = \pi_d(\theta) + \pi_x(\theta) = f_d \left[\left(\frac{\theta}{\theta_d} \right)^{\beta/\Gamma} - 1 \right] + I_x(\theta) f_x \left[\left(\frac{\theta}{\theta_x} \right)^{\beta/\Gamma} - 1 \right],$$

where $I_x(\theta) = 1$ for $\theta \geq \theta_x$ and $I_x(\theta) = 0$ for $\theta < \theta_x$. Using these relationships, we can rewrite the free entry condition as

$$f_d \int_{\theta_d}^{\infty} \left[\left(\frac{\theta}{\theta_d} \right)^{\beta/\Gamma} - 1 \right] dG_\theta + f_x \int_{\theta_x}^{\infty} \left[\left(\frac{\theta}{\theta_x} \right)^{\beta/\Gamma} - 1 \right] dG_\theta = f_e. \tag{S23}$$

S2.5.3. *Expenditure, Mass of Firms, and the Labor Force*

Given the demand shifter for sector i (A_i), the price index for that sector (P_i) can be determined from consumer optimization given prices in all other sectors ($\mathbf{P}_{-i}$) and aggregate income (Ω):

$$A_i = \tilde{A}_i(P_i, \mathbf{P}_{-i}, \Omega), \tag{S24}$$

where the functional form of this relationship depends on the way in which the model is closed in general equilibrium, as shown in Section S6.

Given the demand shifter (A_i) and price index (P_i) for sector i, the real consumption index (Q_i) also follows from consumer optimization, which from (S2) implies

$$Q_i = (A_i/P_i)^{1/(1-\beta)}, \tag{S25}$$

which yields total expenditure within the sector $E = PQ$. The mass of firms within the sector (M) is determined by

$$E = M\int_{\theta_d}^{\infty} r_d(\theta)\,dG_\theta(\theta) + M^*\int_{\theta_x^*}^{\infty} r_x^*(\theta)\,dG_\theta(\theta). \tag{S26}$$

Using the expressions for equilibrium revenue from domestic sales and exports derived above (see (S10) and (S13)), we can rewrite (S26) as

$$\begin{aligned} E = \frac{1+\beta\gamma}{\Gamma}\Bigg[& Mf_d\int_{\theta_d}^{\infty} \Upsilon(\theta)^{(1-\beta)/\Gamma-1}\left(\frac{\theta}{\theta_d}\right)^{\beta/\Gamma} dG_\theta(\theta) \\ & + M^* f_x \frac{\Upsilon_x^{*(1-\beta)/\Gamma}}{\Upsilon_x^{*(1-\beta)/\Gamma}-1}\,\frac{\Upsilon_x^*-1}{\Upsilon_x^*}\int_{\theta_x^*}^{\infty}\left(\frac{\theta}{\theta_x^*}\right)^{\beta/\Gamma} dG_\theta(\theta)\Bigg]. \end{aligned}$$

The sectoral labor force (L) can be determined from the equality between the total sectoral wage bill and workers' share of total sectoral revenue:

$$\omega L = M\int_{\theta_d}^{\infty} w(\theta)h(\theta)\,dG_\theta(\theta) = M\frac{\beta\gamma}{1+\beta\gamma}\int_{\theta_d}^{\infty} r(\theta)\,dG_\theta(\theta). \tag{S27}$$

Using the expressions for equilibrium revenue from domestic sales and exports derived above, we can also rewrite (S27) as

$$\begin{aligned} \omega L &= \frac{\beta\gamma}{\Gamma}M\left[f_d\int_{\theta_d}^{\infty}\left(\frac{\theta}{\theta_d}\right)^{\beta/\Gamma} dG_\theta(\theta) + f_x\int_{\theta_x}^{\infty}\left(\frac{\theta}{\theta_x}\right)^{\beta/\Gamma} dG_\theta(\theta)\right] \\ &= z\gamma f_e M, \end{aligned}$$

where we have evaluated the integrals in the square brackets using the Pareto distribution and applied the free entry condition (S23). This condition implies that L/M is constant in any equilibrium, and L and M are equivalent measures of the size of the differentiated sector. Finally, observe that in a symmetric case, the expression for E above can be considerably simplified and becomes identical to the expression for L above up to a factor of $\beta\gamma/(1+\beta\gamma)$.

S2.6. *Symmetric Countries Closed-Form Solutions*

In this section, we characterize sectoral equilibrium for the case of symmetric countries, where a number of the expressions above simplify further. Evaluating the integrals in the free entry condition (S23) using a Pareto productivity distribution, we obtain

$$\left(\frac{\beta}{z\Gamma-\beta}\right)f_d\left(\frac{\theta_{\min}}{\theta_d}\right)^z\left[1+\frac{f_x}{f_d}\left(\frac{\theta_d}{\theta_x}\right)^z\right]=f_e.$$

Using (S22), we can rewrite this as

$$\text{(S28)}\qquad \left(\frac{\beta}{z\Gamma-\beta}\right)f_d\left(\frac{\theta_{\min}}{\theta_d}\right)^z\left[1+\left(\frac{f_d}{f_x}\right)^{(z\Gamma-\beta)/\beta}\left[Y_x^{(1-\beta)/\Gamma}-1\right]^{z\Gamma/\beta}\right]=f_e.$$

Note that these two expressions do not themselves rely on symmetry. Under the assumption of a symmetric equilibrium, $A=A^*$ and, hence, we have $Y_x = 1+\tau^{-\beta/(1-\beta)}$. Therefore, in a symmetric equilibrium, (S28) defines θ_d and, after solving for θ_d, θ_x, can be obtained from (S22).

Using (S22) and (S28), we can now derive the conditions on the parameters that ensure $\theta_x > \theta_d > \theta_{\min}$ in a symmetric equilibrium. Note that the material in square brackets in (S28) is always greater than 1. Therefore, it is enough to require that

$$f_d > f_e\frac{z\Gamma-\beta}{\beta}$$

to ensure that $\theta_d > \theta_{\min}$ in any symmetric equilibrium. Therefore, a high enough f_d ensures $\theta_d > \theta_{\min}$ in a symmetric equilibrium. Next note from (S22) that since $[Y_x^{(1-\beta)/\Gamma}-1] < 1$, it is enough to require that $f_x \geq f_d$ to ensure $\theta_x > \theta_d$ in any symmetric equilibrium. Therefore, a high enough f_x ensures $\theta_x > \theta_d$ in a symmetric equilibrium. Note that the same condition applies in Melitz (2003). Numerical simulations suggest that a much weaker condition is generally sufficient in this model.

Once we have established the equilibrium value of θ_d, we can determine the equilibrium demand shifter A from the domestic productivity cutoff condition (S20):

$$\text{(S29)}\qquad A=\left(\frac{1+\beta\gamma}{\kappa_r\Gamma}f_d\right)^{\Gamma}c^{\beta(1-\gamma k)/\delta}b^{\beta\gamma}\theta_d^{-\beta}.$$

Note that b and c do not affect θ_d in the symmetric equilibrium. However, these parameters do affect the demand shifter A. In contrast, trade costs do not alter the relationship between A and θ_d in (S29), but higher trade costs do reduce θ_d and, hence, increase A (see (S28)).

Finally, knowing A and following the steps outlined in Section S2.5.3, we can solve for the sectoral price index P, real consumption index Q, and expenditure $E = PQ$. Then the mass of firm entrants M and the measure of workers searching for a job in the differentiated sector L satisfy

$$\frac{\beta\gamma}{1+\beta\gamma}E = L = z\gamma f_e M.$$

This completes the solutions for the case of symmetric countries.

S3. SECTORAL WAGE INEQUALITY

S3.1. *Wage Distribution Among Workers in Exporting and Nonexporting Firms*

The share of workers employed by firms that serve only the domestic market is from (S16) and the Pareto productivity distribution:

$$\text{(S30)}\qquad S_{h,d} = 1 - \frac{\displaystyle\int_{\theta_x}^{\infty} h(\theta)\,dG_\theta(\theta)}{\displaystyle\int_{\theta_d}^{\infty} h(\theta)\,dG_\theta(\theta)} = \frac{1-\rho^{z-\beta(1-k/\delta)/\Gamma}}{1+\rho^{z-\beta(1-k/\delta)/\Gamma}[\Upsilon_x^{(1-\beta)(1-k/\delta)/\Gamma}-1]},$$

where $\rho \equiv \theta_d/\theta_x$. To compute the distribution of wages across workers employed by nonexporting firms, note that the fraction of workers receiving a particular wage $w(\theta) \in [w_d, w_d/\rho^{\beta k/(\delta\Gamma)}]$ is proportional to $h(\theta)\,dG_\theta(\theta)$. In other words, we have

$$\begin{aligned}
G_{w,d}(w) &= \frac{M\displaystyle\int_{\theta_d}^{\theta_{w,d}(w)} h(\theta)\,dG_\theta(\theta)}{M\displaystyle\int_{\theta_d}^{\theta_x} h(\theta)\,dG_\theta(\theta)} \\
&= 1 - \frac{\displaystyle\int_{\theta_{w,d}(w)}^{\theta_x} h(\theta)\,dG_\theta(\theta)}{\displaystyle\int_{\theta_d}^{\theta_x} h(\theta)\,dG_\theta(\theta)} \quad \text{for} \quad w \in \big[w_d, w_d/\rho^{\beta k/(\delta\Gamma)}\big],
\end{aligned}$$

where $\theta_{w,d}(\cdot)$ is the inverse of $w(\cdot)$ and is equal to $\theta_{w,d}(w) = \theta_d(w/w_d)^{\delta\Gamma/(\beta k)}$. Finally, for $w < w_d$, $G_{w,d}(w) = 0$, and for $w > w_d/\rho^{\beta k/(\delta\Gamma)}$, $G_{w,d}(w) = 1$. Using the Pareto productivity distribution, the distribution of wages across workers employed by domestic firms is the truncated Pareto distribution

$$
\begin{aligned}
G_{w,d}(w) &= \frac{1-\left(\dfrac{\theta_d}{\theta_{w,d}(w)}\right)^{z-\beta(1-k/\delta)/\Gamma}}{1-\left(\dfrac{\theta_d}{\theta_x}\right)^{z-\beta(1-k/\delta)/\Gamma}} \\
&= \frac{1-\left(\dfrac{w_d}{w}\right)^{1+1/\mu}}{1-\rho^{z-\beta(1-k/\delta)/\Gamma}} \quad \text{for} \quad w \in \left[w_d, w_d/\rho^{\beta k/(\delta\Gamma)}\right],
\end{aligned}
$$

where $\mu \equiv \beta k/[\delta(z\Gamma - \beta)]$.

The distribution of wages across workers employed by exporters can be computed in the same way:

$$
\begin{aligned}
G_{w,x}(w) &= \frac{M\displaystyle\int_{\theta_x}^{\theta_{w,x}(w)} h(\theta)\,dG_\theta(\theta)}{M\displaystyle\int_{\theta_x}^{\infty} h(\theta)\,dG_\theta(\theta)} \\
&= 1-\frac{\displaystyle\int_{\theta_{w,x}(w)}^{\infty} h(\theta)\,dG_\theta(\theta)}{\displaystyle\int_{\theta_x}^{\infty} h(\theta)\,dG_\theta(\theta)} \\
&\text{for} \quad w \in \left[w_d Y_x^{k(1-\beta)/(\delta\Gamma)}/\rho^{\beta k/(\delta\Gamma)}, \infty\right),
\end{aligned}
$$

where $\theta_{w,x}(\cdot)$ is the inverse of $w(\cdot)$ and is equal to $\theta_{w,x}(w) = \theta_d(w/w_d)^{\delta\Gamma/(\beta k)} \times Y_x^{k(1-\beta)/(\delta\Gamma)}/\rho^{\beta k/(\delta\Gamma)}$. Finally, for $w < w_d Y_x^{k(1-\beta)/(\delta\Gamma)}/\rho^{\beta k/(\delta\Gamma)}$, $G_{w,x}(w) = 0$. Using the Pareto productivity distribution, the distribution of wages across workers employed by exporters is the untruncated Pareto distribution

$$
\begin{aligned}
G_{w,x}(w) &= 1-\left(\frac{\theta_x}{\theta_w(w)}\right)^{z-\beta(1-k/\delta)/\Gamma} \\
&= 1-\left(\frac{w_d}{w} Y_x^{k(1-\beta)/(\delta\Gamma)} \rho^{-\beta k/(\delta\Gamma)}\right)^{1+1/\mu} \\
&\text{for} \quad w \in \left[w_d Y_x^{k(1-\beta)/(\delta\Gamma)}/\rho^{\beta k/(\delta\Gamma)}, \infty\right).
\end{aligned}
$$

Combining $S_{h,d}$, $G_{w,d}(\cdot)$, and $G_{w,x}(\cdot)$ together, we obtain the unconditional wage distribution among workers employed in the differentiated sector,

$G_w(w)$, as defined in the paper:

$$
\text{(S31)} \qquad G_w(w) = \begin{cases} S_{h,d} G_{w,d}(w), \\ \qquad \text{for } w_d \le w \le w_d/\rho^{\beta k/(\delta \Gamma)}, \\ S_{h,d}, \quad \text{for } w_d/\rho^{\beta k/(\delta \Gamma)} \le w \le w_d \Upsilon_x^{k(1-\beta)/(\delta\Gamma)}/\rho^{\beta k/(\delta\Gamma)}, \\ S_{h,d} + (1 - S_{h,d}) G_{w,x}(w), \\ \qquad \text{for } w \ge w_d \Upsilon_x^{k(1-\beta)/(\delta\Gamma)}/\rho^{\beta k/(\delta\Gamma)}. \end{cases}
$$

S3.2. *Proof of Proposition 1*

Consider the closed economy wage distribution

$$
G_w^a(w) = 1 - \left(\frac{w_d}{w} \right)^{1+1/\mu} \quad \text{for} \quad w \ge w_d.
$$

It is a Pareto distribution with a shape parameter $(1 + 1/\mu) > 2$ and lower bound w_d defined in (S16). We now show that with a Pareto distribution, a large class of inequality measures depends only on the shape parameter and not on the lower bound of the distribution.

(i) With this distribution, the mean and variance of wages are given by

$$
\bar{w}^a = (1 + \mu) w_d \quad \text{and} \quad V^a(w) = \frac{1 + \mu}{1 - \mu} \mu^2 w_d^2.
$$

Recall that we require $\mu \in (0, 1)$ so that the variance of the wage distribution is finite. Therefore, the coefficient of variation is given by

$$
\mathrm{CV}_w^a = \frac{\sqrt{V^a(w)}}{\bar{w}^a} = \frac{\mu}{\sqrt{1 - \mu^2}}.
$$

Clearly, μ is a sufficient statistic for the coefficient of variation.

(ii) We now characterize the Lorenz curve for the sectoral wage distribution. To do so, we compute the share in sectoral employment and the share in the sectoral wage bill of firms with productivity below θ':

$$
s_h(\theta) = \frac{M \displaystyle\int_{\theta_d}^{\theta'} h(\theta) \, dG_\theta(\theta)}{M \displaystyle\int_{\theta_d}^{\infty} h(\theta) \, dG_\theta(\theta)} = 1 - \left(\frac{\theta_d}{\theta'} \right)^{z - \beta(1-k/\delta)/\Gamma},
$$

$$
s_w(\theta) = \frac{M \displaystyle\int_{\theta_d}^{\theta'} w(\theta) h(\theta) \, dG_\theta(\theta)}{M \displaystyle\int_{\theta_d}^{\infty} w(\theta) h(\theta) \, dG_\theta(\theta)} = 1 - \left(\frac{\theta_d}{\theta'} \right)^{z - \beta/\Gamma},
$$

where we have used the solution for firm-level variables (S16) and the Pareto productivity distribution. From these expressions we can solve for the wage share s_w as a function of the employment share s_h:

$$s_w = \mathcal{L}^a(s_h) = 1 - (1 - s_h)^{1/(1+\mu)}, \quad \mu = \frac{\beta k}{\delta(z\Gamma - \beta)}, s_h, s_w \in [0, 1]. \tag{S32}$$

This relationship represents the Lorenz curve, because workers' wages are increasing in firm productivity and, hence, this procedure ranks workers by their wages. Note that μ is the only parameter which determines the position of the Lorenz curve. A higher μ makes the Lorenz curve more convex which implies greater wage inequality.

(iii) The Gini coefficient is determined uniquely by the shape of the Lorenz curve and, therefore, μ is also a sufficient statistic for the Gini coefficient. Specifically, the Gini coefficient is defined as

$$\mathcal{G}_w^a \equiv 1 - 2 \int_0^1 \mathcal{L}^a(s_h)\, ds_h = \frac{\mu}{2 + \mu}.$$

(iv) The Theil index of wage inequality is defined as

$$\mathcal{T}_w^a = \int_{w_d}^{\infty} \frac{w}{\bar{w}^a} \ln\left(\frac{w}{\bar{w}^a}\right) dG_w^a(w),$$

where the properties of the Theil index are discussed in Bourguignon (1979). Using the autarky wage distribution, we can compute this integral to obtain

$$\mathcal{T}_w^a = \mu - \ln(1 + \mu).$$

Note that μ is a sufficient statistic for the Theil index, which is decreasing in μ and equal to 0 when $\mu = 0$.

Note that all discussed measures of inequality depend on the parameters of the model only through their effect on μ; all these measures of inequality are increasing in μ. This completes the proof of Proposition 1. *Q.E.D.*

S3.3. *Proof of Proposition 2*

From Proposition 1 we know that μ is a sufficient statistic for sectoral wage inequality in the closed economy. Recall that

$$\mu = \frac{\beta k}{\delta(z\Gamma - \beta)}, \quad \Gamma = 1 - \beta\gamma - \frac{\beta}{\delta}(1 - \gamma k).$$

Evidently, $\partial\mu/\partial z < 0$. The effect of k is more subtle as it increases both the numerator and denominator in the expression for μ. Taking the derivative with respect to k and rearranging we obtain:

$$\operatorname{sign}\left\{\frac{\partial\mu}{\partial k}\right\} = \operatorname{sign}\{1 - z\gamma\mu\} = \operatorname{sign}\{\beta^{-1} - \gamma - \delta^{-1} - z^{-1}\}.$$

This finishes the proof of Proposition 2. *Q.E.D.*

S3.4. *Proof of Proposition 3*

We first prove the second part of the proposition. The two limiting cases for the open economy wage distribution (S31) are (a) autarky (as $\rho = \theta_d/\theta_x \to 0$ and no firms export) and (b) when all firms export ($\rho \to 1$). In the first case, the wage distribution $G_w^a(w)$ is an untruncated Pareto with shape parameter $(1 + 1/\mu)$ and lower bound w_d. In the second case, the wage distribution is again an untruncated Pareto with shape parameter $(1 + 1/\mu)$, but now with a higher lower bound given by $w_d Y_x^{k(1-\beta)/(\delta\Gamma)}$. However, from Proposition 1, a broad class of inequality measures depend only on the shape parameter of the Pareto distribution and do not depend on its lower bound. Therefore, wage inequality is the same in autarky and when all firms export.

Now consider the first part of the proposition. Define the notation

$$\eta_1 \equiv Y_x^{(1-\beta)(1-k/\delta)/\Gamma} - 1, \quad \eta_2 \equiv Y_x^{(1-\beta)/\Gamma} - 1,$$
$$\vartheta_1 \equiv z - \frac{\beta(1-k/\delta)}{\Gamma}, \quad \vartheta_2 \equiv z - \frac{\beta}{\Gamma}.$$

Using this notation, the lowest wage paid by exporters and the highest wage paid by domestic firms can be written as

$$w(\theta_x^+) = w(\theta_x^-)\frac{1+\eta_2}{1+\eta_1} \quad \text{and} \quad w(\theta_x^-) = w_d \rho^{\vartheta_2 - \vartheta_1}.$$

Similarly, using this notation, the actual wage distribution (S31) can be written as

$$G_w(w) = \begin{cases} \dfrac{1}{1+\eta_1\rho^{\vartheta_1}}\left[1 - (w_d/w)^{1+1/\mu}\right], & w_d \le w \le w(\theta_x^-), \\ (1-\rho^{\vartheta_1})/(1+\eta_1\rho^{\vartheta_1}), & w(\theta_x^-) \le w \le w(\theta_x^+), \\ \dfrac{1-\rho^{\vartheta_1}}{1+\eta_1\rho^{\vartheta_1}} + \dfrac{(1+\eta_1)\rho^{\vartheta_1}}{1+\eta_1\rho^{\vartheta_1}}\left[1 - (w(\theta_x^+)/w)^{1+1/\mu}\right], & \\ & w \ge w(\theta_x^+), \end{cases} \tag{S33}$$

and the mean of this distribution can be written as

$$\bar{w} = (1+\mu)w_d\frac{1+\eta_2\rho^{\vartheta_2}}{1+\eta_1\rho^{\vartheta_1}}.$$

The counterfactual wage distribution is defined as

$$G_w^c(w) = 1 - (w_d^c/w)^{1+1/\mu}, \quad w \geq w_d^c, \tag{S34}$$

where, for the mean of the counterfactual distribution to equal $\bar{w}$, its lower limit must satisfy

$$w_d^c = \frac{1+\eta_2\rho^{\vartheta_2}}{1+\eta_1\rho^{\vartheta_1}} w_d.$$

Therefore, we can establish the result

$$w_d^c > w_d \quad \text{since} \quad \frac{1+\eta_2\rho^{\vartheta_2}}{1+\eta_1\rho^{\vartheta_1}} > 1 \quad \text{for} \quad 0 < \rho < 1,$$

as $0 < \eta_1 < \eta_2 < 1$ and $1 < z/2 < \vartheta_2 < \vartheta_1 < z$. Similarly, we can establish

$$w_d^c < w(\theta_x^+) \quad \text{since} \quad \frac{1+\eta_2\rho^{\vartheta_2}}{1+\eta_1\rho^{\vartheta_1}} < \frac{1+\eta_2}{1+\eta_1}\rho^{\vartheta_2-\vartheta_1} = \frac{(1+\eta_2)\rho^{\vartheta_2}}{(1+\eta_1)\rho^{\vartheta_1}},$$

with the inequality being satisfied since

$$\begin{aligned} \frac{1+\eta_2\rho^{\vartheta_2}}{1+\eta_1\rho^{\vartheta_1}} &= \frac{(1+\eta_2)\rho^{\vartheta_2} + (1-\rho^{\vartheta_2})}{(1+\eta_1)\rho^{\vartheta_1} + (1-\rho^{\vartheta_1})} \\ &= \frac{(1+\eta_2)\rho^{\vartheta_2}}{(1+\eta_1)\rho^{\vartheta_1}} \cdot \frac{1 + \dfrac{1-\rho^{\vartheta_2}}{(1+\eta_2)\rho^{\vartheta_2}}}{1 + \dfrac{1-\rho^{\vartheta_1}}{(1+\eta_1)\rho^{\vartheta_1}}} < \frac{(1+\eta_2)\rho^{\vartheta_2}}{(1+\eta_1)\rho^{\vartheta_1}}, \end{aligned}$$

as $\rho^{\vartheta_1} < \rho^{\vartheta_2} < 1$ and $(1+\eta_2) > (1+\eta_1)$. Note that, in general, we can have either $w_d^c > w(\theta_x^-)$ or $w_d^c < w(\theta_x^-)$, but the same arguments apply in each case.

We can also show that the slope of the counterfactual wage distribution is smaller than the slope of the actual wage distribution at $w(\theta_x^+)$: $g_w(w(\theta_x^+)) > g_w^c(w(\theta_x^+))$. Since the truncations of $G_w(w)$ and $G_w^c(w)$ at $w(\theta_x^+)$ are both Pareto with shape parameter $(1 + 1/\mu)$, we can show that $g_w(w(\theta_x^+)) > g_w^c(w(\theta_x^+))$ by establishing that $1 - G_w(w(\theta_x^+)) > 1 - G_w^c(w(\theta_x^+))$. From (S33) and (S34), this implies

$$\begin{aligned} & 1 - \frac{1-\rho^{\vartheta_1}}{1+\eta_1\rho^{\vartheta_1}} > \left(\frac{w_d^c}{w(\theta_x^+)}\right)^{1+1/\mu} \\ \Leftrightarrow \quad & \frac{(1+\eta_1)\rho^{\vartheta_1}}{1+\eta_1\rho^{\vartheta_1}} > \left(\frac{1+\eta_2\rho^{\vartheta_2}}{1+\eta_1\rho^{\vartheta_1}}\frac{1+\eta_1}{1+\eta_2}\right)^{\vartheta_1/(\vartheta_1-\vartheta_2)} \rho^{\vartheta_1} \\ \Leftrightarrow \quad & \phi(\rho) \equiv \left(\frac{1+\eta_1\rho^{\vartheta_1}}{1+\eta_1}\right)^{\vartheta_2} - \left(\frac{1+\eta_2\rho^{\vartheta_2}}{1+\eta_2}\right)^{\vartheta_1} > 0. \end{aligned}$$

To show that $\phi(\rho) > 0$ for all $\rho \in [0, 1)$, note that

$$\phi(0) \equiv \left(\frac{1}{1+\eta_1}\right)^{\vartheta_2} - \left(\frac{1}{1+\eta_2}\right)^{\vartheta_1} > 0,$$

as $\eta_1 < \eta_2$ and $\vartheta_1 > \vartheta_2$. Note also that $\phi(1) = 1 - 1 = 0$. Consider now the derivative of $\phi(\rho)$ for $\rho \in (0, 1]$:

$$\phi'(\rho) = \frac{\eta_1\eta_2}{\rho}\left[\left(\frac{1+\eta_1\rho^{\vartheta_1}}{1+\eta_1}\right)^{\vartheta_2} \frac{\eta_1\rho^{\vartheta_1}}{1+\eta_1\rho^{\vartheta_1}} - \left(\frac{1+\eta_2\rho^{\vartheta_2}}{1+\eta_2}\right)^{\vartheta_1} \frac{\eta_2\rho^{\vartheta_2}}{1+\eta_2\rho^{\vartheta_2}}\right].$$

Note that

$$\frac{\eta_2\rho^{\vartheta_2}}{1+\eta_2\rho^{\vartheta_2}} > \frac{\eta_1\rho^{\vartheta_1}}{1+\eta_1\rho^{\vartheta_1}},$$

since $\eta_1 < \eta_2$ and $\rho^{\vartheta_1} < \rho^{\vartheta_2}$. As a result, whenever $\phi(\rho) \leq 0$, we also necessarily have $\phi'(\rho) < 0$. Therefore, if there exists ρ' such that $\phi(\rho') = 0$, then $\phi(\rho) < 0$ for all $\rho > \rho'$. But since $\phi(1) = 0$, this implies that $\phi(\rho) > 0$ for all $\rho \in (0, 1)$.

We now establish that $G_w^c(w)$ second-order stochastically dominates $G_w(w)$ for $\rho \in (0, 1)$. Using the fact that the truncations of $G_w(w)$ and $G_w^c(w)$ at $w(\theta_x^+)$ are both Pareto with shape parameter $(1 + 1/\mu)$, and the result above that $g_w(w(\theta_x^+)) > g_w^c(w(\theta_x^+))$, we know that this inequality holds for all $w > w(\theta_x^+)$. We have two cases.

Case 1. $w(\theta_x^-) \leq w_d^c < w(\theta_x^+)$:

$$g_w(w) - g_w^c(w) = \begin{cases} > 0, & w_d \leq w < w(\theta_x^-), \\ = 0, & w(\theta_x^-) \leq w < w_d^c, \\ < 0, & w_d^c \leq w < w(\theta_x^+), \\ > 0, & w \geq w(\theta_x^+). \end{cases}$$

Case 2. $w_d^c < w(\theta_x^-)$:

$$g_w(w) - g_w^c(w) = \begin{cases} > 0, & w_d \leq w < w_d^c, \\ \lessgtr 0, & w_d^c \leq w < w(\theta_x^-), \\ < 0, & w(\theta_x^-) \leq w < w(\theta_x^+), \\ > 0, & w \geq w(\theta_x^+). \end{cases}$$

Importantly, $g_w(w) - g_w^c(w)$ takes either only positive or only negative values in the range $[w_d^c, w(\theta_x^-))$, since for this range

$$g_w(w) - g_w^c(w) = (C - C^c)w^{-(2+1/\mu)},$$

where C and C^c are positive constants.

Note that in both cases the above characterization of $g_w(w) - g_w^c(w)$ implies that this difference of density functions is positive for low values of w, negative for intermediate values of w, and again positive for larger values of w. This immediately implies that the cumulative distribution functions intersect only once in the range where the difference of density functions is negative (see Figure 2 in the text), which is a sufficient condition to establish that indeed $G_w^c(w)$ second-order stochastically dominates $G_w(w)$ (see, for example, Mas-Colell, Whinston, and Green (1995, p. 195)).

Therefore, for all measures of inequality that respect second-order stochastic dominance, wage inequality in the open economy when some, but not all, firms export is strictly greater than in the closed economy. This finishes the proof of Proposition 3. *Q.E.D.*

S4. SECTORAL UNEMPLOYMENT

The sectoral unemployment rate is given by

$$u = \frac{L-H}{L} = 1 - \frac{H}{N}\frac{N}{L} = 1 - \sigma x,$$

where $\sigma = H/N$ is the hiring rate and $x = N/L$ is labor market tightness. From (S19), labor market tightness is given by $x = (\omega/\alpha_0)^{1/(1+\alpha_1)}$.

We now derive the expression for the sectoral hiring rate:

$$\begin{aligned}
\sigma &= \frac{M\displaystyle\int_{\theta_d}^{\infty} h(\theta)\,dG_\theta(\theta)}{M\displaystyle\int_{\theta_d}^{\infty} n(\theta)\,dG_\theta(\theta)} \\
&= \frac{h_d\displaystyle\int_{\theta_d}^{\theta_x}\left(\frac{\theta}{\theta_d}\right)^{\beta(1-k/\delta)/\Gamma} dG_\theta(\theta) + h_x\int_{\theta_x}^{\infty}\left(\frac{\theta}{\theta_x}\right)^{\beta(1-k/\delta)/\Gamma} dG_\theta(\theta)}{n_d\displaystyle\int_{\theta_d}^{\theta_x}\left(\frac{\theta}{\theta_d}\right)^{\beta/\Gamma} dG_\theta(\theta) + n_x\int_{\theta_x}^{\infty}\left(\frac{\theta}{\theta_x}\right)^{\beta/\Gamma} dG_\theta(\theta)},
\end{aligned}$$

where $h_x = \Upsilon_x^{(1-\beta)(1-k/\delta)/\Gamma}(\theta_x/\theta_d)^{\beta(1-k/\delta)/\Gamma} h_d$, $n_x = \Upsilon_x^{(1-\beta)/\Gamma}(\theta_x/\theta_d)^{\beta/\Gamma} n_d$, and we have used the solution for firm-specific variables (S16). Evaluating the integrals using the Pareto productivity distribution yields

$$\sigma = \sigma^a \cdot \varphi(\rho, \Upsilon_x),$$

where

$$\sigma^a \equiv \frac{1}{1+\mu}\left[\frac{\Gamma}{\beta(1-\gamma k)}\frac{c a_{\min}^{\delta}}{f_d}\right]^{k/\delta}$$

is the autarky hiring rate and

$$\varphi(\rho, Y_x) \equiv \frac{1 + [Y_x^{(1-\beta)(1-k/\delta)/\Gamma} - 1]\rho^{z-\beta(1-k/\delta)/\Gamma}}{1 + [Y_x^{(1-\beta)/\Gamma} - 1]\rho^{z-\beta/\Gamma}} = \frac{1 + \eta_1 \rho^{\vartheta_1}}{1 + \eta_2 \rho^{\vartheta_2}},$$

where the last equality uses the notation introduced in Section S3.4. Note that $\varphi(0, Y_x) = 1$ and $\varphi(\rho, Y_x) < 1$ for $0 < \rho \leq 1$, since $\eta_1 < \eta_2$ and $\vartheta_1 > \vartheta_2$. Therefore, the hiring rate is lower in any open economy equilibrium than in autarky.

Opening up to trade does not affect sectoral labor market tightness x directly, but does affect it indirectly through ω. In the case when ω is invariant to trade, sectoral unemployment is affected by trade only through hiring rate. As a result, in this case, sectoral unemployment is strictly higher in any open economy equilibrium than in autarky. This constitutes a proof of Proposition 5.

Finally, we look at the determinants of the hiring rate and sectoral unemployment in the autarky equilibrium. From the expressions for u, x, and σ^a, it is evident that holding ω constant, then c, z, and k affect the unemployment rate only through their effects on the hiring rate σ^a, while α_0 affects the unemployment rate only through its effect on labor market tightness x. Specifically, an increase in α_0 decreases x and increases u; an increase in z or c increases σ^a and decreases u; the effect of k on σ^a, and hence u, is ambiguous.

Additional Results on Sectoral Income Inequality

First, consider the Lorenz curve for the sectoral income distribution. When the Lorenz curve for the wage distribution is given by $s_w = \mathcal{L}_w(s_h)$ and the unemployment rate is u, the Lorenz curve for the income distribution can be written as

$$s_\iota = \mathcal{L}_\iota(s_\ell) = \begin{cases} 0, & 0 \leq s_\ell \leq u, \\ \mathcal{L}_w\left(\dfrac{s_\ell - u}{1-u}\right), & u \leq s_\ell \leq 1, \end{cases}$$

and from (S32), $\mathcal{L}_w(s) = 1 - (1-s)^{1/(1+\mu)}$. We can now compute the Gini coefficient of sectoral income inequality:

$$\begin{aligned} \mathcal{G}_\iota &= 1 - 2\int_0^1 \mathcal{L}_\iota(s_\ell)\, ds_\ell = u + (1-u)\mathcal{G}_w \\ &= u + (1-u)\left[1 - \frac{2}{1-u}\int_u^1 \mathcal{L}_w\left(\frac{s_\ell - u}{1-u}\right) ds_\ell\right]. \end{aligned}$$

Substitution of variables $s_h = (s_\ell - u)/(1-u)$ results in

$$\mathcal{G}_\iota = u + (1-u)\mathcal{G}_w,$$

where $\mathcal{G}_w$ is the Gini coefficient of sectoral wage inequality as stated in Section S3.2. Therefore, sectoral income inequality as measured by the Gini coefficient is increasing in the sectoral unemployment rate and sectoral wage inequality.

Now consider the Theil index of sectoral income inequality. The general definition of the Theil index is

$$\mathcal{T}_\varpi = \int \frac{\varpi}{\bar{\varpi}} \ln\left(\frac{\varpi}{\bar{\varpi}}\right) dG_\varpi(\varpi),$$

where ϖ is a measure of income distributed according to the cumulative distribution function $G_\varpi(\varpi)$ and $\bar{\varpi}$ is the mean of this measure. Note that $\varpi \ln \varpi = 0$ at $\varpi = 0$. Then for the sectoral income distribution, we have $G_\iota(\iota) = u$ for $\iota \in [0, w_d)$ and $G_\iota(\iota) = u + (1-u)G_w(\iota)$ for $\iota > w_d$, where $G_w(w)$ is the sectoral wage distribution. The mean of the sectoral income distribution is $\bar{\iota} = (1-u)\bar{w} + u \cdot 0 = (1-u)\bar{w}$, where $\bar{w}$ is the mean of the sectoral wage distribution. We can now compute the Theil index of sectoral income inequality as

$$\begin{aligned}
\mathcal{T}_\iota &= u \cdot 0 + (1-u) \int_{w_d}^{\infty} \frac{w}{\bar{\iota}} \ln\left(\frac{w}{\bar{\iota}}\right) dG_w(w) \\
&= \frac{(1-u)\bar{w}}{\bar{\iota}} \int_{w_d}^{\infty} \frac{w}{\bar{w}} \left[\ln\left(\frac{w}{\bar{w}}\right) - \ln\left(\frac{\bar{\iota}}{\bar{w}}\right)\right] dG_w(w).
\end{aligned}$$

Since $\bar{\iota}/\bar{w} = (1-u)$, we can rewrite this as

$$\mathcal{T}_\iota = \mathcal{T}_w - \ln(1-u),$$

where $\mathcal{T}_w$ is the Theil index of sectoral wage inequality as stated in Section S3.2. The same expression can be derived using the decomposition of the Theil index into within and between-group components, as studied in Bourguignon (1979) and applied to this setting in Helpman, Itskhoki, and Redding (2008a, 2008b). Finally, note that sectoral income inequality as measured by the Theil index is increasing in the sectoral unemployment rate and sectoral wage inequality, consistent with the results for the Gini coefficient above.

S5. OBSERVABLE WORKER HETEROGENEITY

S5.1. *Wage Bargaining With Two Types of Workers*

Consider the extension of the Stole and Zweibel (1996a, 1996b) bargaining game to the case with two groups of workers. Let the revenue function be given by $r = Bh_1^{\zeta_1} h_2^{\zeta_2}$, where, in the case of our model, we have $\zeta_\ell = \lambda_\ell \beta \gamma_\ell$ for $\ell = 1, 2$ and $B = Y(\theta)^{1-\beta} A(\kappa_y \theta \bar{a}_1^{\lambda_1} \bar{a}_2^{\lambda_2})^\beta$. Every worker of type ℓ receives a wage w_ℓ

which is a function of employment levels of both types of workers, while the firm receives the residual, $r - w_1 h_1 - w_2 h_2$. Assuming equal bargaining weights, the surplus from an extra worker of type ℓ is divided equally between the firm and the worker:

$$\frac{\partial[r(h_1, h_2) - w_1(h_1, h_2)h_1 - w_2(h_1, h_2)h_2]}{\partial h_\ell} = w_\ell, \quad \ell = 1, 2, \tag{S35}$$

where we assumed that the outside options of both types of workers are 0. This yields a system of differential equations in (w_1, w_2). The initial conditions are $w_\ell(0, 0) = 0$ for $\ell = 1, 2$. It is immediate to verify that there exists a solution which satisfies these initial conditions in the form

$$w_\ell(h_1, h_2) = \phi_\ell \frac{r(h_1, h_2)}{h_\ell}, \quad \phi_\ell = \frac{\zeta_\ell}{1 + \zeta_1 + \zeta_2}, \quad \ell = 1, 2,$$

and the share of the firm in the revenues is $1/(1 + \zeta_1 + \zeta_2)$.

S5.2. *Heckscher–Ohlin Extension*

Consider the model of Section 5.1 with an outside sector which uses a Cobb–Douglas technology combining the two types of labor:

$$y_0 = \left(\frac{L_{01}}{\lambda_{01}}\right)^{\lambda_{01}} \left(\frac{L_{02}}{\lambda_{02}}\right)^{\lambda_{02}}, \quad \lambda_{01} + \lambda_{02} = 1,$$

where $L_{0\ell}$ is the employment of type-ℓ workers in the outside sector. Denote by ω_ℓ the wage rates of type-ℓ workers in the outside sector and normalize the price of the outside good to 1 ($p_0 = 1$). Therefore, competitive production (at any scale) in this sector requires $\omega_1^{\lambda_{01}} \omega_2^{\lambda_{02}} = 1$ which ensures zero profits, while the relative labor demand is given by

$$\frac{L_{01}}{L_{02}} = \frac{\lambda_{01}}{\lambda_{02}} \frac{\omega_2}{\omega_1}.$$

From the equilibrium conditions in the differentiated sector, one can see that the relative number of workers of the two types searching for jobs in the differentiated sector is also proportional to relative outside-sector wage rate:

$$\frac{L_1}{L_2} \propto \frac{\omega_2}{\omega_1}.$$

Furthermore, the two labor balances have to hold:

$$L_{0\ell} + L_\ell = \bar{L}_\ell, \quad \ell = 1, 2,$$

where $\bar{L}_\ell$ is the total supply of type-ℓ labor in the economy. This implies that there is only one ratio ω_2/ω_1 consistent with the equilibrium in the labor market. Together with the zero-profit condition in the outside sector, this pins down the value of ω_1 and ω_2. Given these values, we can solve for L_1 and L_2 from the equilibrium conditions in the differentiated sector. It then only remains to make sure that $\bar{L}_\ell$ is large enough to ensure $L_{0\ell} > 0$ for $\ell = 1, 2$. This constitutes a general equilibrium in this model.

If, upon opening up trade, there is positive intersectoral trade, ω_2/ω_1 would be different in an open economy. If the countries are symmetric in all respects other than endowments of the two types of labor, the intersectoral patterns of trade and changes in relative rewards ω_2/ω_1 will be determined by Heckscher–Ohlin forces, which directly affect between-group wage inequality, but have no effect on within-group inequality in the differentiated sector.

S5.3. *Model With Technology Choice*

Consider a model in which a firm can choose at no cost between two technologies with different $\lambda_1 = 1 - \lambda_2$. Also let $\gamma_1 > \gamma_2$ and $k_1 < k_2 < \delta$, and interpret group 1 to be high skilled. Consider the solution to the firm's problem for a given value of λ_1 in the closed economy:

$$\pi(\theta) = \frac{\Gamma}{1 + \beta[\lambda_1\gamma_1 + \lambda_2\gamma_2]} r(\theta) - f_d,$$

$$r(\theta) = \kappa_r \theta^{\beta/\Gamma},$$

$$\kappa_r \equiv A^{1/\Gamma} \prod_{\ell=1,2} \left(b_\ell^{-\gamma_\ell} c^{-(1-\gamma_\ell k_\ell)/\delta} \frac{k_\ell a_{\min \ell}^{\gamma_\ell k_\ell}}{k_\ell - 1} \right.$$
$$\left. \times \left(\frac{\lambda_\ell \beta \gamma_\ell}{1 + \beta\bar{\gamma}} \right)^{\gamma_\ell} \left(\frac{\lambda_\ell \beta (1 - \gamma_\ell k_\ell)}{\delta(1 + \beta\bar{\gamma})} \right)^{(1-\gamma_\ell k_\ell)/\delta} \right)^{\lambda_\ell \beta/\Gamma}.$$

Furthermore, note from the definition of Γ that

$$\left.\frac{\partial \Gamma}{\partial \lambda_1}\right|_{\lambda_1+\lambda_2=1} = -\beta[\gamma_1(1 - k_1/\delta) - \gamma_2(1 - k_2/\delta)] < 0.$$

Therefore, a technology with a larger λ_1 results in a larger Γ. Note that this increases the log slope of the revenue and profit functions in productivity θ. However, it reduces the ratio of variable profits to revenues and also has an ambiguous effect on κ_r. This implies that the most productive firms (as $\theta \to \infty$) always choose the high-λ_1 technology, but κ_r has to be small enough for the low-productivity firms ($\theta \to \theta_d$) to choose the low-λ_1 technology. This can be always guaranteed when b_1 is sufficiently greater than b_2 (which is the case when ω_1 is sufficiently greater than ω_2).

When low-productivity firms are better off with a low-λ_1 technology, there exists a threshold θ_t such that firms below this threshold choose a low-λ_1 technology, while firms above this threshold choose a high-λ_1 technology. The profit function has a kink at this point, while revenues jump up since the share of variable profits in revenues decreases with λ_1:

$$\frac{\partial}{\partial \lambda_1} \frac{\Gamma}{1+\beta[\lambda_1\gamma_1+\lambda_2\gamma_2]} < 0.$$

Using the solution for firm-level variables, we can characterize the jump in employment and wages at the technological threshold θ_t:

$$\Delta \ln h_\ell(\theta_t) = \left(1-\frac{k_\ell}{\delta}\right)\Delta \ln\left(\lambda_\ell \frac{r(\theta_t)}{1+\beta\bar{\gamma}}\right),$$

$$\Delta \ln w_\ell(\theta_t) = \frac{k_\ell}{\delta}\Delta \ln\left(\lambda_\ell \frac{r(\theta_t)}{1+\beta\bar{\gamma}}\right).$$

For the group-1 workers, both employment and wages unambiguously jump up at θ_t because both λ_1 and $r(\theta_t)/(1+\beta\bar{\gamma})$ discontinuously increase. For the group-2 workers, the effect is ambiguous because λ_2 discontinuously decreases. On both sides of θ_t, the behavior of employment and wages is just like in the model without technology choice.

As a result, for both groups the closed-economy wage distribution is a mix of two Pareto distributions with shape parameter $1+1/\mu_\ell$. Since the jump in wages is larger for group-1 workers and the employment share of these workers is larger in more productive firms, one can show analytically that within-group wage inequality can be larger for this group of workers provided that k_1 is not much smaller than k_2 and relatively few firms choose the high-λ_1 technology. Although this model remains analytically tractable, the analysis in the open economy is complicated by the interaction between opening up and the response in the technology choice cutoff.

S5.4. *Model With CES Production Function*

With the CES production function defined in the text, the revenue of the firm is given by

$$r = Y^{1-\beta}A[\lambda_1(\theta_1\bar{a}_1h_1^\gamma)^\nu + \lambda_2(\theta_2\bar{a}_2h_2^\gamma)^\nu]^{\beta/\nu}, \quad 0 \le \nu \le 1, \lambda_1+\lambda_2 = 1,$$

where θ_1 and θ_2 are productivities of the two types of labor. Our baseline assumption is technology–skill complementarity: $\theta_1 = \theta$ and $\theta_2 \equiv 1$, where θ is the productivity draw of the firm and we interpret group 1 as skilled.

As we discuss below, to match observed features of the data, we also assume $\nu \le \beta$. Note that $\nu = 0$ corresponds to the Cobb–Douglas limit studied above, while $\nu = \beta$ results in a separable revenue function in the two type of labor. In the CES case, we make additional symmetry assumptions for the two groups of workers: $\gamma_1 = \gamma_2 = \gamma$, $k_1 = k_2 = k$, and $\delta_1 = \delta_2 = \delta$. The first of these symmetry assumptions ensures a closed-form solution to the bargaining game, while the latter two assumptions allow generalization of the general equilibrium conditions of the model.

For derivations below, it is useful to introduce the notation

$$\varphi \equiv \frac{\lambda_1(\theta_1 \bar{a}_1 h_1^{\gamma})^{\nu}}{\lambda_2(\theta_2 \bar{a}_2 h_2^{\gamma})^{\nu}}, \quad \phi_1 \equiv \frac{\varphi}{1+\varphi}, \quad \phi_2 \equiv \frac{1}{1+\varphi}.$$

Note that $\phi_1 + \phi_2 \equiv 1$ and ϕ_1 and ϕ_2 are generalizations of λ_1 and λ_2 in the Cobb–Douglas case.

Wage Bargaining

The wage schedules which result from the Stole and Zweibel bargaining game still satisfy the system of differential equations in (S35). One can verify that the following wage schedules are solutions to this system in this case:

$$w_\ell = \frac{\beta\gamma}{1+\beta\gamma}\frac{\phi_\ell r}{h_\ell}, \tag{S36}$$

where ϕ_ℓ is as we have just defined. Note that these wage schedules imply that the firm again receives a constant fraction $1/(1+\beta\gamma)$ of revenues (since $\phi_1 + \phi_2 \equiv 1$). At the same time, the relative wage bill of the skilled group, $(w_1 h_1)/(w_2 h_2) = \phi_1/\phi_2 = \varphi$, increases with φ.

Problem of the Firm

The firm maximizes

$$\pi(\theta) = \max_{n_\ell, a_{c\ell}, I_x} \left\{ \frac{1}{1+\beta\gamma} r - b_1 n_1 - b_2 n_2 - \frac{c}{\delta} a_{c1}^{\delta} - \frac{c}{\delta} a_{c2}^{\delta} - f_d - f_x I_x \right\},$$

where

$$\begin{aligned} r &= \left[1 + I_x \tau^{-\beta/(1-\beta)} \left(\frac{A^*}{A} \right)^{1/(1-\beta)} \right]^{1-\beta} \\ &\quad \times A \kappa_y^{\beta} [\lambda_1(\theta_1 a_{c1}^{1-\gamma k} n_1^{\gamma})^{\nu} + \lambda_2(\theta_2 a_{c2}^{1-\gamma k} n_2^{\gamma})^{\nu}]^{\beta/\nu}, \\ \kappa_y &\equiv \frac{k a_{\min}^{\gamma k}}{k-1}. \end{aligned}$$

Note that we have allowed b_ℓ to vary across the two groups, but have assumed that $a_{\min \ell}$ and c_ℓ are the same. It is straightforward to allow for variation in any of these parameters across the two groups.

The first-order conditions for the firm's problem can be written as

$$\frac{\beta\gamma}{1+\beta\gamma}\phi_\ell r = b_\ell n_\ell,$$
$$\frac{\beta(1-\gamma k)}{1+\beta\gamma}\phi_\ell r = c a_{c\ell}^{\delta}.$$

Using these first-order conditions, we obtain

$$\varphi = \left(\frac{\lambda_1}{\lambda_2}\right)^{1/\Lambda}\left(\frac{b_1}{b_2}\right)^{-\gamma\nu/\Lambda}\left(\frac{\theta_1}{\theta_2}\right)^{\nu/\Lambda}, \quad \Lambda \equiv 1-\nu\gamma-\frac{\nu}{\delta}(1-\gamma k) > 0, \tag{S37}$$

and

$$\frac{h_1(\theta)}{h_2(\theta)} = \frac{b_2}{b_1}\varphi^{1-k/\delta} \quad \text{and} \quad \frac{w_1(\theta)}{w_2(\theta)} = \frac{b_1}{b_2}\varphi^{k/\delta}. \tag{S38}$$

Recall that we assume $\theta_1 = \theta$ and $\theta_2 \equiv 1$; under these circumstances, φ is monotonically increasing in θ. Note that, in contrast to the Cobb–Douglas case, both relative wages and relative employment of skilled labor are increasing in the productivity of the firm.

Finally, note that it is still the case that profits can be written as

$$\pi(\theta) = \frac{\Gamma}{1+\beta\gamma}r(\theta) - f_d - f_x I_x,$$

where Γ is defined as before (this step requires common k and δ across the two groups). Therefore, the sectoral equilibrium analysis can be handled as before and we omit it for brevity.

Firm-Level Variation

Solving the model, one arrives at[2]

$$r(\theta) = \kappa_r \Upsilon(\theta)^{(1-\beta)/\Gamma}(1+\varphi)^{\beta\Lambda/(\nu\Gamma)},$$
$$h_2(\theta) = h_{d2}\Upsilon(\theta)^{(1-\beta)(1-k/\delta)/\Gamma}(1+\varphi)^{[\beta\Lambda/(\nu\Gamma)-1](1-k/\delta)},$$
$$w_2(\theta) = w_{d2}\Upsilon(\theta)^{(1-\beta)(k/\delta)/\Gamma}(1+\varphi)^{[\beta\Lambda/(\nu\Gamma)-1](k/\delta)},$$

where φ as a function of θ is given in (S37), and $h_1(\theta)$ and $w_1(\theta)$ can be immediately recovered from (S38). Note that $\nu < \beta$ implies $\Lambda > \Gamma$ and $\nu = \beta$ implies $\Lambda = \Gamma$. Therefore, when $\nu < \beta$, unskilled employment and wages increase with

[2]To obtain this solution, we can express

$$r(\theta) = \kappa_y^{\beta}\Upsilon(\theta)^{1-\beta}A[1+\varphi]^{\beta/\nu}\left(\lambda_2^{1/\nu}a_{c2}(\theta)^{1-\gamma k}n_2(\theta)^{\gamma}\right)^{\beta}$$

and then use the first-order conditions.

productivity of the firm, which is the empirically relevant pattern. The case of $\nu = \beta$ implies a constant unskilled employment and wage rate across firms of different productivity (while both employment and wages still increase with export status). Finally, $\nu > \beta$ results in a decreasing unskilled employment and wage rate in firm productivity.

We start by discussing the two limiting cases. When $\nu = 0$, we are back in the Cobb–Douglas case studied above. In this case, employment and wages for both groups of labor are power functions of θ; therefore, the results of Proposition 3 immediately generalize for inequality within each group, and under our symmetry assumptions there is an equal amount of inequality within each group.

The second limiting case is $\nu = \beta$. In this case, $w_2(\theta) = w_{d2} Y(\theta)^{k(1-\beta)/(\delta\Gamma)}$ and is independent of firm productivity. Therefore, unskilled workers face no wage inequality in autarky and positive wage inequality in a trade equilibrium. In fact, Proposition 3 applies in this case as well, as both employment and wages of the unskilled are trivial power functions of θ (with a power of 0).

Next consider the group of skilled workers. Equation (S38) implies $h_1(\theta) = h_{d1} Y(\theta)^{(1-\beta)/\Gamma(1-k/\delta)} \varphi^{1-k/\delta}$ and $w_2(\theta) = w_{d2} Y(\theta)^{k(1-\beta)/(\delta\Gamma)} \varphi^{k/\delta}$. From (S37) it follows that φ is a power function of θ. Therefore, both employment and wages of skilled workers are a power function of firm productivity, so that Proposition 3 again applies. To summarize, when $\nu = \beta$, trade increases inequality within both skilled and unskilled groups and there is more inequality within the skilled group than within the unskilled group.

Finally, having discussed what happens at the limits, we study the intermediate case of $0 < \nu < \beta$. In this case, employment and wages in the two groups are no longer simple power functions of firm productivity θ. Therefore, the sharp characterization which leads to Proposition 3 does not generalize to this case.[3] Nevertheless, the numerical solution is still straightforward in this case. Our numerical exploration suggests that the qualitative patterns depicted in Figure 3 also arise in the general CES case with $0 < \nu < \beta$. Moreover, when $\nu > 0$, inequality in the skilled group exceeds that in the unskilled group.

S6. GENERAL EQUILIBRIUM

S6.1. *Outside Sector and Risk Neutrality*

As noted in the paper, the Harris–Todaro condition that equates ex ante expected indirect utility in the two sectors takes the form

$$\text{(S39)} \qquad \mathbb{V} = \frac{1}{1-\eta} \mathbb{E}\left(\frac{w}{\mathcal{P}}\right)^{1-\eta} = \frac{\mathbb{E}w}{\mathcal{P}} = \frac{1}{\mathcal{P}} \quad \text{for} \quad \eta = 0,$$

[3] Specifically, the distribution of employment and wages across firms is generalized Pareto in this case. Recall that the property of Pareto distribution is that the shape parameter of the distribution is a sufficient statistic for inequality. This property no longer holds for the generalized Pareto distribution.

which, given the probability of matching x and the expected wage conditional on matching of $w(\theta)h(\theta)/n(\theta) = b$, implies

$$xb = \omega = 1.$$

From this expression, a sufficient condition for $\alpha_0 > \omega$, and hence $0 < x < 1$ in (S19), is $\alpha_0 > 1$.

Ex post indirect utility depends on the aggregate price index ($\mathcal{P}$) and ex post wages, which depend on a worker's sector and firm of employment:

$$\text{(S40)} \qquad \mathbb{V} = \begin{cases} 1/\mathcal{P}, & \text{if employed in the outside sector,} \\ w(\theta)/\mathcal{P}, & \text{if employed by a } \theta \text{ firm in the differentiated sector,} \\ 0, & \text{if unemployed,} \end{cases}$$

where $\mathcal{P}$ is the dual of the aggregate consumption index ($\mathcal{C}$),

$$\text{(S41)} \qquad \mathcal{P} = \left[\vartheta + (1-\vartheta)P^{-\zeta/(1-\zeta)}\right]^{-(1-\zeta)/\zeta},$$

and depends on the differentiated sector price index (P), which can be determined from the expression for the demand shifter (A) with CES preferences,

$$\text{(S42)} \qquad A^{1/(1-\beta)} = \frac{(1-\vartheta)P^{(\beta-\zeta)/((1-\beta)(1-\zeta))}\bar{L}}{\vartheta + (1-\vartheta)P^{-\zeta/(1-\zeta)}},$$

where we have used $\Omega = \bar{L}$. From the solutions for firm-specific variables (S16), ex post wages in the differentiated sector in the closed and open economies are

$$w^a(\theta) = w_d\left(\frac{\theta}{\theta_d^a}\right)^{\beta k/(\delta\Gamma)} \quad \text{for} \quad \theta \geq \theta_d^a,$$

$$w^t(\theta) = \begin{cases} w_d\left(\dfrac{\theta}{\theta_d^t}\right)^{\beta k/(\delta\Gamma)}, & \text{for } \theta \in [\theta_d^t, \theta_x^t), \\ \Upsilon_x^{k(1-\beta)/(\delta\Gamma)} w_d\left(\dfrac{\theta}{\theta_d^t}\right)^{\beta k/(\delta\Gamma)}, & \text{for } \theta \geq \theta_x^t, \end{cases}$$

where the superscripts a and t denote closed and open economy variables, respectively.

To characterize the impact of the opening of trade on ex post wages at domestic and exporting firms, note that $w^t(\theta) < w^a(\theta)$ for $\theta \in [\theta_d^t, \theta_x^t)$ because $\theta_d^t > \theta_d^a$. Note also that $w^t(\theta) > w^a(\theta)$ for $\theta \geq \theta_x^t$ whenever $\Upsilon_x^{1-\beta} > (\theta_d^t/\theta_d^a)^{\beta}$.

To show that this inequality must be satisfied, note that the free entry conditions in the closed and open economies (S28) together imply

$$\left(\frac{\theta_d^t}{\theta_d^a}\right)^z = \left[1+\left(\frac{f_d}{f_x}\right)^{(z\Gamma-\beta)/\beta}\left[\Upsilon_x^{(1-\beta)/\Gamma}-1\right]^{z\Gamma/\beta}\right].$$

We can rewrite this condition as

$$\left(\frac{\theta_d^t}{\theta_d^a}\right)^z - 1 = \left[\Upsilon_x^{(1-\beta)/\Gamma}-1\right]^{z\Gamma/\beta}\left(\frac{f_d}{f_x}\right)^{z\Gamma/\beta-1} < \left[\Upsilon_x^{(1-\beta)/\Gamma}\right]^{z\Gamma/\beta}-1,$$

where the above inequality holds because $\Upsilon_x > 1$, $z\Gamma > \beta$, and $f_d/f_x < 1$. This immediately implies $(\theta_d^t/\theta_d^a)^\beta < \Upsilon_x^{1-\beta}$.

Proposition 6 can be proved as follows. The Harris–Todaro condition under risk neutrality implies $xb = \omega = 1$, which together with the search technology (S17) implies that search costs and labor market tightness are invariant to trade: $b = \alpha_0^{1/(1+\alpha_1)}$ and $x = \alpha_0^{-1/(1+\alpha_1)}$. From the free entry condition (S28), θ_d is higher in the open economy than in the closed economy. From the zero-profit productivity cutoff condition (S20), a higher value of θ_d and an unchanged value of b imply a lower value of A in the open economy than in the closed economy. From (S42) above, a lower value of A implies a lower value of P in the open economy than in the closed economy. From (S41), this reduction in P in turn implies a lower value of $\mathcal{P}$ and higher ex ante expected welfare (S39) in the open economy than in the closed economy, which establishes the proposition.

S6.2. *Single Differentiated Sector and Risk Neutrality*

The conditions for sectoral equilibrium remain the same as in (S17)–(S27). With a single differentiated sector, the expression for the demand shifter in (S24) becomes

$$A \equiv Q^{1-\beta}P = Q^{1-\beta}, \tag{S43}$$

where we have used the choice of numeraire $P = 1$. Therefore, total revenues are $E = Q$.

The solutions for (θ_d, θ_x) under the assumption of country symmetry follow from (S28) and (S22), as discussed above. Using symmetric countries, equilibrium labor force (S27), and labor market clearing ($L = \bar{L}$), we obtain

$$\frac{\beta\gamma}{1+\beta\gamma}Q = \bar{L}\omega.$$

Using search technology (S17), expected worker income (S18), and the zero-profit cutoff condition (S20), we obtain

$$Q = \left(\frac{f_d}{\kappa_r}\frac{1+\beta\gamma}{\Gamma}\right)^{\Gamma/(1-\beta)} \times c^{\beta(1-\gamma k)/((1-\beta)\delta)}\alpha_0^{\beta\gamma/((1-\beta)(1+\alpha_1))}\theta_d^{-\beta/(1-\beta)}\omega^{(\beta\gamma/(1-\beta))(\alpha_1/(1+\alpha_1))}.$$

These equations define two upward-sloping relationships in (Q, ω) space that determine the equilibrium values of Q and ω. One can show that a necessary and sufficient condition for the stability of the equilibrium is

$$\text{(S44)} \qquad \frac{\beta\gamma}{1-\beta}\frac{\alpha_1}{1+\alpha_1} > 1,$$

which is satisfied for sufficiently large α_1 (sufficiently convex hiring costs) and sufficiently large β (a sufficiently high elasticity of substitution between varieties). As $0 < \alpha_1/(1+\alpha_1) < 1$, a necessary condition for this parameter restriction to hold is

$$\frac{\beta\gamma}{1-\beta} > 1,$$

which is satisfied for values of β sufficiently close to but less than 1. Assuming parameter values satisfying these inequalities, the conditions for sectoral equilibrium in (S17)–(S27) (where (S43) replaces (S24)) can be solved to yield the following solutions for the other endogenous variables of the model:

$$\text{(S45)} \qquad Q = Q^* = \frac{1+\beta\gamma}{\beta\gamma}\alpha_0^{\gamma\beta/((1-\beta)\Delta)}c^{\beta(1-\gamma k)(1+\alpha_1)/(\delta(1-\beta)\Delta)}\theta_d^{-\beta(1+\alpha_1)/((1-\beta)\Delta)} \times \bar{L}^{-\gamma\beta/((1-\beta)\Delta)}\kappa_b^{(1+\alpha_1)/\Delta},$$

$$\omega = \omega^* = \alpha_0^{\gamma\beta/((1-\beta)\Delta)}c^{\beta(1-\gamma k)(1+\alpha_1)/(\delta(1-\beta)\Delta)}\theta_d^{-\beta(1+\alpha_1)/((1-\beta)\Delta)} \times \bar{L}^{-(1+\alpha_1)/\Delta}\kappa_b^{(1+\alpha_1)/\Delta},$$

$$b = b^* = \alpha_0^{1/\Delta}c^{\beta(1-\gamma k)\alpha_1/(\delta(1-\beta)\Delta)}\theta_d^{-\beta\alpha_1/((1-\beta)\Delta)}\bar{L}^{-\alpha_1/\Delta}\kappa_b^{\alpha_1/\Delta},$$

$$x = x^* = \alpha_0^{(\gamma\beta/(1-\beta)-1)/\Delta}c^{\beta(1-\gamma k)/(\delta(1-\beta)\Delta)}\theta_d^{-\beta/((1-\beta)\Delta)}\bar{L}^{-1/\Delta}\kappa_b^{1/\Delta},$$

$$M = M^* = \left(\frac{z\Gamma-\beta}{z}\right)\frac{\theta_{\min}^{-z}}{f_d\beta\gamma}\alpha_0^{\gamma\beta/((1-\beta)\Delta)}c^{\beta(1-\gamma k)(1+\alpha_1)/(\delta(1-\beta)\Delta)} \times \theta_d^{z-\beta(1+\alpha_1)/((1-\beta)\Delta)}\bar{L}^{-\gamma\beta/((1-\beta)\Delta)}\kappa_b^{(1+\alpha_1)/\Delta},$$

where

$$\Delta \equiv \frac{(1-\beta)(1+\alpha_1) - \beta\gamma\alpha_1}{(1-\beta)} < 0,$$

$$\kappa_b \equiv \frac{\beta\gamma}{(1+\beta\gamma)} \left[\frac{f_d}{\kappa_r} \frac{(1+\beta\gamma)}{\Gamma} \right]^{\Gamma/(1-\beta)} > 0.$$

From the above, a sufficient condition for $\alpha_0 > \omega$, and hence $0 < x < 1$ in (S19), is

$$\alpha_0^{(1+\alpha_1)[(1-\beta)-\beta\gamma]/((1+\alpha_1)(1-\beta)-\alpha_1\beta\gamma)} > c^{\beta(1-\gamma k)(1+\alpha_1)/(\delta(1-\beta)\Delta)} \theta_d^{-\beta(1+\alpha_1)/((1-\beta)\Delta)} \times \bar{L}^{-(1+\alpha_1)/\Delta} \kappa_b^{(1+\alpha_1)/\Delta}.$$

Finally, under the assumption of risk neutrality, expected indirect utility is

$$\text{(S46)} \qquad \mathbb{V} = \mathbb{E}\Big(\frac{w}{\mathcal{P}}\Big) = \omega \quad \text{for} \quad \eta = 0.$$

The proof of Proposition 7 follows immediately from the closed-form solutions for (θ_d, θ_x) in (S28) and (S22) and from the closed-form solutions for $(Q, \omega, b, x, M, \mathbb{V})$ in (S45) and (S46).

S6.3. *Outside Sector and Risk Aversion*

We first show how the introduction of worker risk aversion affects the equilibrium share of revenue received by workers in the bargaining game. Specifically, with CRRA-CES preferences, the solution to the bargaining game under risk aversion takes a similar form as when there are differences in bargaining weight between the firm and its workers. Given workers' revenue share, we next show that the determination of sectoral equilibrium remains unchanged except for the Harris–Todaro condition equating expected utility in the two sectors.

S6.3.1. *Stole and Zweibel Bargaining*

The bargaining game takes the same form as discussed in the paper for risk neutrality, except that workers are assumed to be risk averse ($0 < \eta < 1$), and instead of assuming equal bargaining weights, we allow for differences in bargaining weight between the firm and its workers. The bargaining power of workers is denoted by $\lambda \in (0, 1)$ and their outside option of unemployment involves zero income.

We start with the case of a discrete number of workers h and then take the limit of continuous divisibility of the workforce. Denote by $R(h)$ the revenue function when a firm employs h workers and denote by $w(h)$ the equilibrium wage schedule that the firm pays to each worker when the employment level is h. Now consider that the firm is separated with $\delta > 0$ workers (the special case is $\delta = 1$ and we will later take the limit $\delta \to 0$). This will reduce revenue to $R(h-\delta)$ and the wage rate to $w(h-\delta)$.

Now consider the firm's bargaining with the marginal δ workers when employment is h. Denote by t the wage that the firm pays to these δ workers, while other workers receive $w(h)$. The utility of these marginal workers is $t^{1-\eta}/(1-\eta)$. If the bargaining breaks down and they leave, the revenues will fall by $R(h) - R(h-\delta)$ and the firm will be paying the remaining $h-\delta$ workers the wage rate $w(h-\delta)$. Therefore, the incremental payoff to the firm from employing these marginal δ workers is

$$[R(h) - R(h-\delta)] - (h-\delta)[w(h) - w(h-\delta)] - \delta t.$$

Assuming zero outside option for workers, the payoff from employment to each of the marginal workers is equal to $t^{1-\eta}/(1-\eta)$. Therefore, we can write Stole and Zweibel's bargaining solution as

$$\begin{aligned} t(h) = \arg\max_t \Big\{ \big([R(h) - R(h-\delta)] \\ - (h-\delta)[w(h) - w(h-\delta)] - \delta t \big)^{1-\lambda} \left(\frac{1}{1-\eta} t^{1-\eta} \right)^{\lambda} \Big\}. \end{aligned}$$

The equilibrium requirement is that $t(h) \equiv w(h)$. That is, for every employment level h, as a result of bargaining with marginal workers, the firm pays them accordingly to the equilibrium wage schedule.

Denote by

$$\phi \equiv \frac{1-\lambda}{\lambda(1-\eta)}$$

the effective relative bargaining weight of the firm. Then we can implicitly write the bargaining solution, taking into account the equilibrium requirement, as

$$(1+\phi)\delta w(h) = [R(h) - R(h-\delta)] - (h-\delta)[w(h) - w(h-\delta)].$$

This holds for every $h > 0$. We now take the limit as $\delta \to 0$ to obtain the differential equation for the wage schedule:

$$(1+\phi) w(h) = R'(h) - w'(h) h \quad \Rightarrow \quad w(h) = \frac{1}{h^{1+\phi}} \int_0^h R'(\xi) \xi^{\phi} \, d\xi.$$

This is the generalized Stole and Zweibel (1996a, 1996b) condition for the case of asymmetric bargaining weights. Therefore, with CRRA-CES preferences, risk aversion is equivalent to an adjustment in the bargaining weights (greater risk aversion of workers reduces their bargaining weight). When revenue is a power function of employment with power $\beta\gamma$ ($R(h) = Ah^{\beta\gamma}$), the solution to this differential equation is given by

$$w(h) = \frac{\beta\gamma}{\phi + \beta\gamma}\frac{R(h)}{h}.$$

Note that the wage rate is again a constant fraction of average revenues. The fraction of revenues accruing to workers is increasing in $\beta\gamma$ (decreasing in the concavity of revenues) and decreasing in ϕ (the relative effective bargaining power of the firm). In turn, ϕ is decreasing in λ (the primitive bargaining power of workers) and increasing in η (the degree of risk aversion of workers). The case of symmetric bargaining weights and no risk aversion is a special case of this more general formulation. Specifically, when $\lambda = 1$ and $\eta = 0$, we have $\phi = 1$ and arrive at our baseline formulation in the paper.

S6.3.2. *Sectoral Equilibrium*

After taking into account the change in workers' revenue share, the only equilibrium condition that changes when risk aversion is introduced is the Harris–Todaro condition of worker indifference between searching for employment in the two sectors. Specifically, this condition now equates the utility from a certain income of 1 in the homogenous sector with the expected utility from being hired and receiving a wage drawn from the equilibrium wage distribution in the differentiated sector:

$$x\sigma\mathbb{E}w^{1-\eta} = x\sigma\int_{w_d}^{\infty} w^{1-\eta}\,dG_w(w) = 1. \tag{S47}$$

Evaluating the integral using the open economy wage distribution (S31), this condition becomes

$$\begin{aligned} x\sigma\mathbb{E}w^{1-\eta} &= x\sigma\frac{1+\mu}{1+\mu\eta}w_d^{1-\eta}\left[\frac{1+\rho^{z-\beta(1-\eta k/\delta)/\Gamma}[Y_x^{(1-\beta)(1-\eta k/\delta)/\Gamma}-1]}{1+\rho^{z-\beta(1-k/\delta)/\Gamma}[Y_x^{(1-\beta)(1-k/\delta)/\Gamma}-1]}\right] \\ &= 1. \end{aligned}$$

Recall that the open economy hiring rate can be expressed as

$$\sigma = \frac{1+[Y_x^{(1-\beta)(1-k/\delta)/\Gamma}-1]\rho^{z-\beta(1-k/\delta)/\Gamma}}{1+[Y_x^{(1-\beta)/\Gamma}-1]\rho^{z-\beta/\Gamma}}\cdot\frac{1}{1+\mu}\cdot\frac{1}{\phi_w},$$

where

$$\phi_w \equiv \left[\frac{\Gamma}{\beta(1-\gamma k)}\frac{ca_{\min}^{\delta}}{f_d}\right]^{k/\delta}.$$

In addition, the lowest wage in the differentiated sector can be written as $w_d = b\phi_w$. Therefore, we can rewrite the Harris–Todaro condition as

$$\Lambda(\rho, Y_x)\frac{b^{1-\eta}x}{\phi_w^\eta(1+\mu\eta)} = 1, \tag{S48}$$

where

$$\Lambda(\rho, Y_x) \equiv \frac{1+\rho^{z-\beta(1-\eta k/\delta)/\Gamma}[Y_x^{(1-\beta)(1-\eta k/\delta)/\Gamma} - 1]}{1+\rho^{z-\beta/\Gamma}[Y_x^{(1-\beta)/\Gamma} - 1]}.$$

Evidently, we have $\Lambda(0, Y_x) = 1$ and $0 < \Lambda(\rho, Y_x) < 1$ for $0 < \rho \le 1$.

Using the Harris–Todaro condition (S48) and the hiring cost function (S17), we obtain the following expressions for (x, b):

$$x = \left(\frac{(1+\mu\eta)\phi_w^\eta}{\alpha_0^{1-\eta}\Lambda(\rho, Y_x)}\right)^{1/(1+(1-\eta)\alpha_1)}, \tag{S49}$$

$$b = \alpha_0^{1/(1+(1-\eta)\alpha_1)}\left(\frac{(1+\mu\eta)\phi_w^\eta}{\Lambda(\rho, Y_x)}\right)^{\alpha_1/(1+(1-\eta)\alpha_1)}. \tag{S50}$$

Expected worker income is, therefore,

$$\omega = xb = \alpha_0^{\eta/(1+(1-\eta)\alpha_1)}\left(\frac{(1+\mu\eta)\phi_w^\eta}{\Lambda(\rho, Y_x)}\right)^{(1+\alpha_1)/(1+(1-\eta)\alpha_1)}. \tag{S51}$$

A sufficient condition for $0 < x < 1$ is now given by

$$\alpha_0^{1-\eta} > \frac{(1+\mu\eta)\phi_w^\eta}{\Lambda(\rho, Y_x)}.$$

The proof of Proposition 8 follows from the above. In the closed economy, $\rho = 0$ and, therefore, $\Lambda(0, Y_x) = 1$. Opening the economy to trade leads to $\rho = \theta_d/\theta_x > 0$ and, therefore, $\Lambda(\rho, Y_x) < 1$. Consequently, equations (S49)–(S51) imply that x, b, and ω are higher in the open economy than in autarky.

S7. SUPPLEMENTARY DERIVATIONS

S7.1. *Derivation of the Production Technology*

We assume the production function

$$y = \theta h^\gamma \bar{a} = \theta\left(\frac{1}{h}\right)^{1-\gamma}\int_0^h a_i\,di, \tag{S52}$$

where $i \in [0, h]$ indexes the workers employed by the firm. One way to think about this technology is the following: A manager with productivity θ has 1 unit of time which he allocates equally among his employees. Thus, the manager allocates $1/h$ of his time to each worker and, as a result, a worker with match-specific ability a can contribute $\theta(1/h)^{1-\gamma}a$ to the total output of the firm, where $(1-\gamma)$ measures the importance of the managerial time input. Aggregating across workers yields the assumed production function. We further assume, following a large literature on moral hazard in teams, that the contributions of individual workers to total output are unobservable as production is done in teams and the production process is nonseparable. As a result, the match-specific ability of workers cannot be deduced from observing output. This justifies the assumption that the manager splits his time equally among the workers since they all appear homogeneous because of the unobservable and nonverifiable nature of their match-specific ability. Alternatively, an equal managerial time allocation among workers can be rationalized by assuming that the productivity of each worker depends on average worker ability as a result of human capital externalities across workers within firms.[4]

S7.2. *Marginal Product of Labor*

Given the production technology (S52), the marginal product of a worker with match-specific ability a is[5]

$$\mathrm{MP}(a) \equiv \mathrm{MP}(a|\bar{a}, h; \theta) = \theta h^{-(1-\gamma)}[a - (1-\gamma)\bar{a}].$$

Let match-specific ability in the pool of candidate employees be distributed according to cumulative distribution function $G_a(a)$. Then, if the firm only hires workers with ability above a_c, the mean ability of its workers is

$$\bar{a}(a_c) = \frac{1}{1 - G_a(a_c)} \int_{a_c}^{\infty} a\, dG_a(a).$$

[4]For empirical evidence on human capital externalities within plants, see, for example, Moretti (2004). See also related work on O-ring production technologies following Kremer (1993).

[5]To define the marginal product, rewrite the production function as

$$y = \theta \left[\int_0^h di\right]^{-(1-\gamma)} \int_0^h a_i\, di.$$

Then the marginal product of adding worker h with productivity a_h is

$$\mathrm{MP}_h \equiv \frac{dy}{dh} = \theta h^{-(1-\gamma)}[a_h - (1-\gamma)\bar{a}].$$

Note that the production function does not depend on the ordering of workers and, hence, the marginal product depends only on the ability of the worker in the sense that $\mathrm{MP}_h = \mathrm{MP}(a_h) = \mathrm{MP}(a_h|\bar{a}, h; \theta)$.

The marginal product of the threshold-ability worker is thus

$$\mathrm{MP}(a_c) = \theta h^{-(1-\gamma)}[a_c - (1-\gamma)\bar{a}(a_c)],$$

which is negative whenever $(1-\gamma)\bar{a}(a_c) > a_c$. Since $\bar{a}(a_c) > a_c$ for any nondegenerate distribution, $\mathrm{MP}(a_c) < 0$ can be guaranteed by choosing γ small enough.

Specifically, consider the case of Pareto-distributed ability as we assume in the paper. In this case, $\bar{a}(a_c) = ka_c/(k-1)$ and the marginal product is

$$\mathrm{MP}(a_c) = -\theta h^{-(1-\gamma)}\frac{1-\gamma k}{k-1}a_c,$$

which is negative whenever $\gamma k < 1$. Recall that this is the same parameter restriction which ensures that output is increasing in the cutoff ability. It also guarantees that the workers not hired by the firm have negative marginal product. The firm will not want to retain these workers even at a zero wage, because the resulting reduction in output from lower average worker ability would dominate the increase in output from a greater measure of hired workers.

Note further that given $\gamma k < 1$, the firm hires some workers with ability in the range $[a_c, \hat{a})$, where

$$\hat{a} = (1-\gamma)\bar{a} = \frac{(1-\gamma)k}{k-1}a_c > a_c,$$

and these workers have a negative marginal product. The firm would prefer not to hire these workers, but costly search and screening make it optimal to hire all workers with match-specific ability above a_c. Finally, note that the average marginal product of workers employed by a firm with productivity θ is always positive:

$$\overline{\mathrm{MP}}(\theta) = \gamma\theta h(\theta)^{-(1-\gamma)}\bar{a}(a_c(\theta)) > 0.$$

S7.3. *Division of Revenue in the Bargaining Game*

In our model, the firm and workers bargain about the division of revenues, as the outside option of workers is normalized to zero. As in Stole and Zweibel (1996a, 1996b), the firm bargains with every worker, taking into account the effect of his departure on the bargaining game with remaining workers. All the firm's other decisions—sampling, screening, production, exporting—are sunk by the bargaining stage. Therefore, from (S3) and (S10), revenues $r(\theta, h)$ are a continuous, increasing, and concave function of employment h, and all other arguments of firm revenue (θ, $\bar{a}(\theta)$, $Y(\theta)$, A) are fixed. Let $w(\theta, h)$ be the bargained wage rate that a θ firm pays as a function of the measure of workers hired h. This function has to satisfy the differential equation

$$\frac{\partial}{\partial h}[r(\theta,h) - w(\theta,h)h] = w(\theta,h) \tag{S53}$$

so that the surplus of workers from employment (the wage rate) is equal to the marginal surplus of the firm from employing the worker.[6] Using the assumed functional forms for revenues, this differential equation yields the solution

$$w(\theta, h) = \frac{\beta\gamma}{1+\beta\gamma}\,\frac{r(\theta, h)}{h},$$

so that each worker gets a fraction $\beta\gamma/(1+\beta\gamma)$ of average revenue and the firm gets the remaining $1/(1+\beta\gamma)$ share of revenue. The worker's share of the surplus is increasing in $\beta\gamma$, where β captures the concavity of demand and γ captures the concavity of the production technology. Therefore, the worker's share of the surplus is decreasing in the concavity of the revenue function in h, because a more concave revenue function implies a smaller effect of the departure of any given worker on firm revenue.

S7.4. *Derivation of the Search Cost (b)*

Following Blanchard and Gali (2010), search costs (S17) can be derived from a constant returns to scale matching technology and a cost of posting vacancies. Suppose there is a cost of posting a vacancy of ψ_0 units of the numeraire. Suppose also that the measure of matched workers is a Cobb–Douglas function of vacancies and workers searching for employment:

$$N = \psi_1 V^{\psi_2} L^{1-\psi_2}, \quad \psi_1 > 0, 0 < \psi_2 < 1.$$

Given this search technology, a firm choosing to match with n workers has to post $v > n$ vacancies:

$$v = \psi_1^{-1/\psi_2} x^{(1-\psi_2)/\psi_2} n, \quad x \equiv N/L.$$

Given the cost of posting each vacancy, a firm choosing to match with n workers incurs a per worker search cost of

$$b = \alpha_0 x^{\alpha_1}, \quad \alpha_0 \equiv \left(\frac{\psi_0}{\psi_1^{1/\psi_2}}\right), \alpha_1 \equiv \frac{(1-\psi_2)}{\psi_2}.$$

[6]Since individual abilities are unobservable, the expressions on both sides of (S53) are evaluated holding $\bar{a} = \bar{a}(\theta)$ constant for all h. Indeed, when a marginal worker departs, the productivity of the remaining workers is still $\bar{a}$ in expectation. The firm evaluates the expected loss from the departure of a worker as

$$\frac{\partial r}{\partial h} + \frac{\partial r}{\partial \bar{a}}\frac{\partial \bar{a}}{\partial h},$$

where $\partial \bar{a}/\partial h = 0$ in expectation and, hence, there remains only the direct effect of h on revenues. As a result, the bargaining solution in this environment with symmetric uncertainty is the same as if all workers had a productivity of $\bar{a}$.

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