Econometrica Supplementary Material

SUPPLEMENT TO "MEDIA COMPETITION AND SOCIAL DISAGREEMENT"

(*Econometrica*, Vol. 90, No. 1, January 2022, 223–265)

JACOPO PEREGO
Economics Division, Graduate School of Business, Columbia University

SEVGI YUKSEL
Department of Economics, UC Santa Barbara

APPENDIX B: ADDITIONAL MATERIAL

B.1. *Proof of Proposition 5*

THE PROOF OF PROPOSITION 5 is divided into five lemmas, structured as follows: Lemma B.1 proves claim (a) in the proposition; Lemmas B.2 proves claim (b); Lemmas B.3 and B.4 are interim results that we use in the proof of Lemma B.5; finally, Lemma B.5 proves claim (c).

LEMMA B.1—Existence: *Let f be regular, let $N \geq 1$, and let $I \geq 1$. An equilibrium of the game exists.*

PROOF: We first establish that an equilibrium of the game exists. As in Section 3, we solve the game by backward induction. In the last stage of the game, each agent t_i observe the *realized* profile of (pure) editorial strategies and prices $(x_n, t_n, p_n)_{n=1}^N$. The agents' equilibrium strategies are determined by Lemmas 1 and 2. These results are independent of the distribution f and, thus, they equally apply to the case under consideration. In the second stage of the game, each firm observes the *realized* profile of (pure) editorial strategies and the vector of realized types $(t_1, \dots, t_I)$, and chooses a price $p_n(t_i)$ for each type. Since firms observe types and can set discriminatory prices, the equilibrium profile of prices is independent of the distribution f. As for the uniform case, given the *realized* profile of (pure) editorial strategies, the prevailing equilibrium price for firm n is $\max\{0, v((x_n, t_n)|t) - V((x_{n'}, t_{n'})_{n' \neq n}|t)\}$. Therefore, the expected profit for firm n is

$$\Pi_n\big((x_n, t_n)_{n=1}^N\big) = I \int_{-\pi}^{\pi} \max\big\{0, v\big((x_n, t_n)|t\big) - V\big((x_{n'}, t_{n'})_{n' \neq n}|t\big)\big\}\, dF(t). \tag{B.1}$$

Next, we argue that for all n, $\Pi_n((x_n, t_n)_{n=1}^N)$ is continuous in $(x_n, t_n)_{n=1}^N$. To see this, let us consider an arbitrary sequence of editorial-strategy profiles $((x_n^k, t_n^k)_{n=1}^N)_k \subset ([1/2, 1] \times T)^N$ converging to $(x_n, t_n)_{n=1}^N$ as $k \to \infty$. We want to show that $\lim_{k\to\infty} \Pi_n((x_n^k, t_n^k)_{n=1}^N) = \Pi_n((x_n, t_n)_{n=1}^N)$. Clearly, the set $([1/2, 1] \times T)^N$ is compact. Moreover, $0 \leq \max\{0, v((x_n^k,$

Jacopo Perego: jacopo.perego@columbia.edu
Sevgi Yuksel: sevgi.yuksel@ucsb.edu

 https://doi.org/10.3982/ECTA16417

$t_n^k)|t) - V((x_{n'}^k, t_{n'}^k)_{n'\neq n}|t)\} \leq \lambda\sqrt{2}$ for all k and t. We have

$$\begin{aligned}\lim_{k\to\infty} \Pi_n\big((x_n^k, t_n^k)_{n=1}^N\big) &= I\int_{-\pi}^{\pi} \lim_{k\to\infty} \max\{0, v\big((x_n^k, t_n^k)|t\big) - V\big((x_{n'}^k, t_{n'}^k)_{n'\neq n}|t\big)\}\, dF(t) \\ &= I\int_{-\pi}^{\pi} \max\Big\{0, \lim_{k\to\infty} v\big((x_n^k, t_n^k)|t\big) - V\big((x_{n'}^k, t_{n'}^k)_{n'\neq n}|t\big)\Big\}\, dF(t) \\ &= I\int_{-\pi}^{\pi} \max\{0, v\big((x_n, t_n)|t\big) - V\big((x_{n'}, t_{n'})_{n'\neq n}|t\big)\}\, dF(t) \\ &= \Pi_n\big((x_n, t_n)_{n=1}^N\big).\end{aligned}$$

The first equality follows from the dominated convergence theorem. The second equality holds as the max operator is continuous. The third equality follows because $v((x_n^k, t_n^k)|t) = \lambda(\sqrt{x_n^k} + \sqrt{1 - x_n^k}\cos(t - t_n^k))$ is continuous in (x_n^k, t_n^k) for all n. Therefore, $v((x_n, t_n)|t) - V((x_{n'}, t_{n'})_{n'\neq n}|t)$ is continuous. Therefore, for all n, the strategy space is compact and the payoff function $\Pi_n((x_n, t_n)_{n=1}^N)$ is continuous. By Glicksberg's theorem, the first stage of the game admits a Nash equilibrium in mixed editorial strategies. Therefore, by backward induction, the game admits an equilibrium. *Q.E.D.*

LEMMA B.2—Daily-Me: I: *Let F be regular and let $I \geq 1$. Fix an arbitrary sequence of equilibria, one for each N. Denote by $\mathcal{V}(N|t_i)$ the expected value of information for type t_i in the equilibrium with N firms. Then $\lim_{N\to\infty} \mathcal{V}(N|t_i) = \bar{\mathcal{V}}$ for all t_i.*

PROOF: Fix $\delta > 0$ and let $\xi_1 = \frac{\delta}{2\lambda}$. Let $\bar{\mathcal{V}} = \max_{(x_n,t_n)} v(x_n, t_n|t_i) = \lambda\sqrt{2}$. This is the highest possible value that $v(x_n, t_n|t_i)$ can achieve and it is independent of t_i. We show that there exists $\bar{N}$ such that for all $N > \bar{N}$ and any equilibrium profile of possibly mixed editorial strategies $\chi \in (\Delta([1/2, 1] \times T))^N$, we have $\mathcal{V}(N|t_i) = \mathbb{E}_\chi(\max_n\{v(x_n, t_n|t_i)\}) > \bar{\mathcal{V}} - \delta$ for all $t_i \in T$. Suppose not. That is, suppose that for all N, there is an equilibrium profile of possibly mixed editorial strategies χ and a type $\bar{t}_i$ such that $\mathbb{E}_\chi(\max_n\{v(x_n, t_n|t_i)\}) \leq \bar{\mathcal{V}} - \delta$. This implies that for all $t_j \in [\bar{t}_i - \xi_1, \bar{t}_i + \xi_1]$, $\mathbb{E}_\chi(\max_n\{v(x_n, t_n|t_j)\}) \leq \bar{\mathcal{V}} - \frac{\delta}{2}$. To see this, suppose, by way of contradiction, that $\mathbb{E}_\chi(\max_n\{v(x_n, t_n|t_j)\}) > \bar{\mathcal{V}} - \frac{\delta}{2}$. Denote by $n(t_j)$ the random variable that, for each realization of χ, indicates the firm from which t_j acquires information. Note that for all $t_n \in T$, $\cos(\bar{t}_i - t_n) \geq \cos(t_j - t_n) - \xi_1$, since $\frac{d}{dt}\cos(t - t_n) \leq 1$. We have that

$$\begin{aligned}\mathbb{E}_\chi\Big(\max_n\big\{v\big((x_n, t_n)|t_i\big)\big\}\Big) &\geq \mathbb{E}_\chi\big(v\big((x_{n(t_j)}, t_{n(t_j)})|t_i\big)\big) \\ &= \lambda\mathbb{E}_\chi\big(\sqrt{x_{n(t_j)}} + \sqrt{1 - x_{n(t_j)}}\cos(\bar{t}_i - t_{n(t_j)})\big) \\ &\geq \lambda\mathbb{E}_\chi\big(\sqrt{x_{n(t_j)}} + \sqrt{1 - x_{n(t_j)}}\big(\cos(t_j - t_{n(t_j)}) - \xi_1\big) \\ &\geq \lambda\mathbb{E}_\chi\big(\sqrt{x_{n(t_j)}} + \sqrt{1 - x_{n(t_j)}}\cos(t_j - t_{n(t_j)})\big) - \lambda\xi_1 \\ &\geq \mathbb{E}_\chi\Big(\max_n\big\{v(x_n, t_n|t_j)\big\}\Big) - \lambda\xi_1\end{aligned}$$

$$> \bar{\mathcal{V}} - \frac{\delta}{2} - \lambda \xi_1$$
$$= \bar{\mathcal{V}} - \delta.$$

The first inequality holds as, in the right-hand side, agent $\bar{t}_i$ chooses the firm $n(t_j)$ that is optimal for t_j. The second inequality holds since $\cos(\bar{t}_i - t_n) \geq \cos(t_j - t_n) - \xi_1$ for all t_n. In summary, this contradicts our assumption that $\mathbb{E}_\chi(\max_n\{v(x_n, t_n|t_i)\}) \leq \bar{\mathcal{V}} - \delta$. Therefore, it must be that $\mathbb{E}_\chi(\max_n\{v(x_n, t_n|t_j)\}) \leq \bar{\mathcal{V}} - \frac{\delta}{2}$.

Note that, by continuity of $v((x_n, t_n)|t_j)$ in t_j, there exists $\xi_2 > 0$ such that for all $t_j \in [\bar{t}_i - \xi_2, \bar{t}_i + \xi_2]$ such that $v((1/2, \bar{t}_i)|t_j) \geq \bar{\mathcal{V}} - \frac{\delta}{4}$. Moreover, such ξ_2 is independent of N. Let $\xi = \min\{\xi_1, \xi_2\}$, which in turn is independent of N. We have established that for all $t_j \in [\bar{t}_i - \xi, \bar{t}_i + \xi]$,

$$\mathbb{E}_\chi\Big(\max_{n' \neq n}\big\{v(x_{n'}, t_{n'}|t_j)\big\}\Big) \leq \mathbb{E}_\chi\Big(\max_{n}\big\{v(x_n, t_n|t_j)\big\}\Big) \leq \bar{\mathcal{V}} - \frac{\delta}{2} < \bar{\mathcal{V}} - \frac{\delta}{4} \leq v\big((1/2, \bar{t}_i)|t_j\big). \quad \text{(B.2)}$$

Consider an arbitrary firm n that deviates from its equilibrium editorial strategy (χ_n) in favor of the pure strategy $(x_n = 1/2, t_n = \bar{t}_i)$. Its expected profits are

$$\begin{aligned}
\Pi_n\big((x_n, t_n), (\chi_{n'})_{n' \neq n}\big) &= I \int_{-\pi}^{\pi} \mathbb{E}_\chi\Big(\max\big\{v\big((x_n, t_n)|t_j\big) - V\big((x_{n'}, t_{n'})_{n' \neq n}|t_j\big), 0\big\}\Big)\, dF(t_j) \\
&\geq I \int_{\bar{t}_i - \xi}^{\bar{t}_i + \xi} \mathbb{E}_\chi\Big(\max\big\{v\big((x_n, t_n)|t_j\big) - V\big((x_{n'}, t_{n'})_{n' \neq n}|t_j\big), 0\big\}\Big)\, dF(t_j) \\
&\geq I \int_{\bar{t}_i - \xi}^{\bar{t}_i + \xi} \mathbb{E}_\chi\Big(v\big((x_n, t_n)|t_j\big) - V\big((x_{n'}, t_{n'})_{n' \neq n}|t_j\big)\Big)\, dF(t_j) \\
&= I \int_{\bar{t}_i - \xi}^{\bar{t}_i + \xi} v\big((x_n, t_n)|t_j\big) - \mathbb{E}_\chi\Big(V\big((x_{n'}, t_{n'})_{n' \neq n}|t_j\big)\Big)\, dF(t_j) \\
&= I \int_{\bar{t}_i - \xi}^{\bar{t}_i + \xi} v\big((x_n, t_n)|t_j\big) - \mathbb{E}_\chi\Big(\max_{n' \neq n}\big\{v(x_{n'}, t_{n'}|t_j)\big\}\Big)\, dF(t_j) \\
&\geq I \int_{\bar{t}_i - \xi}^{\bar{t}_i + \xi} \left(\bar{\mathcal{V}} - \frac{\delta}{4} - \bar{\mathcal{V}} + \frac{\delta}{2}\right) f(t_j)\, dt_j \\
&\geq \frac{IC\delta\xi}{2}.
\end{aligned}$$

The first inequality holds since the integrand function is everywhere positive. The second inequality holds by monotonicity of the operator $\mathbb{E}_\chi$. The second-to-last inequality obtains as a consequence of Equation (B.2). The last inequality, instead, obtains because $f(t_j) \geq C > 0$ for all t_j. We established that firm n can secure an expected profit of at least $\frac{IC\delta\xi}{2}$ by deviating to $(x_n = 1/2, t_n = \bar{t}_i)$. This lower bound is strictly positive and independent of N. To conclude the proof, note that the industry profits are bounded above by $I\bar{\mathcal{V}}$. Therefore, when N firms are competing, there is at least one firm, which we denote by n, whose expected equilibrium profits is $\Pi_n(\chi) \leq I\bar{\mathcal{V}}/N$. When N is large, $\frac{IC\delta\xi}{2} > I\bar{\mathcal{V}}/N$ and firm n is a strictly profitable deviation from its equilibrium editorial strategy χ_n in the first stage of the game. Therefore, χ is not an equilibrium—a contradiction. *Q.E.D.*

LEMMA B.3—Daily-Me: II: *Let F be regular and let $I \geq 1$. For any t_i, denote by $(x^N_{n(t_i)}, t^N_{n(t_i)})$ the random variable specifying the information structure that agent t_i acquires in an equilibrium with N firms. Then $(x^N_{n(t_i)}, t^N_{n(t_i)}) \to (1/2, t_i)$ in probability as $N \to \infty$.*

PROOF: Fix t_i, $\epsilon > 0$, and a sequence of equilibria. For any N, denote by $(x^N_{n(t_i)}, t^N_{n(t_i)})$ the random variable specifying the information structure that agent t_i acquires in equilibrium. We want to show that for all $\delta > 0$, there exists $\bar{N}$ such that for all $N > \bar{N}$, $\Pr(\|(x^N_{n(t_i)}, t^N_{n(t_i)}) - (1/2, t_i)\| > \epsilon) < \delta$. Suppose not. Then there is $\delta > 0$ such that for all $\bar{N}$, there is $N > \bar{N}$ such that $\Pr(\|(x^N_{n(t_i)}, t^N_{n(t_i)}) - (1/2, t_i)\| > \epsilon) \geq \delta$. Let (x_n, t_n) be a realization of $(x^N_{n(t_i)}, t^N_{n(t_i)})$ such that $\|(x_n, t_n) - (1/2, t_i)\| > \epsilon$. That is, $\sqrt{(x_n - 1/2)^2 + (t_n - t_i)^2} > \epsilon$. This implies that

$$\max\{|x_n - 1/2|, |t_n - t_i|\} > \frac{\epsilon}{\sqrt{2}}.$$

Consider the difference $\bar{\mathcal{V}} - v((x_n, t_n)|t_i) = \lambda(\sqrt{2} - (\sqrt{x_n} + \sqrt{1 - x_n}\cos(t_n - t_i)))$. Suppose $|t_n - t_i| > \frac{\epsilon}{\sqrt{2}}$. Then

$$\bar{\mathcal{V}} - v\big((x_n, t_n)|t_i\big) \geq \frac{\lambda}{\sqrt{2}}\big(1 - \cos(t_n - t_i)\big) > \frac{\lambda}{\sqrt{2}}\left(1 - \cos\left(\frac{\epsilon}{\sqrt{2}}\right)\right) =: K_1(\epsilon) > 0.$$

Conversely, suppose that $|x_n - 1/2| > \frac{\epsilon}{\sqrt{2}}$. Then

$$\begin{aligned}\bar{\mathcal{V}} - v\big((x_n, t_n)|t_i\big) &\geq \lambda(\sqrt{2} - \sqrt{x_n} - \sqrt{1 - x_n}) \\ &> \lambda\left(\sqrt{2} - \frac{1}{2}\big(\sqrt{1 + \epsilon\sqrt{2}} + \sqrt{1 - \epsilon\sqrt{2}}\big)\right) =: K_2(\epsilon) > 0.\end{aligned}$$

Let $K(\epsilon) = \min\{K_1(\epsilon), K_2(\epsilon)\}$. We established that for all realizations of the random variable $(x^N_{n(t_i)}, t^N_{n(t_i)})$ that satisfy $\|(x^N_{n(t_i)}, t^N_{n(t_i)}) - (1/2, t_i)\| > \epsilon$, we have $\bar{\mathcal{V}} - v((x_n, t_n)|t_i) > K(\epsilon) > 0$. This implies that

$$\Pr\big(\bar{\mathcal{V}} - v\big((x^N_{n(t_i)}, t^N_{n(t_i)})|t_i\big) > K(\epsilon)\big) \geq \delta.$$

Since δ and ϵ are independent of N, we conclude that $\mathcal{V}(N|t_i) = \mathbb{E}_\chi(v((x^N_{n(t_i)}, t^N_{n(t_i)})|t_i))$ does not converge to $\bar{\mathcal{V}}$, a contradiction to Lemma B.2. *Q.E.D.*

LEMMA B.4: *Let F be regular and let $I \geq 1$. For any sequence of equilibria indexed by N,*

$$\mathcal{U}(N) \to \frac{I-1}{I}\mathbb{E}_{\omega,t_i,t_j}\left(\Phi\left(\frac{1}{\sqrt{2}}u_j(\omega, t_j)\right)u_i(\omega, t_i)\right) + \bar{\mathcal{V}}.$$

PROOF: Fix $N \geq 1$. Let χ be an equilibrium profile of (possibly mixed) editorial strategies. Denote by $(x_{n(t_i)}, t_{n(t_i)})$ the equilibrium random variable which specifies the information structure that is chosen by agent t_i among those that are offered by the N firms. As shown in the proof of Lemma 2 and Proposition 4, the agent's expected welfare can be written as

$$\mathcal{U}(N) = \mathbb{E}_\chi\mathbb{E}_{(t_1,\dots,t_I)}\big(\mathbb{E}_\omega\big(A_{-i}(\omega, t_{-i})u(\omega, t_i)\big) + v\big((x_{n(t_i)}, t_{n(t_i)}|t_i) - p_{n(t_i)}(t_i)\big). \tag{B.3}$$

We begin by showing that

$$\lim_{N\to\infty} \mathbb{E}_\chi \mathbb{E}_{t_i}\big(v\big((x_{n(t_i)}, t_{n(t_i)}|t_i) - p_{n(t_i)}(t_i)\big) = \bar{\mathcal{V}}. \tag{B.4}$$

To see this, fix t_i. We want to show that $\lim_{N\to\infty} \mathbb{E}_\chi(p_{n(t_i)}(t_i)) = 0$. For each N, recall that $p_{n(t_i)}(t_i) = v(x_{n(t_i)}, t_{n(t_j)}) - \max_{m\neq n(t_i)} v((x_m, t_m)|t_i)$. Thanks to Lemma B.2, it is enough to show that

$$\lim_{N\to\infty} \mathbb{E}_\chi\Big(\max_{m\neq n(t_i)} v\big((x_m, t_m)|t_i\big)\Big) = \bar{\mathcal{V}}.$$

In Lemma B.3, we established that for all t_i, $(x_{n(t_i)}, t_{n(t_i)}) \to (1/2, t_i)$ in probability. This implies that for any $t_j \neq t_i$, as N goes to infinity, $\Pr(n(t_i) = n(t_j)) \to 0$. This implies that the value generated for type t_i by the firm acquired by t_j should be a lower bound for $\max_{m\neq n(t_i)} v((x_m, t_m)|t_i)$ in the limit. Formally,

$$\lim_{N\to\infty} \Pr\Big(v\big((x_{n(t_j)}, t_{n(t_j)})|t_i\big) \leq \max_{n\neq n(t_i)} v\big((x_n, t_n)|t_i\big)\Big) = 1.$$

By the continuous mapping theorem, the fact that $(x_{n(t_j)}, t_{n(t_j)})$ converges to $(1/2, t_j)$ in probability implies that $v((x_{n(t_j)}, t_{n(t_j)})|t_i) \to v((1/2, t_j)|t_i)$ in probability. Now fix any $\epsilon > 0$ and $\delta > 0$. There exists t_j close enough to t_i such that $\bar{\mathcal{V}} - v((1/2, t_j)|t_i) < \epsilon$. Therefore,

$$\Pr\Big(\bar{\mathcal{V}} - \max_{n\neq n(t_i)} v\big((x_n, t_n)|t_i\big) < \epsilon\Big) > 1 - \delta.$$

That is, $\max_{n\neq n(t_i)} v((x_n, t_n)|t_i)$ converges in probability to $\bar{\mathcal{V}}$. Since $|v|$ is bounded, this implies that $\mathbb{E}_\chi(\max_{n\neq n(t_i)} v((x_n, t_n)|t_i))$ converges to $\bar{\mathcal{V}}$. Together with Lemma B.2, this shows that for any t_i, $\lim_{N\to\infty} \mathbb{E}_\chi(v((x_{n(t_i)}, t_{n(t_i)})|t_i) - p_{n(t_i)}(t_i)) = \bar{\mathcal{V}}$. Since t_i was arbitrary and its distribution is independent of χ, Equation (B.4) holds.

We are left to show that the first term in Equation (B.3) converges to

$$\frac{I-1}{I}\mathbb{E}_{\omega,t_i,t_j}\bigg(\Phi\bigg(\frac{1}{\sqrt{2}}u_j(\omega, t_j)\bigg)u_i(\omega, t_i)\bigg).$$

To this purpose, recall that

$$\begin{aligned} A_{-i}(\omega, t_{-i}) &= \frac{1}{I}\sum_{j\neq i} a_j(\omega, t_j) \\ &= \frac{1}{I}\sum_{j\neq i} \Phi\big(\sqrt{x_{n(t_j)}}\omega_0 + \sqrt{1 - x_{n(t_j)}}\big(\cos(t_{n(t_j)})\omega_1 + \sin(t_{n(t_j)})\omega_2\big)\big). \end{aligned}$$

Moreover, note that χ, ω, and $(t_1, \ldots, t_I)$ are mutually independent random variables. Therefore, by swapping the order of integration and defining $U_i(\omega) = \mathbb{E}_{t_i} u(\omega, t_i)$ to simplify notation, we obtain

$$\begin{aligned}\lim_{N\to\infty}\mathbb{E}_\chi\mathbb{E}_{(t_1,\dots,t_I)}\big(\mathbb{E}_\omega\big(A_{-i}(\omega,t_{-i})u(\omega,t_i)\big)\big) &= \frac{1}{I}\lim_{N\to\infty}\mathbb{E}_\omega\bigg(\sum_{j\neq i}\mathbb{E}_{t_j}\mathbb{E}_\chi a_j(\omega,t_j)\mathbb{E}_{t_i}u(\omega,t_i)\bigg)\\ &= \frac{(I-1)}{I}\lim_{N\to\infty}\mathbb{E}_\omega\big(\mathbb{E}_{t_j}\mathbb{E}_\chi a_j(\omega,t_j)\mathbb{E}_{t_i}u(\omega,t_i)\big)\\ &= \frac{(I-1)}{I}\lim_{N\to\infty}\mathbb{E}_\omega\big(\mathbb{E}_{t_j}\mathbb{E}_\chi a_j(\omega,t_j)U_i(\omega)\big).\end{aligned}$$

Fix (ω, t_j). By Lemma B.3, the random variable $(x_{n(t_i)}, t_{n(t_i)})$ converges in probability to the constant $(\frac{1}{2}, t_j)$. Since $\Phi(\cdot)$ is continuous,

$$\Phi\big(\sqrt{x_{n(t_j)}}\omega_0+\sqrt{1-x_{n(t_j)}}\big(\cos(t_{n(t_j)})\omega_1+\sin(t_{n(t_j)})\omega_2\big)\big)\to\Phi\bigg(\frac{1}{\sqrt{2}}u_j(\omega,t_j)\bigg)$$

in probability, by the continuous mapping theorem. Moreover, since $\Phi(\cdot)\in[0,1]$, convergence in probability implies convergence in expectation. That is,

$$\lim_{N\to\infty}\mathbb{E}_\chi a(\omega,t_i)=\Phi\bigg(\frac{1}{\sqrt{2}}u_j(\omega,t_j)\bigg).$$

Moreover, since $a_j(\omega,t_j)\le 1$, for all χ, $U_i(\omega)\mathbb{E}_\chi a_j(\omega,t_j)\le U_i(\omega)$, and $\mathbb{E}_\omega U_i(\omega)\in\mathbb{R}$. Therefore, by the dominated convergence theorem,

$$\begin{aligned}\lim_{N\to\infty}\mathbb{E}_\chi\mathbb{E}_{(t_1,\dots,t_I)}\big(\mathbb{E}_\omega\big(A_{-i}(\omega,t_{-i})u(\omega,t_i)\big)\big) &= \frac{(I-1)}{I}\lim_{N\to\infty}\mathbb{E}_\omega\big(\mathbb{E}_{t_j}\mathbb{E}_\chi a_j(\omega,t_j)U_i(\omega)\big)\\ &= \frac{(I-1)}{I}\mathbb{E}_\omega\Big(\mathbb{E}_{t_j}\lim_{N\to\infty}\mathbb{E}_\chi a_j(\omega,t_j)U_i(\omega)\Big)\\ &= \frac{(I-1)}{I}\mathbb{E}_{\omega,t_i,t_j}\bigg(\Phi\bigg(\frac{1}{\sqrt{2}}u_j(\omega,t_j)\bigg)u_i(\omega,t_i)\bigg),\end{aligned}$$

which concludes the proof. *Q.E.D.*

LEMMA B.5: *Let F be regular. There exists $\bar{I}$ such that for all $I>\bar{I}$, the agent's expected welfare is higher under monopoly than perfect competition. That is,* $\mathcal{U}(1)>\lim_{N\to\infty}\mathcal{U}(N)$.

PROOF: We first compute $\mathcal{U}(1)$ and then compute $\lim_{N\to\infty}\mathcal{U}(N)$.

Monopoly, $N=1$. Fix f and let $N=1$. The monopolistic firm chooses $(x^\star, t^\star)$ to maximize $I\int_{-\pi}^{\pi}v((x^\star,t^\star)|t_i)f(t_i)\,dt_i=\lambda I\int_{-\pi}^{\pi}\sqrt{x^\star}+\sqrt{1-x^\star}\cos(t^\star-t_i)f(t_i)\,dt_i$. The first-order condition with respect to t implies $-\int\sin(t^\star-t_i)f(t_i)\,dt_i=0$. By symmetry of f around t^m, the first-order condition is met at $t^\star\in\{t^m, t^m+\pi(\operatorname{mod}\pi)\}\subset[-\pi,\pi]$. The second-order condition with respect to t implies $-\int\cos(t^\star-t_i)f(t_i)\,dt_i\le 0$. Since $\cos(t+\pi)=-\cos(t)$, we have that either $\int\cos(t^m-t_i)f(t_i)\,dt_i\ge 0$ or $\int\cos(t^m+\pi-t_i)f(t_i)\,dt_i\ge 0$ (or both). Without loss of generality, let t^m be the type at which $\int\cos(t^m-t_i)f(t_i)\,dt_i\ge 0$. Therefore, the monopolist locates at $t^\star=t^m$. Define $\beta_F=\int\cos(t^m-t_i)f(t_i)\,dt_i\in[0,1]$. Given this, we can rewrite the monopoly profits for an arbitrary x as $I(\sqrt{x}+\sqrt{1-x}\beta_F)$. The first-order condition with respect to x gives $\sqrt{1-x^\star}=\beta_F\sqrt{x^\star}$, which implies $x^\star=$

$\frac{1}{1+\beta_F^2}$. Hence, we established that the equilibrium editorial strategy chosen by a monopolist is $(\frac{1}{1+\beta_F^2}, t^m)$.

We now compute $\mathcal{U}(1)$. We begin by establishing that

$$\mathbb{E}_{t_i}\big(u(\omega, t_i)\big) = \omega_0 + \beta_F\big(\cos\big(t^m\big)\omega_1 + \sin\big(t^m\big)\omega_2\big). \tag{B.5}$$

To see this, notice that

$$\begin{aligned}
\mathbb{E}_{t_i}\big(u(\omega, t_i)\big) &= \omega_0 + \int_{-\pi}^{\pi} \big(\cos(t_i)\omega_1 + \sin(t_i)\omega_2\big) f(t_i)\, dt_i \\
&= \omega_0 + \left(\cos\big(t^m\big) \int \cos\big(t_i - t^m\big) f(t_i)\, dt_i - \sin\big(t^m\big) \int \sin\big(t_i - t^m\big) f(t_i)\, dt_i\right)\omega_1 \\
&\quad + \left(\sin\big(t^m\big) \int \cos\big(t_i - t^m\big) f(t_i)\, dt_i - \cos\big(t^m\big) \int \sin\big(t_i - t^m\big) f(t_i)\, dt_i\right)\omega_2 \\
&= \omega_0 + \beta_F\big(\cos\big(t^m\big)\omega_1 + \sin\big(t^m\big)\omega_2\big).
\end{aligned}$$

In the first equality, we used $t_i = t^m + (t_i - t^m)$ and the following two trigonometric identities: $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$ and $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$. In the second equality, we used that f is symmetric around t_m (implying $\int \sin(t^m - t) f(t)\, dt = 0$) and the definition of β_F.

Equation (B.3) characterizes the expected welfare for a typical agent. When $N = 1$, $p^\star(t_i) = v((x^\star, t^\star)|t_i)$ for all t_i. That is, the monopolist extracts all surplus from each type. Therefore, the last two terms of Equation (B.3) cancel out. Thus, using the mutual independence between ω and $(t_1, \dots, t_I)$, we have

$$\begin{aligned}
\mathcal{U}(1) &= \mathbb{E}_{(t_1,\dots,t_I)}\big(\mathbb{E}_\omega\big(A^\star_{-i}(\omega, t_{-i}) u(\omega, t_i)\big)\big) \\
&= \mathbb{E}_\omega\big(\mathbb{E}_{t_{-i}}\big(A^\star_{-i}(\omega, t_{-i})\big)\mathbb{E}_{t_i}\big(u(\omega, t_i)\big)\big) \\
&= \mathbb{E}_\omega\big(\mathbb{E}_{t_{-i}}\big(A^\star_{-i}(\omega, t_{-i})\big)\big(\omega_0 + \beta_F\big(\cos\big(t^m\big)\omega_1 + \sin\big(t^m\big)\omega_2\big)\big)\big) \\
&= \mathbb{E}_\omega\left(\frac{I-1}{I}\mathbb{E}_{t_j}\big(a^\star(\omega, t_j)\big)\big(\omega_0 + \beta_F\big(\cos\big(t^m\big)\omega_1 + \sin\big(t^m\big)\omega_2\big)\big)\right) \qquad \text{(B.6)} \\
&= \mathbb{E}_\omega\left(\frac{I-1}{I}\Phi\big(\sqrt{x^\star}\omega_0 + \sqrt{1-x^\star}\big(\cos\big(t^m\big)\omega_1 + \sin\big(t^m\big)\omega_2\big)\big)\right. \\
&\qquad \left.\times \big(\omega_0 + \beta_F\big(\cos\big(t^m\big)\omega_1 + \sin\big(t^m\big)\omega_2\big)\big)\right).
\end{aligned}$$

Next, denote $y = \cos(t^m)\omega_1 + \sin(t^m)\omega_2 \sim \mathcal{N}(0, 1)$, $a = \sqrt{x^\star}$, and $b = \sqrt{1 - x^\star}$. We have

$$\begin{aligned}
\mathcal{U}(1) &= \frac{I-1}{I}\mathbb{E}_{\omega_0}\big(\mathbb{E}_y\big(\Phi(a\omega_0 + by)(\omega_0 + \beta_F y)\big)\big) \\
&= \frac{I-1}{I}\mathbb{E}_{\omega_0}\mathbb{E}_y\big(\Phi(a\omega_0 + by)(\omega_0 + \beta_F y)\big)
\end{aligned}$$

$$= \frac{I-1}{I}\mathbb{E}_{\omega_0}\big(\omega_0\mathbb{E}_y\big(\Phi(a\omega_0+by)\big)+\beta_F\mathbb{E}_y\big(y\Phi(a\omega_0+by)\big)\big)$$
$$= \frac{I-1}{I}\mathbb{E}_{\omega_0}\left(\omega_0\Phi\left(\frac{a\omega_0}{\sqrt{1+b^2}}\right)+\frac{\beta_F b}{\sqrt{1+b^2}}\phi\left(\frac{a\omega_0}{\sqrt{1+b^2}}\right)\right).$$

The last line makes use of the properties of the normal distribution. Specifically, since $y \sim \mathcal{N}(0,1)$, we have that, for all $\alpha, \gamma \in \mathbb{R}$, $\mathbb{E}_y(\Phi(\alpha+\gamma y)) = \Phi(\frac{\alpha}{\sqrt{1+\gamma^2}})$ and $\mathbb{E}_y(y\Phi(\alpha+\gamma y)) = \frac{\gamma}{\sqrt{1+\gamma^2}}\phi(\frac{\alpha}{\sqrt{1+\gamma^2}})$ (for both, see Patel and Read (1996)).

Finally, we integrate with respect to w_0. Define $\tilde{b} = \frac{a}{\sqrt{1+b^2}}$. We have

$$\begin{aligned}
\mathcal{U}(1) &= \frac{I-1}{I}\mathbb{E}_{\omega_0}\left(\omega_0\Phi(\tilde{b}\omega_0)+\frac{\beta_F b}{\sqrt{1+b^2}}\phi(\tilde{b}\omega_0)\right)\\
&= \frac{I-1}{I}\left(\frac{\tilde{b}}{\sqrt{1+\tilde{b}^2}}\phi(0)+\frac{\beta_F b}{\sqrt{1+b^2}}\frac{1}{\sqrt{1+\tilde{b}^2}}\phi(0)\right)\\
&= \frac{I-1}{I2\sqrt{\pi}}\big(\sqrt{x^\star}+\beta_F\sqrt{1-x^\star}\big)\\
&= (I-1)\lambda\sqrt{1+\beta_F^2}.
\end{aligned}$$

In the second line, we used once again the integral properties of the normal distribution listed before. In the third line, we substituted the definitions of $\tilde{b}$, b, and a, and used the fact that $\phi(0) = 1/\sqrt{2\pi}$. In the last line, we used $x^\star = \frac{1}{1+\beta_F^2}$ and $\lambda = \frac{1}{2I\sqrt{\pi}}$. In passing, note that $\beta_F = 0$ if f is uniform. In such a case, the value of $\mathcal{U}(1)$ matches the one computed in the Proof of Proposition 4.

Perfect Competition, $N = \infty$. Lemma B.4 showed that for any sequence of equilibria indexed by N,

$$\lim_{N\to\infty}\mathcal{U}(N) = \frac{I-1}{I}\mathbb{E}_{\omega,t_i,t_j}\left(\Phi\left(\frac{1}{\sqrt{2}}u_j(\omega,t_j)\right)u_i(\omega,t_i)\right)+\bar{\mathcal{V}}.$$

We begin by focusing on the first term of the right-hand side. Note that

$$\begin{aligned}
&\mathbb{E}_{\omega,t_i,t_j}\left(\Phi\left(\frac{1}{\sqrt{2}}u_j(\omega,t_j)\right)u_i(\omega,t_i)\right)\\
&\quad= \mathbb{E}_{\omega,t_j}\left(\Phi\left(\frac{1}{\sqrt{2}}u_j(\omega,t_j)\right)\mathbb{E}_{t_i}u_i(\omega,t_i)\right)\\
&\quad= \mathbb{E}_{\omega,t_j}\left(\Phi\left(\frac{1}{\sqrt{2}}\big(\omega_0+\cos(t_j)\omega_1+\sin(t_j)\omega_2\big)\right)\big(\omega_0+\beta_F\big(\cos\big(t^m\big)\omega_1+\sin\big(t^m\big)\omega_2\big)\big)\right),
\end{aligned}$$

where we used Equation (B.5) and the fact that (ω, t_i, t_j) are independent.

Fix any t_j and consider first the expectation with respect to ω. To simplify notation, let us write $t_j = t$ and $a = b = 1/\sqrt{2}$. Then

$$\begin{aligned}
&\mathbb{E}_\omega\big(\Phi\big(a\omega_0 + b\big(\cos(t)\omega_1 + \sin(t)\omega_2\big)\big)\big(\omega_0 + \beta_F\big(\cos\big(t^m\big)\omega_1 + \sin\big(t^m\big)\omega_2\big)\big)\big) \\
&\quad = \mathbb{E}_\omega\big(\omega_0\Phi\big(a\omega_0 + b\big(\cos(t)\omega_1 + \sin(t)\omega_2\big)\big)\big) \\
&\qquad + \beta_F \cos\big(t^m\big)\mathbb{E}_\omega\big(\omega_1\Phi\big(a\omega_0 + b\big(\cos(t)\omega_1 + \sin(t)\omega_2\big)\big)\big) \\
&\qquad + \beta_F \sin\big(t^m\big)\mathbb{E}_\omega\big(\omega_2\Phi\big(a\omega_0 + b\big(\cos(t)\omega_1 + \sin(t)\omega_2\big)\big)\big).
\end{aligned}$$

We compute this expectation term by term. We begin with the first term. Let $y_t = \cos(t_j)\omega_1 + \sin(t_j)\omega_2$ and note that $y_t \sim \mathcal{N}(0, 1)$. Define $\tilde{b} = \frac{a}{\sqrt{1+b^2}}$. Then, using the independence of $(\omega_0, \omega_1, \omega_2)$, we have

$$\begin{aligned}
\mathbb{E}_\omega\big(\omega_0\Phi\big(a\omega_0 + b\big(\cos(t)\omega_1 + \sin(t)\omega_2\big)\big)\big) &= \mathbb{E}_{\omega_0}\big(\omega_0\mathbb{E}_{y_t}\Phi\big(a\omega_0 + by_t\big)\big) \\
&= \mathbb{E}_{\omega_0}\left(\omega_0\Phi\left(\frac{a}{\sqrt{1+b^2}}\omega_0\right)\right) \\
&= \frac{\tilde{b}}{\sqrt{1+\tilde{b}^2}}\phi(0) \\
&= \frac{1}{2}\phi(0).
\end{aligned}$$

We now focus on the second term. We first integrate ω_1, then ω_2, and finally ω_0. As we have done before, we use the integral identity $\mathbb{E}_z(z\Phi(\alpha + \gamma z)) = \frac{\gamma}{\sqrt{1+\gamma^2}}\phi(\frac{\alpha}{\sqrt{1+\gamma^2}})$ for all $\alpha, \gamma \in \mathbb{R}$ and $z \sim \mathcal{N}(0, 1)$. Moreover, we use a new integral identity that gives us $\mathbb{E}(\phi(\alpha + \gamma z)) = \frac{1}{\sqrt{1+\gamma^2}}\phi(\frac{\alpha}{\sqrt{1+\gamma^2}})$ (see Patel and Read (1996)). We obtain

$$\begin{aligned}
&\mathbb{E}_\omega\big(\omega_1\Phi\big(a\omega_0 + b\big(\cos(t)\omega_1 + \sin(t)\omega_2\big)\big)\big) \\
&\quad = \mathbb{E}_{\omega_0,\omega_2}\big(\mathbb{E}_{\omega_1}\big(\omega_1\Phi\big(\big(a\omega_0 + b\sin(t)\omega_2\big) + b\cos(t)\omega_1\big)\big)\big) \\
&\quad = \frac{b\cos(t)}{\sqrt{1+b^2\cos^2(t)}}\mathbb{E}_{\omega_0,\omega_2}\phi\left(\frac{a\omega_0 + b\sin(t)\omega_2}{\sqrt{1+b^2\cos^2(t)}}\right) \\
&\quad = \frac{b\cos(t)}{\sqrt{1+b^2}}\mathbb{E}_{\omega_0}\phi\left(\frac{a\omega_0}{\sqrt{1+b^2}}\right) \\
&\quad = \frac{b\cos(t)}{\sqrt{1+b^2}}\frac{1}{\sqrt{1+\tilde{b}^2}}\phi(0) \\
&\quad = \frac{1}{2}\cos(t)\phi(0).
\end{aligned}$$

We now focus on the last term. We first integrate ω_2, then ω_1, and finally ω_0. Otherwise, the steps and properties we follow are identical to those from the second term:

$$\begin{aligned}
&\mathbb{E}_\omega\big(\omega_2\Phi\big(a\omega_0+b\big(\cos(t)\omega_1+\sin(t)\omega_2\big)\big)\big)\\
&\quad=\mathbb{E}_{\omega_0,\omega_1}\big(\mathbb{E}_{\omega_2}\big(\omega_2\Phi\big(\big(a\omega_0+b\cos(t)\omega_1\big)+b\sin(t)\omega_2\big)\big)\big)\\
&\quad=\frac{b\sin(t)}{\sqrt{1+b^2\sin^2(t)}}\mathbb{E}_{\omega_0,\omega_1}\phi\left(\frac{a\omega_0+b\cos(t)\omega_1}{\sqrt{1+b^2\sin^2(t)}}\right)\\
&\quad=\frac{b\sin(t)}{\sqrt{1+b^2}}\mathbb{E}_{\omega_0}\phi\left(\frac{a\omega_0}{\sqrt{1+b^2}}\right)\\
&\quad=\frac{b\sin(t)}{\sqrt{1+b^2}}\frac{1}{\sqrt{1+\tilde{b}^2}}\phi(0)\\
&\quad=\frac{1}{2}\sin(t)\phi(0).
\end{aligned}$$

Putting all together, we have that

$$\begin{aligned}
\lim_{N\to\infty}\mathcal{U}(N)&=\frac{I-1}{I}\mathbb{E}_{\omega,t_i,t_j}\left(\Phi\left(\frac{1}{\sqrt{2}}u_j(\omega,t_j)\right)u_i(\omega,t_i)\right)+\bar{\mathcal{V}}\\
&=\frac{I-1}{I}\mathbb{E}_{t_j}\left(\frac{1}{2}\phi(0)+\frac{1}{2}\beta_F\phi(0)\big(\cos(t_j)\cos\big(t^m\big)+\sin(t_j)\sin\big(t^m\big)\big)\right)+\bar{\mathcal{V}}\\
&=\frac{I-1}{I}\frac{1}{2}\phi(0)\mathbb{E}_{t_j}\big(1+\beta_F\cos\big(t_j-t^m\big)\big)+\bar{\mathcal{V}}\\
&=\frac{I-1}{I}\frac{1}{2}\frac{1}{\sqrt{2\pi}}\big(1+\beta_F^2\big)+\lambda\sqrt{2}\\
&=\lambda(I-1)\frac{1}{\sqrt{2}}\big(1+\beta_F^2\big)+\lambda\sqrt{2}.
\end{aligned}$$

For the fourth equality, we use the definition β_F. Moreover, we used the fact that $\bar{\mathcal{V}}=\lambda\sqrt{2}$ and $\phi(0)=\frac{1}{\sqrt{2\pi}}$. In passing, note that $\beta_F=0$ if f is uniform. In such a case, the value of $\lim_{N\to\infty}\mathcal{U}(N)$ matches the one computed in Equation (A.9).

Comparison Between Monopoly and Perfect Competition. We established that

$$\mathcal{U}(1)-\lim_{N\to\infty}\mathcal{U}(N)=\lambda(I-1)\sqrt{1+\beta_F^2}\left(1-\sqrt{\frac{1+\beta_F^2}{2}}\right)-\lambda\sqrt{2}.$$

Note that for all nondegenerate distributions F, $\beta_F\in[0,1)$. Therefore, $1>\sqrt{\frac{1+\beta_F^2}{2}}$. That is, for any distribution F, there exists a $\bar{I}$ such that for all $I>\bar{I}$, $\mathcal{U}(1)>\lim_{N\to\infty}\mathcal{U}(N)$. That is, the expected welfare of a typical agent is higher when $N=1$ than when $N\to\infty$. *Q.E.D.*

REMARK B.1: Fix a regular F. For all N, the equilibrium editorial strategy of the monopolist maximizes $\mathcal{G}(N)$.

PROOF: Fix N. Let $(x_{n(i)}, t_{n(i)})$ denote the editorial strategy associated with the signal acquired by type t_i and let $a^\star(\omega, t_i)$ denote the optimal approval decision for type t_i given the signal induced by $(x_{n(i)}, t_{n(i)})$:

$$\begin{aligned}
\mathcal{G}(N) &= \mathbb{E}_{(t_1,\dots,t_I)}\big(\mathbb{E}_\omega\big(A^\star_{-i}(\omega, t_{-i})u(\omega, t_i)\big)\big) \\
&= \mathbb{E}_\omega\bigg(\frac{I-1}{I}\mathbb{E}_{t_j}\big(a^\star(\omega, t_j)\big)\big(\omega_0 + \beta_F\big(\cos\big(t^m\big)\omega_1 + \sin\big(t^m\big)\omega_2\big)\big)\bigg) \\
&= \mathbb{E}_\omega\bigg(\frac{I-1}{I}\mathbb{E}_{t_j}\big(\Phi\big(\sqrt{x} + \sqrt{1-x}\big(\cos(t)\omega_1 + \sin(t)\omega_2\big)\big)\big) \\
&\qquad \times \big(\omega_0 + \beta_F\big(\cos\big(t^m\big)\omega_1 + \sin\big(t^m\big)\omega_2\big)\big)\bigg) \\
&= \frac{I-1}{I}\frac{1}{\sqrt{2}}\phi(0)\big(\sqrt{x} + \sqrt{1-x}\beta_F\cos\big(t-t^m\big)\big).
\end{aligned}$$

The second line is established from Equation (B.6) in the proof of Lemma B.5. Suppose that each agent's approval decision depends on the sign of the signal she receives (to be confirmed below). If everyone follows the signal, then the optimal solution involves providing the same information structure to all agents. That is, the solution is independent of N. Therefore, the approval probability can written as a function of ω and a single editorial strategy (x, t). The last line follows from implementing the same steps as the second part (i.e., perfect competition, $N \to \infty$) of the proof of Lemma B.5. Given this derivation, it is immediate to see that $\mathcal{G}(N)$ is maximized when $t = t^m$ and $x = (1 + \beta_F^2)^{-1}$. As shown in the first part (i.e., monopoly, $N = 1$) of the proof of Lemma B.5, this coincides with the equilibrium editorial strategy of the monopolist. To conclude, note that $x > 1/2$, since $\beta_F < 1$, which implies that the signal induced is positively correlated with $u(\omega, t_i)$ for any t_i, confirming that all types would indeed vote according to the sign of the signal. *Q.E.D.*

B.2. *A Model of Multimedia*

In the baseline version of the model, we assumed that agents can acquire at most one signal. This section discusses an extension of our main result to the case when agents can simultaneously acquire information from multiple firms. One obvious effect of increasing the number of competing firms—for example, from $N = 1$ to $N = 2$—is that agents can acquire more signals. This can, in principle, affect the results of the paper. Indeed, if agents can process an unlimited number of signals at no cost and the price of these signals converges to zero with N, then agents could learn the state as the market becomes perfectly competitive.

While extending the main result to the multimedia case, we maintain the assumption that agents are constrained in how many signals they can acquire or process. In particular, we assume that each agent is endowed with a unit of time that she can divide among N firms. That is, agent i chooses α subject to $\sum_n \alpha_n \leq 1$ with $\alpha_n \in [0, 1]$ for all n. The term α_n represents the fraction of time that the agent spends on the signal supplied by firm n. It is convenient to model firms' editorial strategies using the vector notation introduced in Section 2. In particular, firm n chooses $b_n \in \mathbb{R}^3$ such that $\|b_n\| \leq 1$. Fix a profile of editorial strategies $(b_1, \dots, b_N)$ and suppose that agent i's information-acquisition strategy is α. We assume that agent i observes the realization from a mixture signal characterized by

b_α, where $b_{\alpha,k} = \sum_{n=1}^N \alpha_n b_{n,k}$ for $k = \{0, 1, 2\}$:

$$s_i(\omega, b|\alpha) = \left(\sum_{n=1}^N \alpha_n b_n\right) \cdot \omega + \varepsilon_i = b_\alpha \cdot \omega + \varepsilon_i. \tag{B.7}$$

Note that when α is degenerate, this reduces to our baseline model. Moreover, the value of information given α, denoted by $v(b_\alpha|t_i)$, is still characterized by Lemma 2.[26] Since ε_i does not scale with N, this formulation preserves a key feature of the baseline model, namely that the agent is constrained in how much she can learn about the state. At the same time, the ability to mix among multiple signals allows an agent to "construct" signals that are better tailored to her own needs.[27] Nonetheless, the best mixture that type t_i can acquire is $b_{t_i} := \frac{1}{\sqrt{2}}(1, \cos(t_i), \sin(t_i))$, leading to the same first-best value $\bar{\mathcal{V}}$.

The main challenge in such a model is to determine how profits of the firms are linked to the value of information created for each agent and the competition in the market. To make the model tractable, we make a reduced-form assumption on how a firm's profit from an agent depends on the *surplus* generated by the firm for the agent, that is, the difference between the agent's first-best value and the second-best value she could have obtained in the absence of this firm. We assume that firm n's revenue from agent t_i is

$$p_n^\star(t_i|b) = \frac{1}{N}\Big(\max_\alpha v(b_\alpha|t_i) - \max_{\alpha':\alpha'_n=0} v(b_{\alpha'}|t_i)\Big). \tag{B.8}$$

When α is degenerate, that is, agent i acquires information from a single firm, then firm n's revenue is the same as in our baseline model, net of weight $\frac{1}{N}$.[28] Overall, a profile of editorial strategies b induces profits for firm n that are $\Pi_n(b_n, b_{-n}) = \int_T p_n^\star(t_i|b)\, dF(t_i)$.

Agents choose α to maximize $v(b_\alpha|t_i) - \sum_{n:\alpha_n>0} p_n^\star(t_i|b)$. Note that the solution of the agent's maximization problem depends on $\max_\alpha v(b_\alpha|t_i)$ via $p_n^\star(t_i|b)$. Remark B.2, which we present after the proof of the main result of this section, shows that if $\hat{\alpha}_i \in \arg\max_\alpha v(b_\alpha|t_i)$, then it also solves the agent's maximization problem. Therefore, just like in the baseline model, we can interpret $p_n^\star(t_i|b)$ as a *price* that the agent has to pay to firm n to acquire its signal.

The next result shows that Proposition 5 extends to the multimedia model.

PROPOSITION 6—Multimedia: *Fix any regular distribution F.*

(a) *Existence. An equilibrium exists for any $N \geq 1$ and $I \geq 1$.*
(b) *Daily-me. Fix any t_i. As $N \to \infty$, the equilibrium expected value of information for type t_i, $\mathcal{V}(N|t_i)$, converges in probability to the first-best value $\bar{\mathcal{V}}$.*
(c) *Inefficiency. There exists $\bar{I}$ such that for all $I > \bar{I}$, the agent's welfare in the multimedia model is higher under the monopoly than perfect competition, that is, $\mathcal{U}(1) > \lim_{N\to\infty} \mathcal{U}(N)$.*

[26] Lemma 2 uses the notation of θ_i—instead of t_i as we do in this section—to denote an agent's type. Remark 1 establishes how one variable can be transformed into the other. For each t_i, there is an equivalent $\theta_i = (1, \cos(t_i), \sin(t_i))$.

[27] For example, suppose that $t_i = \pi/4$, $b_1 = (0, 1, 0)$, and $b_2 = (0, 0, 1)$. Fix $\alpha_i(1) = \alpha_i(2) = \frac{1}{2}$. Then $v(b_{\alpha_i}|t_i) > v(b_1|t_i) = v(b_2|t_i)$. That is, the agent does strictly better by mixing than by acquiring a single signal.

[28] Any weighting vector $(w_i(1|b), \dots, w_i(N|b))$ that possibly depends on i and b in a continuous manner would generate the same results.

PROOF: This proof is divided into five steps that closely follow and leverage on the proofs of Lemmas B.1–B.5.

Step 1: *Equilibrium Existence*. Fix N and I. The time line in the multimedia model is as in Figure 1, with the difference that prices are being set exogenously as a function of the chosen profile of editorial strategies b. Using backward induction, we argue that an equilibrium of the game exists. Fix an arbitrary profile of *information-acquisition* strategies $(\alpha_1, \dots, \alpha_I)$. Then Lemma 1 still characterizes the agents' equilibrium *approval* decisions. Now consider an arbitrary profile of editorial strategies b and the profile prices $(p_n^\star(t_i|b))_n$ that ensues. Agent t_i's equilibrium *information-acquisition* strategy consists of choosing α to maximize $v(b_\alpha|t_i) - \sum_{n:\alpha_n>0} p_n^\star(t_i|b)$. Note that $v(b_\alpha|t_i) - \sum_{n:\alpha_n>0} p_n^\star(t_i|b)$ is continuous in $\alpha \in \mathbb{R}^N$ and that $\{\alpha \mid \alpha_n \in [0,1], \sum \alpha_n \leq 1\}$ is compact. Therefore, the agent's problem admits a solution. In the first stage of the game, firms simultaneously choose b_n. Their payoff function is $\Pi_n(b_n, b_{-n}) = \int_T p_n^\star(t_i|b)\, dF(t_i) = \frac{1}{N}\int_T \max_\alpha v(b_\alpha|t_i) - \max_{\alpha':\alpha'_n=0} v(b_{\alpha'}|t_i)\, dF(t_i)$. By the theorem of the maximum, $\max_\alpha v(b_\alpha|t_i)$ is continuous is b. By a similar argument, one can show that $\max_{\alpha':\alpha'_n=0} v(b_{\alpha'}|t_i)$ is also continuous is b. Therefore, $\Pi_n(b_n, b_{-n})$ is continuous in b for all n. As in Lemma B.1, we invoke Glicksberg's theorem to argue that, in the first stage of the game, a Nash equilibrium exists in (possibly mixed) editorial strategies. By backward induction, we have shown that the game admits an equilibrium.

Step 2: *Convergence of* $\mathbb{E}_\chi(\max_\alpha v(b_\alpha|t_i))$. Fix $\delta > 0$ and let $\xi_1 = \frac{\delta}{2\underline{\lambda}}$, where $\underline{\lambda} = \frac{1}{2I\sqrt{\pi}}$. We show that there exists $\bar{N}$ such that for all $N > \bar{N}$ and any equilibrium profile of possibly mixed editorial strategies χ, we have $\mathbb{E}_\chi(\max_\alpha v(b_\alpha|t_i)) > \bar{\mathcal{V}} - \delta$ for all $t_i \in T$. Suppose not. That is, suppose that for all N, there is an equilibrium profile of possibly mixed editorial strategies χ and a type $\bar{t}_i$ such that $\mathbb{E}_\chi(\max_\alpha v(b_\alpha|t_i)) \leq \bar{\mathcal{V}} - \delta$. This implies that for all $t_j \in [\bar{t}_i - \xi_1, \bar{t}_i + \xi_1]$, $\mathbb{E}_\chi(\max_\alpha\{v(b_\alpha|t_j)\}) \leq \bar{\mathcal{V}} - \frac{\delta}{2}$. To see this, suppose, by way of contradiction, that $\mathbb{E}_\chi(\max_\alpha\{v(b_\alpha|t_j)\}) > \bar{\mathcal{V}} - \frac{\delta}{2}$. Denote by $\alpha(t_i)$ the random variable that, for each realization of χ, indicates the information acquisition strategy of type t_i. That is, $\alpha(t_i) \in \arg\max_\alpha v(b_\alpha|t_i)$. Note that there exists a $t_{\alpha(t_j)} \in T$ and $y_{\alpha(t_j)} \in [0,1]$ and $\lambda_{\alpha(t_j)}$ such that $b_{\alpha(t_j)} = (b_{\alpha(t_j),0}, \sqrt{y_{\alpha(t_j)}}\cos(t_{\alpha(t_j)}), \sqrt{y_{\alpha(t_j)}}\sin(t_{\alpha(t_j)}))$, $y_{\alpha(t_j)} = b^2_{\alpha(t_j),1} + b^2_{\alpha(t_j),2}$ and $\lambda_{\alpha(t_j)} = \frac{1}{I\sqrt{2\pi(1+\|b_{\alpha(t_j)}\|^2)}}$. Note that for all $t_{\alpha(t_j)} \in T$, $\cos(\bar{t}_i - t_{\alpha(t_j)}) \geq \cos(t_j - t_{\alpha(t_j)}) - \xi_1$, since $\frac{d}{dt}\cos(t - t_{\alpha(t_j)}) \leq 1$. We have that

$$\begin{aligned}
\mathbb{E}_\chi\Big(\max_\alpha\big\{v(b_\alpha|t_i)\big\}\Big) &\geq \mathbb{E}_\chi\big(v(b_{\alpha(t_j)}|t_i)\big) \\
&= \lambda_{\alpha(t_j)}\mathbb{E}_\chi\big(b_{\alpha(t_j),0} + \sqrt{y_{\alpha(t_j)}}\cos(\bar{t}_i - t_{\alpha(t_j)})\big) \\
&\geq \lambda_{\alpha(t_j)}\mathbb{E}_\chi\big(b_{\alpha(t_j),0} + \sqrt{y_{\alpha(t_j)}}\big(\cos(t_j - t_{\alpha(t_j)}) - \xi_1\big) \\
&\geq \lambda_{\alpha(t_j)}\mathbb{E}_\chi\big(b_{\alpha(t_j),0} + \sqrt{y_{\alpha(t_j)}}\cos(t_j - t_{\alpha(t_j)})\big) - \lambda_{\alpha(t_j)}\xi_1 \\
&\geq \mathbb{E}_\chi\Big(\max_\alpha\big\{v(b_\alpha|t_j)\big\}\Big) - \lambda_{\alpha(t_j)}\xi_1 \\
&> \bar{\mathcal{V}} - \frac{\delta}{2} - \lambda_{\alpha(t_j)}\xi_1 \\
&\geq \bar{\mathcal{V}} - \delta.
\end{aligned}$$

The first inequality holds as, in the right-hand side, agent $\bar{t}_i$ chooses the information acquisition strategy $\alpha(t_j)$ that is optimal for t_j. The second inequality holds since $\cos(\bar{t}_i - t_{\alpha(t_j)}) \geq \cos(t_j - t_{\alpha(t_j)}) - \xi_1$ for all $t_{\alpha(t_j)}$ and $\sqrt{y_{\alpha(t_j)}} < 1$. The last inequality uses that $\lambda_{\alpha(t_j)}/\underline{\lambda} \geq 1$. In summary, this contradicts our assumption that $\mathbb{E}_\chi(\max_\alpha\{v(b_\alpha|t_i)\}) \leq \bar{\mathcal{V}} - \delta$. Therefore, it must be that $\mathbb{E}_\chi(\max_\alpha\{v(b_\alpha|t_j)\}) \leq \bar{\mathcal{V}} - \frac{\delta}{2}$.

Note that by continuity of $v(b_{\bar{t}_i}|t_j)$ in t_j, there exists $\xi_2 > 0$ such that for all $t_j \in [\bar{t}_i - \xi_2, \bar{t}_i + \xi_2]$ such that $v(b_{\bar{t}_i}|t_j) \geq \bar{\mathcal{V}} - \frac{\delta}{4}$. Moreover, such ξ_2 is independent of N. Let $\xi = \min\{\xi_1, \xi_2\}$, which in turn is independent of N. We have established that for all $t_j \in [\bar{t}_i - \xi, \bar{t}_i + \xi]$,

$$\mathbb{E}_\chi\Big(\max_{\alpha':\alpha'_n=0} v(b_{\alpha'}|t_j)\Big) \leq \mathbb{E}_\chi\Big(\max_\alpha v(b_\alpha|t_j)\Big) \leq \bar{\mathcal{V}} - \frac{\delta}{2} < \bar{\mathcal{V}} - \frac{\delta}{4} \leq v(b_{\bar{t}_i}|t_j), \tag{B.9}$$

Now fix an arbitrary firm n that deviates from its equilibrium editorial strategy χ_n in favor of the pure editorial strategy $b_{\bar{t}_i}$. Its expected profits are

$$\begin{aligned}
\Pi_n\big(b_{\bar{t}_i}, (\chi_{n'})_{n'\neq n}\big) &= \frac{I}{N}\int_{-\pi}^{\pi} \mathbb{E}_\chi\Big(\max\Big\{v(b_{\bar{t}_i}|t_j) - \max_{\alpha':\alpha'_n=0} v(b_{\alpha'}|t_j), 0\Big\}\Big)\, dF(t_j) \\
&\geq \frac{I}{N}\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi} \mathbb{E}_\chi\Big(\max\Big\{v(b_{\bar{t}_i}|t_j) - \max_{\alpha':\alpha'_n=0} v(b_{\alpha'}|t_j), 0\Big\}\Big)\, dF(t_j) \\
&\geq \frac{I}{N}\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi} \mathbb{E}_\chi\Big(v(b_{\bar{t}_i}|t_j) - \max_{\alpha':\alpha'_n=0} v(b_{\alpha'}|t_j)\Big)\, dF(t_j) \\
&= \frac{I}{N}\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi} v(b_{\bar{t}_i}|t_j) - \mathbb{E}_\chi\Big(\max_{\alpha':\alpha'_n=0} v(b_{\alpha'}|t_j)\Big)\, dF(t_j) \\
&\geq \frac{I}{N}\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi} \Big(\bar{\mathcal{V}} - \frac{\delta}{4} - \bar{\mathcal{V}} + \frac{\delta}{2}\Big) f(t_j)\, dt_j \\
&\geq \frac{IC\delta\xi}{2N}.
\end{aligned}$$

The first inequality holds since the integrand function is everywhere positive. The second inequality holds by monotonicity of the operator $\mathbb{E}_\chi$. The second-to-last inequality obtains as a consequence of Equation (B.9). The last inequality, instead, obtains because $f(t_j) \geq C > 0$ for all t_j. We established that firm n can secure an expected profit of at least $\frac{IC\delta\xi}{2N}$ by deviating to $b_{\bar{t}_i}$. This lower bound is strictly positive and decreasing in N at rate $1/N$. To conclude the proof, note that by Lemma B.6 the maximum amount paid for information by any agent can at most be $\bar{\mathcal{V}}/N$. This implies that the industry profits are bounded above by $I\bar{\mathcal{V}}/N$. Therefore, when N firms are competing, there is at least one firm, which we denote by n, whose expected equilibrium profit is $\Pi_n(\chi) \leq I\bar{\mathcal{V}}/N^2$. When N is large, $\frac{IC\delta\xi}{2N} > \frac{I\bar{\mathcal{V}}}{N^2}$ and firm n is a strictly profitable deviation from its equilibrium editorial strategy χ_n in the first stage of the game—a contradiction.

Step 3: *Convergence of* $b^N_{\alpha(t_i)}$. Building on the previous argument, we now show that $b^N_{\alpha(t_i)} \to b_{t_i}$ in probability as $N \to \infty$. More formally, fix t_i, $\epsilon > 0$, and a sequence of equilibria. For any N, denote by $b^N_{\alpha(t_i)}$ the random variable specifying the information structure that agent t_i acquires in equilibrium. As above, let $b_{t_i} := \frac{1}{\sqrt{2}}(1, \cos(t_i), \sin(t_i))$ be type t_i's first-best, that is, $v(b_{t_i}|t_i) = \bar{\mathcal{V}}$. We want to show that for all $\delta > 0$, there exists $\bar{N}$ such that for all $N > \bar{N}$, $\Pr(\|b^N_{\alpha(t_i)} - b_{t_i}\| > \epsilon) < \delta$. Suppose not. Then there is $\delta > 0$ such that for all $\bar{N}$, there is $N > \bar{N}$ such that $\Pr(\|b^N_{\alpha(t_i)} - b_{t_i}\| > \epsilon) \geq \delta$. Consider any particular realization b_i of the random variable $b^N_{\alpha(t_i)}$ for which $\|b_i - b_{t_i}\| > \epsilon$. Then there exists $K(\epsilon) > 0$ such that $\bar{\mathcal{V}} - v((x_n, t_n)|t) \geq K(\epsilon)$ (see Lemma B.3 for more details on $K(\epsilon)$). Thus, $\Pr(\|b^N_{\alpha(t_i)} - b_{t_i}\| > \epsilon) \geq \delta$ implies that $\Pr(\bar{\mathcal{V}} - v(b_{\alpha(t_i)}|t_i) \geq K(\epsilon)) \geq \delta$. Since δ and ϵ were fixed independently of N, we conclude that $\mathbb{E}(v(b_{\alpha(t_i)}|t_i))$ does not converge to $\bar{\mathcal{V}}$—a contradiction to Step 2 above.

Step 4: *Convergence of* $\mathcal{U}(N)$. Fix $N \geq 1$. Let $B_n = \{b_n \in \mathbb{R} : \|b_n\| = 1\}$. Let $\chi \in \prod_n \Delta(B_n)$ be a (possibly mixed) equilibrium profile of editorial strategy. Denote by $b^N_{\alpha(t_i)}$ the equilibrium random variable which specifies the signal that is acquired by type t_i, possibly by mixing those that are offered by the N firms. As usual, denote by $A_{-i}(\omega, t_{-i})$ the equilibrium approval rate excluding agent i. Following the proof of Lemma B.4, an agent's welfare can be written as

$$\mathcal{U}(N) = \mathbb{E}_\chi \mathbb{E}_{(t_1,\dots,t_I)}\left(\mathbb{E}_\omega\big(A_{-i}(\omega, t_{-i})u(\omega, t_i)\big) + v\big(b^N_{\alpha(t_i)}|t_i\big) - \sum_{n|\alpha_n(t_i)>0} p^\star_n\big(t_i|b^N\big)\right).$$

To compute the limit of $\mathcal{U}(N)$, we split the above expression into three parts. Fix an arbitrary t_i. First, note that by Lemma B.6, the total amount paid for information by an agent is at most $\bar{\mathcal{V}}/N$. Therefore, $\lim_N \sum_{n|\alpha_n(t_i)>0} p^\star_n(t_i|b^N) = 0$. Second, as shown in Step 3, $b^N_{\alpha(t_i)} \to b_{t_i}$ in probability as $N \to \infty$, where b_{t_i} is the first-best information structure for agent t_i. By the continuous mapping theorem, $v(b^N_{\alpha(t_i)}|t_i) \to \bar{\mathcal{V}} = \lambda\sqrt{2}$ in probability. Since $|v|$ is bounded, convergence in probability implies convergence in expectation. Together with the first step, we have that for any t_i, $\lim_{N\to\infty} \mathbb{E}_\chi(v(b_{\alpha(t_i)}|t_i) - \sum_{n|\alpha_n(t_i)>0} p^\star_n(t_i|b^N)) = \bar{\mathcal{V}}$. Since t_i was arbitrary and is independent of χ, we have that $\lim_{N\to\infty} \mathbb{E}_\chi \mathbb{E}_{t_i}(v(b_{\alpha(t_i)}|t_i) - \sum_{n|\alpha_n(t_i)>0} p^\star_n(t_i|b^N)) = \bar{\mathcal{V}}$. The third and final step consists of computing the limit of $\mathbb{E}_{\chi,t,\omega}(A_{-i}(\omega, t_{-i})u(\omega, t_i))$. To this purpose, note that χ, ω, and $(t_1, \dots, t_I)$ are mutually independent random variables. Therefore, by swapping the order of integration and defining $U_i(\omega) = \mathbb{E}_{t_i} u(\omega, t_i)$ to simplify notation, we obtain

$$\begin{aligned}
\lim_{N\to\infty} \mathbb{E}_\chi \mathbb{E}_{(t_1,\dots,t_I)}\big(\mathbb{E}_\omega\big(A_{-i}(\omega, t_{-i})u(\omega, t_i)\big)\big) &= \frac{1}{I} \lim_{N\to\infty} \mathbb{E}_\omega\left(\sum_{j\neq i} \mathbb{E}_{t_j}\mathbb{E}_\chi a_j(\omega, t_j)\mathbb{E}_{t_i} u(\omega, t_i)\right) \\
&= \frac{(I-1)}{I} \lim_{N\to\infty} \mathbb{E}_\omega\big(\mathbb{E}_{t_j}\mathbb{E}_\chi a_j(\omega, t_j)\mathbb{E}_{t_i} u(\omega, t_i)\big) \\
&= \frac{(I-1)}{I} \lim_{N\to\infty} \mathbb{E}_\omega\big(\mathbb{E}_{t_j}\mathbb{E}_\chi a_j(\omega, t_j)U_i(\omega)\big).
\end{aligned}$$

Recall that $a_j(\omega, t_j) = \Phi(b^N_{\alpha(t_j)}\omega)$. Once again, as shown in Step 3, $b^N_{\alpha(t_i)} \to b_{t_j}$ in probability as $N \to \infty$. By the continuous mapping theorem, $\Phi(b^N_{\alpha(t_j)}\omega) \to \Phi(\frac{1}{\sqrt{2}}u_j(\omega, t_j))$ in probability. Moreover, since Φ is bounded, convergence in probability implies convergence in expectation: $\lim_{N\to\infty} \mathbb{E}_\chi a(\omega, t_i) = \Phi(\frac{1}{\sqrt{2}}u_j(\omega, t_j))$. Following one-to-one the last few steps in the proof of Lemma B.4, we conclude that

$$\lim_{N\to\infty} \mathbb{E}_\chi \mathbb{E}_{(t_1,\dots,t_I)}\big(\mathbb{E}_\omega\big(A_{-i}(\omega, t_{-i})u(\omega, t_i)\big)\big) = \frac{(I-1)}{I}\mathbb{E}_{\omega,t_i,t_j}\left(\Phi\left(\frac{1}{\sqrt{2}}u_j(\omega, t_j)\right)u_i(\omega, t_i)\right).$$

Therefore, $\mathcal{U}(N) \to \frac{I-1}{I}\mathbb{E}_{\omega,t_i,t_j}(\Phi(\frac{1}{\sqrt{2}}u_j(\omega, t_j))u_i(\omega, t_i)) + \bar{\mathcal{V}}$.

Step 5: *Monopoly Versus Perfect Competition*. When $N = 1$, the multimedia model is identical to the baseline model: all agents acquire information from a single firm. Therefore, $\mathcal{U}(1) = (I-1)\lambda\sqrt{1+\beta_F}$ as computed in Lemma B.5. The same proof shows $\frac{I-1}{I}\mathbb{E}_{\omega,t_i,t_j}(\Phi(\frac{1}{\sqrt{2}}u_j(\omega, t_j))u_i(\omega, t_i)) + \bar{\mathcal{V}} = \lambda(I-1)\frac{1}{\sqrt{2}}(1+\beta_F) + \lambda\sqrt{2}$. Therefore, by Step 3, we have that $\lim_N \mathcal{U}(N) = \lambda(I-1)\frac{1}{\sqrt{2}}(1+\beta_F) + \lambda\sqrt{2}$. In summary, we argued that when both $N = 1$ and $N \to \infty$, the baseline model and the multimedia model generate identical expected utilities. Given this, the statement of Proposition 6 follows directly from Proposition 5(c). *Q.E.D.*

REMARK B.2: Fix t_i and a profile of editorial strategies b. Let $\hat{\alpha} \in \arg\max_\alpha v(b_\alpha|t_i)$. Then

$$\hat{\alpha} \in \arg\max_{\alpha}\left(v(b_\alpha|t_i) - \sum_{n|\alpha_n>0} p_n^\star\big(t_i|b^N\big)\right).$$

PROOF: Suppose the statement is not true. That is, there is $\tilde{\alpha} \neq \hat{\alpha}$ such that

$$v(b_{\tilde{\alpha}}|t_i) - \sum_{n|\tilde{\alpha}_n>0} p_n^\star\big(t_i|b^N\big) > v(b_{\hat{\alpha}}|t_i) - \sum_{n:\hat{\alpha}_n>0} p_n^\star(t_i|b).$$

Let $\hat{N} = \{n : \hat{\alpha}_n > 0\} \setminus \{n : \tilde{\alpha}_n > 0\}$. Note that $\hat{N} \neq \emptyset$. Indeed, given that $\hat{\alpha} \in \arg\max_\alpha v(b_\alpha| t_i)$, $\hat{N} = \emptyset$ would contradict the definition of $\tilde{\alpha}$. Thus, we can rewrite the inequality above as

$$v(b_{\hat{\alpha}}|t_i) - \max_{\alpha|\alpha_n=0\ \forall n\in\hat{N}} v(b_\alpha|t_i) < \sum_{n\in\hat{N}} p_n^\star(t_i|b).$$

However, a contradiction is reached by noting that

$$\begin{aligned}
\sum_{n\in\hat{N}} p_n^\star(t_i|b) &= \sum_{n'\in\hat{N}} \frac{1}{N}\Big(v(b_{\hat{\alpha}}|t_i) - \max_{\alpha|\alpha_{n'}=0} v(b_\alpha|t_i)\Big) \\
&\leq \sum_{n'\in\hat{N}} \frac{1}{N}\Big(v(b_{\hat{\alpha}}|t_i) - \max_{\alpha|\alpha_n=0\ \forall n\in\hat{N}} v(b_\alpha|t_i)\Big) \\
&= \Big(v(b_{\hat{\alpha}_i}|t_i) - \max_{\alpha_i|\alpha_i(n)=0\ \forall n\in\hat{N}} v(b_\alpha|t_i)\Big)\left(\sum_{n'\in\hat{N}} \frac{1}{N}\right) \\
&\leq v(b_{\hat{\alpha}}|t_i) - \max_{\alpha|\alpha_n=0\ \forall n\in\hat{N}} v(b_\alpha|t_i).
\end{aligned}$$

The first inequality holds since, for all $n' \in \hat{N}$, $\max_\alpha\{v(b_\alpha|t_i)|\alpha_n = 0, \forall n \in \hat{N}\} \leq \max_\alpha\{v(b_\alpha| t_i)|\alpha_{n'} = 0\}$. The last inequality holds since $\sum_{n'\in\hat{N}} \frac{1}{N} \leq 1$. *Q.E.D.*

LEMMA B.6: *For any* t_i *and* b, $\sum_n p^\star(t_i|b) \leq \frac{\bar{\mathcal{V}}}{N}$.

PROOF: We switch to notation θ_i to denote an agent's type. By Lemma 2, $v(b_{\alpha(\theta_i)}|\theta_i) = \lambda_{\alpha(\theta_i)}|\theta_i \cdot b_{\alpha(\theta_i)}|$, where $\lambda_{\alpha(\theta_i)} = \frac{1}{I\sqrt{2\pi(1+\|b_{\alpha(\theta_i)}\|^2)}}$. We establish below that for all n with $\alpha_n(\theta_i) > 0$, the sign of $\theta_i \cdot b_n$ is the same as the sign of $\theta_i \cdot b_{\alpha(\theta_i)}$. This allows us to move the absolute value sign inside: $v(b_{\alpha(\theta_i)}|\theta_i) = \lambda_{\alpha(\theta_i)} \sum_n \alpha_n(\theta_i)|\theta_i \cdot b_n|$.

Assume $\theta_i \cdot b_{\alpha(\theta_i)} > 0$. Now we show that for any n with $\alpha_n(\theta_i) > 0$, $\theta_i \cdot b_n \geq 0$. The proof for the other case, $\theta_i \cdot b_{\alpha(\theta_i)} < 0$, follows a symmetric argument and, thus, it is not replicated here. Assume for contradiction that there exists an n' with $\alpha_{n'}(\theta_i) > 0$, but $\theta_i \cdot b_{n'} < 0$. Consider $\tilde{\alpha}$ such that $\tilde{\alpha}_n = \alpha_n(\theta_i)$ for all $n \neq n'$ with $\tilde{\alpha}_{n'} = 0$. Note that $|\theta_i \cdot b_{\tilde{\alpha}}| = |\sum_{n\neq n'} \alpha_n(\theta_i)(\theta_i \cdot b_n)| = \theta_i \cdot b_{\alpha(\theta_i)} + \alpha_{n'}(\theta_i)|\theta_i \cdot b_{n'}| > |\theta_i \cdot b_{\alpha(\theta_i)}|$. Also note that $\|b_{\tilde{\alpha}}\| < \|b_{\alpha(\theta_i)}\|$, implying $\lambda_{\tilde{\alpha}} > \lambda_{\alpha(\theta_i)}$. Therefore, we have $v(b_{\tilde{\alpha}}|\theta_i) > v(b_{\alpha(\theta_i)}|\theta_i)$, contradicting $\alpha(\theta_i) \in \arg\max_\alpha v(b_\alpha|\theta_i)$.

Using similar techniques, we can also show that $\max_{\alpha:\alpha_n=0} v(b_\alpha|t_i) > \lambda_{\alpha(\theta_i)} \times \sum_{n'\neq n} \alpha_{n'}(\theta_i)|\theta_i \cdot b_{n'}|$. This implies that $p_n^\star(\theta_i|b) = \max_\alpha v(b_\alpha|\theta_i) - \max_{\alpha:\alpha_n=0} v(b_\alpha|\theta_i) \leq \lambda_{\alpha_i}\alpha_n(\theta_i)|\theta_i \cdot b_n|$. We conclude the proof by summing over n:

$$\sum_n p_n^\star(\theta_i|b) = \frac{1}{N}\sum_n \Big(\max_\alpha v(b_\alpha|\theta_i) - \max_{\alpha:\alpha_n=0} v(b_\alpha|\theta_i)\Big)$$

$$\leq \frac{1}{N}\sum_n \lambda_{\alpha(\theta_i)}\alpha_n(\theta_i)|\theta_i \cdot b_n| = \frac{1}{N}\max_\alpha v(b_\alpha|\theta_i) \leq \frac{\bar{\mathcal{V}}}{N}.$$

Q.E.D.

B.3. *Remaining Proofs*

LEMMA B.7—Inequality. I: *Let* $x^\star = 1/(1 + (\sin(\pi/N)/\pi/N)^2)$ *and* $x_n \in [1/2, 1]$. *For all* $N \geq 3$, $v((x_n, 0)|2\pi/N) < v((x^\star, 0)|\pi/N)$.

PROOF: We begin by noting that

$$\sqrt{x_n} + \sqrt{1-x_n}\cos(2\pi/N) \leq \max_{x_n\in[1/2,1]} v\big((x_n,0)|2\pi/N\big) = \begin{cases} 1 & \text{if } N \leq 4, \\ \sqrt{1+\cos^2(2\pi/N)} & \text{if } N \geq 5. \end{cases}$$

Moreover, by substituting the definition of $x^\star$ in $v((x^\star, 0)|\pi/N)$, we obtain

$$\sqrt{x^\star} + \sqrt{1-x^\star}\cos(\pi/N) = \frac{2\pi/N + \sin(2\pi/N)}{2\sqrt{(\pi/N)^2 + \sin^2(\pi/N)}}.$$

Let $N \in \{3, 4\}$. In such a case, it is enough to show that

$$1 < \frac{2\pi/N + \sin(2\pi/N)}{2\sqrt{(\pi/N)^2 + \sin^2(\pi/N)}}.$$

Rearranging and simplifying, $2\pi/N\cos(\pi/N) - \sin^3(\pi/N) > 0$. It is straightforward to verify that this holds when $N \in \{3, 4\}$. Thus, let $N \geq 5$. In such a case, it is enough to show that

$$\sqrt{1+\cos^2(2\pi/N)} < \frac{2\pi/N+\sin(2\pi/N)}{2\sqrt{(\pi/N)^2+\sin^2(\pi/N)}}.$$

Denoting $y = \pi/N \in (0, \pi/5]$ and simplifying the above inequality, we obtain

$$G(y) = \frac{1}{4}\sin^2(2y) + y\sin(2y) - \sin^2(y) - \cos^2(2y)\big(y^2+\sin^2(y)\big) > 0.$$

Note that $G(0) = 0$. To conclude the proof, we will show that $G'(y) > 0$ for all $y \in (0, \pi/5]$. Note that

$$\begin{aligned} G'(y) = {} & \sin(2y)\cos(2y) + \sin(2y) + 2y\cos(2y) - 2\sin(y)\cos(y) \\ & + 4\cos(2y)\sin(2y)\big(y^2+\sin^2(y)\big) - \cos^2(2y)\big(2y + 2\sin(y)\cos(y)\big). \end{aligned}$$

Since $2\sin(y)\cos(y) = \sin(2y)$, the second and fourth terms cancel. Moreover,

$$\begin{aligned} G'(y) &= \sin(2y)\cos(2y) + 2y\cos(2y) \\ &\quad + 4\cos(2y)\sin(2y)\big(y^2+\sin^2(y)\big) - \cos^2(2y)\big(2y+\sin(2y)\big) \\ &> \sin(2y)\cos(2y) + 2y\cos(2y) \\ &\quad + 4\cos(2y)\sin(2y)\big(y^2+\sin^2(y)\big) - \cos(2y)\big(2y+\sin(2y)\big) \\ &= 4\cos(2y)\sin(2y)\big(y^2+\sin^2(y)\big) \\ &> 0. \end{aligned}$$

The first inequality follows from the fact that, since $\cos(2y) \in (0, 1)$, $\cos(2y) > \cos^2(2y)$. The last inequality follows trivially, as all terms of the expression are strictly positive. *Q.E.D.*

LEMMA B.8: *For all $t_2 \in [-\pi, \pi]$ and $x_1 \le x_2$, the set $\{t \in [-\pi, \pi] | v((x_1, t_1 = 0)|t) \ge v((x_2, t_2)|t)\}$ is an interval in $[-\pi, \pi]$.*

PROOF: Let $\Gamma(t) = v((x_1, 0)|t) - v((x_2, t_2)|t)$. If $x_1 = x_2$ and $t_1 = t_2$, $\{\Gamma(t) \ge 0\} = [-\pi, \pi]$ and the claim follows. If $x_1 < x_2$ and $t_1 = t_2$, instead, we have that

$$\{\Gamma(t) \ge 0\} = \left\{t : \cos(t) \ge \frac{\sqrt{x_2}-\sqrt{x_1}}{\sqrt{1-x_1}-\sqrt{1-x_2}} > 0\right\}.$$

It is immediate to see this is an interval in $[-\pi, \pi]$. Therefore, let $t_2 \neq t_1 = 0$ and $1/2 \le x_1 \le x_2$. Suppose $t_2 > 0$. It is immediate to see that $\Gamma(0) > 0$ and $\Gamma(\pi) = \Gamma(-\pi) < 0$. Consider the interval $[0, \pi]$. By continuity of $\Gamma(t)$, there exists at least one $\bar{t} \in (0, \pi)$ such that $\Gamma(\bar{t}) = 0$. We want to show that there is only one such $\bar{t}$. Note that if $t \in (0, \pi/2]$, the derivative of Γ is

$$\Gamma'(t) = -\sqrt{1-x_1}\sin(t) + \sqrt{1-x_2}\sin(t-t_2) < 0.$$

Indeed, $\sqrt{1-x_1} \geq \sqrt{1-x_2}$ and $\sin(t) > \sin(t-t_2)$ if $t \in (0, \pi/2]$ (note that while $\sin(t)$ is necessarily positive, $\sin(t-t_2)$ is either negative or positive but smaller than $\sin(t)$). For a similar argument, note that if $t \in [\pi/2, \pi)$, the second derivative of Γ is

$$\Gamma''(t) = -\sqrt{1-x_1}\cos(t) + \sqrt{1-x_2}\cos(t-t_2) > 0.$$

Therefore, $\Gamma'(t)$ is strictly increasing in $[\pi/2, \pi)$ and, hence, it is single-crossing. Since $\Gamma(\pi) < 0$, this implies that $\bar{t}$ is the unique type in $[0, \pi]$ such that $\Gamma(\bar{t}) = 0$.

We now apply a parallel argument for the interval $[-\pi, 0]$. By continuity, there exists at least one $\underline{t} \in (-\pi, 0)$ such that $\Gamma(\underline{t}) = 0$. We need to establish its uniqueness. Note that if $t \in (-\pi, -\pi/2]$, $\Gamma'(t) > 0$. Similarly, if $t \in [-\pi/2, 0)$, $\Gamma''(t) < 0$. Following the argument made above, we can conclude that there exists a unique $\underline{t} \in [-\pi, 0]$ such that $\Gamma(\underline{t}) = 0$.

Therefore, since $\Gamma(0) > 0$, we have that $\{\Gamma(t) \geq 0\} = [\underline{t}, \bar{t}]$. We omit the discussion of the case $t_2 < 0$ as it follows trivially from the argument above. *Q.E.D.*

LEMMA B.9: *The function* $G(\delta) = \frac{2\delta+\sin(2\delta)}{2\sqrt{\delta^2+\sin^2(\delta)}}$ *is strictly decreasing in* $\delta \in (0, \pi/2)$.

PROOF: Note that

$$G'(\delta) = \frac{2+2\cos(2\delta)}{2\sqrt{\delta^2+\sin^2(\delta)}} - \frac{\big(2\delta+\sin(2\delta)\big)\big(2\delta+2\sin(\delta)\cos(\delta)\big)}{4\big(\delta^2+\sin^2(\delta)\big)^{3/2}}.$$

We want to show that $G'(\delta) < 0$ for $\delta \in (0, \pi/2)$. Multiplying both sides by $(\delta^2 + \sin^2(\delta))^{3/2}$ and using $\sin(2\delta) = 2\sin(\delta)\cos(\delta)$, we get that the sign of $G'(\delta)$ is equal to the sign of

$$\begin{aligned}
&\big(1+\cos(2\delta)\big)\big(\delta^2+\sin^2(\delta)\big) - \big(\delta+\sin(\delta)\cos(\delta)\big)^2 \\
&\quad = \delta^2 + \sin^2(\delta) + \cos(2\delta)\delta^2 + \cos(2\delta)\sin^2(\delta) - \delta^2 - 2\delta\sin(\delta)\cos(\delta) - \sin^2(\delta)\cos^2(\delta) \\
&\quad = \sin^2(\delta) + \cos(2\delta)\delta^2 + \cos(2\delta)\sin^2(\delta) - \delta\sin(2\delta) - \sin^2(\delta)\cos^2(\delta) \\
&\quad = \sin^2(\delta) + \cos(2\delta)\delta^2 - \sin^4(\delta) - \delta\sin(2\delta) \\
&\quad < \sin^2(\delta) + \cos(2\delta)\delta^2 - \delta\sin(2\delta) \\
&\quad = H(\delta),
\end{aligned}$$

where we used the identity $\cos(2\delta)\sin^2(\delta) = \sin^2(\delta)\cos^2(\delta) - \sin^4(\delta)$ for the second-to-last equality and the fact that $\sin^4(\delta) > 0$ for $\delta \in (0, \pi/2)$ for the last inequality. Note that $H(0) = 0$ and, for all $\delta \in (0, \pi/2)$,

$$\begin{aligned}
H'(\delta) &= 2\sin(\delta)\cos(\delta) - 2\sin(2\delta)\delta^2 + \cos(2\delta)2\delta - 2\delta\cos(2\delta) - \sin(2\delta) \\
&= \sin(2\delta) - 2\sin(2\delta)\delta^2 - \sin(2\delta) \\
&= -2\sin(2\delta)\delta^2 \\
&< 0.
\end{aligned}$$

Therefore, $H(\delta) < 0$ and, hence, $G'(\delta) < 0$ for all $\delta \in (0, \pi/2)$. *Q.E.D.*

REMARK B.3: Fix an arbitrary sequence of equilibria $((x_n^N, t_n^N)_{n=1}^N)_{N\in\mathbb{N}}$ and a type t_i. There exists a subsequence (N_k) such that the equilibrium value of information for agent t_i is strictly increasing in k.

PROOF: Fix $((x_n^N, t_n^N)_{n=1}^N)_{N\in\mathbb{N}}$ and a type t_i. Let $v_{t_i,N}$ be the equilibrium value of information for type t_i when N firms are competing. The proof of Proposition 2 part (b) shows that the sequence $(v_{t_i,N})_N$ is converging to $\lambda\sqrt{2}$. Therefore, it admits a monotone subsequence. Since $v_{t_i,N} \le \lambda\sqrt{2}$ for all N, such subsequence must be increasing. *Q.E.D.*

LEMMA B.10: *Fix N and let $(x^\star(N), t_n^\star)_{n=1}^N$ be an equilibrium profile of editorial strategies. For all ω_0,*

$$\bar{a}_i(\omega_0) := \mathbb{E}_{\omega_1,\omega_2,t_i}\big(a_i\big((\omega_0,\omega_1,\omega_2),t_i\big)\big) = \Phi\left(\frac{\sqrt{x^\star}(N)}{\sqrt{2-x^\star(N)}}\omega_0\right).$$

PROOF: Fix N and an equilibrium profile of editorial strategies $(x^\star(N), t_n^\star)_{n=1}^N$. Suppose that in this equilibrium, agent t_i acquires information from firm n. Conditional on a signal realization $\bar{s} = s(\omega, (x^\star(N), t_n^\star))$, the agent's equilibrium approval strategy is characterized in Lemma 1 and depends on $\mathbb{E}_\omega(u(\omega, t_i)|\bar{s})$. Using Equation (A.1) and Remark 1, we have that

$$\mathbb{E}_\omega\big(u(\omega,t_i)|\bar{s}\big) = \frac{v\big((x^\star(N),t_n^\star)|t_i\big)}{\lambda}\Big(\sqrt{x^\star(N)}\omega_0 + \sqrt{1-x^\star(N)}\big(\omega_1\cos\big(t_n^\star\big) + \omega_2\sin\big(t_n^\star\big)\big) + \varepsilon_i\Big).$$

Since, in equilibrium, $\frac{v((x^\star(N),t_n^\star)|t_i)}{\lambda} > 0$, the agent approves if and only if the signal she observes is positive. Since $\varepsilon_i \sim \mathcal{N}(0,1)$, the probability that agent t_i approves policy ω before ε_i realizes is given by $\bar{a}_i^\star(\omega, t_i) = \Phi(\sqrt{x^\star(N)}\omega_0 + \sqrt{1-x^\star(N)}(\cos(t_n^\star)\omega_1 + \sin(t_n^\star)\omega_2))$, where Φ denotes the cdf of the standard normal distribution. Thanks to the symmetry in the equilibrium location (Theorem 2) and the uniformity of the distribution of t_i, we have that

$$\mathbb{E}_{t_i}\big(a_i(\omega,t_i)\big) = \frac{1}{N}\sum_n \Phi\Big(\sqrt{x^\star(N)}\omega_0 + \sqrt{1-x^\star(N)}\big(\cos\big(t_n^\star\big)\omega_1 + \sin\big(t_n^\star\big)\omega_2\big)\Big).$$

We need to compute the expectation of the expression above with respect to ω_1 and ω_2. Since ω_1 and ω_2 are independent, we do so in two separate steps. For both steps, we use the identity $\int_{\mathbb{R}} \Phi(\alpha + \gamma y)\, d\Phi(y) = \Phi(\alpha/\sqrt{1+\gamma^2})$ (see Patel and Read (1996)). We begin by integrating with respect to ω_2. Let $y = \omega_2$, and for each n, let $\alpha_n = \sqrt{x^\star}\omega_0 + \sqrt{1-x^\star}\cos(t_n^\star)\omega_1$ and $\gamma_n = \sqrt{1-x^\star}\sin(t_n^\star)$. Using the integral identity, we obtain

$$\mathbb{E}_{\omega_2,t_i}\big(a_i(\omega,t_i)\big) = \frac{1}{N}\sum_n \Phi\left(\frac{\alpha_n}{\sqrt{1+\gamma_n^2}}\right) = \frac{1}{N}\sum_n \Phi\left(\frac{\sqrt{x^\star}\omega_0 + \sqrt{1-x^\star}\cos\big(t_n^\star\big)\omega_1}{\sqrt{1+\big(1-x^\star\big)\sin^2\big(t_n^\star\big)}}\right).$$

Next, we integrate the above with respect to ω_1. Let $y = \omega_1$ and

$$\alpha_n' = \frac{\sqrt{x^\star}\omega_0}{\sqrt{1+\big(1-x^\star\big)\sin^2\big(t_n^\star\big)}}, \qquad \gamma_n' = \frac{\sqrt{1-x^\star}\cos\big(t_n^\star\big)}{\sqrt{1+\big(1-x^\star\big)\sin^2\big(t_n^\star\big)}}.$$

Using again the integral identity, we obtain

$$\mathbb{E}_{\omega_1,\omega_2,t_i}\big(a_i(\omega,t_i)\big)=\frac{1}{N}\sum_n \Phi\bigg(\frac{\alpha'_n}{\sqrt{1+\gamma_n'^2}}\bigg)=\Phi\bigg(\frac{\sqrt{x^\star}(N)}{\sqrt{2-x^\star(N)}}\omega_0\bigg),$$

where we used the fact that $\cos^2(t_n^\star)+\sin^2(t_n^\star)=1$ for all n. *Q.E.D.*

PROOF OF REMARK 3: Fix $I\geq 1$ and $N\geq 1$. Let $(x^\star, t_n^\star)_{n=1}^N$ be the equilibrium profile of editorial strategies. Let $A^\star(\omega)$ be the equilibrium rate of approval. By assumption, it is equal to the probability that the society implements ω. We want to show that the total probability of implementing a policy in Ω^+, namely $\int_{\Omega^+} A^\star(\omega)\phi(\omega)\,d\omega$, decreases in N. To do so, we divide the proof into three steps. First, we partition the set of policies Ω^+. Second, we compute the integral, restricting attention on an arbitrary cell of such a partition. Third, we show that such an integral is decreasing in N.

Step 1. For any $\omega_0\in\mathbb{R}$ and $K\geq 0$, define the set of policies

$$\Omega^\circ(\omega_0,K)=\Big\{\tilde\omega\in\Omega:\tilde\omega_0=\omega_0\text{ and }\sqrt{\tilde\omega_1^2+\tilde\omega_2^2}=K\Big\}.$$

Fix $\tilde\omega\in\Omega^\circ(\omega_0,K)$. Note that by letting $t_{\tilde\omega}=\arctan(\tilde\omega_2/\tilde\omega_1)\in[-\pi,\pi]$, we can write $u(\tilde\omega,t_i)=\omega_0+K\cos(t_i-t_{\tilde\omega})$. Moreover, all $\tilde\omega\in\Omega^\circ(\omega_0,K)$ are equally likely. To see this, note that $\Pr(\tilde\omega)=\phi(\tilde\omega_0)\phi(\tilde\omega_1)\phi(\tilde\omega_2)=\phi(\omega_0)\frac{1}{2\pi}e^{\frac{K^2}{2}}$, which only depends on (ω_0,K). Therefore, $t_{\tilde\omega}$ is uniformly distributed in $[-\pi,\pi]$.

Clearly, $(\omega_0',K')\neq(\omega_0'',K'')$ and $\Omega^\circ(\omega_0',K')\cap\Omega^\circ(\omega_0'',K'')=\emptyset$. Moreover, let $C=\{(\omega_0,K)\in\mathbb{R}^2\mid\omega_0>K\geq 0\}$. We have that $\Omega^+=\bigcup_{(\omega_0,K)\in C}\Omega^\circ(\omega_0,K)$. Too see this, let first $\omega\in\Omega^+$. Define $K=\sqrt{\omega_1+\omega_2}\geq 0$. There is $t_\omega\in[-\pi,\pi]$ such that $u(\omega,t_i)=\omega_0+K\cos(t_\omega-t_i)>0$ for all t_i. Moreover, there is $\bar t_i\in[-\pi,\pi]$ such that $\cos(\bar t_i-t_\omega)=-1$. Therefore, $u(\omega,\bar t_i)=\omega_0-K>0$. Therefore, $(\omega_0,K)\in C$ and, thus, $\omega\in\Omega^\circ(\omega_0,K)$. Conversely, suppose $\hat\omega\in\Omega^\circ(\omega_0,K)$ for some $\omega_0>K$. Since for all t_i $\cos(t_{\hat\omega}-t_i)\geq -1$, we have $0<\omega_0-K\leq\hat\omega-K\cos(t_{\hat\omega}-t_i)=u(\hat\omega,t_i)$ for all t_i, therefore, $\hat\omega\in\Omega^+$. We conclude that $\{\Omega^\circ(\omega_0,K)\}_{(\omega_0,K)\in C}$ partitions Ω^+.

Step 2. Next assume $\omega_0>K\geq 0$, and focus on the set of policies $\Omega^\circ(\omega_0,K)\subset\Omega^+$. We want to show that the total probability of implementing these policies, namely $\frac{1}{2\pi}\int_{\Omega^\circ(\omega_0,K)}A^\star(\omega)\,d\omega$, decreases in N. We have that

$$\begin{aligned}\frac{1}{2\pi}\int_{\Omega^\circ(\omega_0,K)}A^\star(\omega)\,d\omega&=\frac{1}{2\pi I}\sum_i\int_{\Omega^\circ(\omega_0,K)}\bar a_i^\star(\omega,t_i)\,d\omega\\&=\frac{1}{2\pi I}\sum_i\int_{-\pi}^{\pi}\Phi\big(\sqrt{x^\star}\omega_0+\sqrt{1-x^\star}K\cos\big(t^\star_{n_i^\star}-t_\omega\big)\big)\,dt_\omega\\&=\frac{1}{2\pi I}\sum_i\int_0^{2\pi}\Phi\big(\sqrt{x^\star}\omega_0+\sqrt{1-x^\star}K\cos(y)\big)\,dy\\&=\frac{1}{2\pi}\int_0^{\pi}\Phi\big(\sqrt{x^\star}\omega_0+\sqrt{1-x^\star}K\cos(y)\big)\\&\quad+\Phi\big(\sqrt{x^\star}\omega_0-\sqrt{1-x^\star}K\cos(y)\big)\,dy.\end{aligned}$$

The first and second equalities follow from the definition of approval rate and the proof of Lemma B.10. In the second equality, we use notation $n_i^\star$ to indicate the firm from which agent i acquires information in equilibrium. To obtain the third equality, we used $y = t_{n_i^\star} - t_{\tilde{\omega}}$ and the fact that for any l and u such that $u = l + 2\pi$, $\int_l^u -\cos(y)\,dy = \int_0^{2\pi}\cos(y)\,dy$. Finally, to obtain the fourth equality, we used $\cos(y+\pi) = -\cos(y)$.

Step 3. In order to show that $\frac{1}{2\pi}\int_{\Omega^\circ(\omega_0,K)} A^\star(\omega)\,d\omega$ is decreasing in N, it is sufficient to show that for all $y \in [0, \pi]$, $\Phi(\sqrt{x^\star}\omega_0 + \sqrt{1-x^\star}K\cos(y)) + \Phi(\sqrt{x^\star}\omega_0 - \sqrt{1-x^\star}K\cos(y))$ is decreasing in N. To this purpose, fix $y \in [0, \pi]$. For notational convenience, let $\alpha = \sqrt{x^\star}$ and $\beta = \sqrt{1-x^\star}K$. We want to show that

$$\frac{d}{dN}\big(\Phi\big(\alpha\omega_0 + \beta\cos(y)\big) + \Phi\big(\alpha\omega_0 - \beta\cos(y)\big)\big) < 0. \tag{B.10}$$

This derivative is equal to

$$\begin{aligned}&\big(\phi\big(\alpha\omega_0 + \beta\cos(y)\big) + \phi\big(\alpha\omega_0 - \beta\cos(y)\big)\big)\omega_0\alpha' \\ &\quad + \big(\phi\big(\alpha\omega_0 + \beta\cos(y)\big) - \phi\big(\alpha\omega_0 - \beta\cos(y)\big)\big)\cos(y)\beta'.\end{aligned}$$

We show that both terms of these derivatives are negative. Let us start from the first term. By assumption $\omega_0 > 0$, since $\omega_0 - K > 0$ and $K \geq 0$. Moreover, the probability density function $\phi(\cdot)$ is everywhere strictly positive. Finally, by Proposition 1, $\alpha' < 0$. Therefore, the first term is strictly negative. Next, we analyze the second term of the derivative. Suppose $\cos(y) \geq 0$. Then since $\omega_0 > 0$, $\alpha\omega_0 + \beta\cos(y) \geq \alpha\omega_0 - \beta\cos(y)$. Moreover, $\alpha\omega_0 > 0$. This implies that $\phi(\alpha\omega_0 + \beta\cos(y)) - \phi(\alpha\omega_0 - \beta\cos(y)) \leq 0$. Conversely, suppose $\cos(y) \leq 0$. Then $\alpha\omega_0 + \beta\cos(y) \leq \alpha\omega_0 - \beta\cos(y)$. Since $\alpha\omega_0 > 0$, this implies that $\phi(\alpha\omega_0 + \beta\cos(y)) - \phi(\alpha\omega_0 - \beta\cos(y)) \geq 0$. In summary, we showed that

$$\big(\phi\big(\alpha\omega_0 + \beta\cos(y)\big) - \phi\big(\alpha\omega_0 - \beta\cos(y)\big)\big)\cos(y) \leq 0.$$

Since $\beta' > 0$ (Proposition 1), this implies that the second term of the derivative is weakly negative. We conclude that the derivative in Equation (B.10) is strictly negative, as we wanted to show. Since y was chosen arbitrarily, this implies that $\frac{1}{2\pi}\int_{\Omega^\circ(\omega_0,K)} A^\star(\omega)\,d\omega$ is decreasing in N. Moreover, since $\Omega^\circ(\omega_0, K)$ is an arbitrary cell in the partition of Ω^+, we conclude that $\int_{\Omega^+} A^\star(\omega)\phi(\omega)\,d\omega$ is decreasing in N.

A similar argument can be made to prove that $\int_{\Omega^-} A^\star(\omega)\phi(\omega)\,d\omega$ is increasing in N. The only differences being that in Step 1, we define $C' = \{(\omega_0, K) \in \mathbb{R}^2 \mid \omega_0 + K < 0, K \geq 0\}$, and in Step 3, we use the fact that $\omega_0 < 0$. *Q.E.D.*

APPENDIX C: ADDITIONAL EXTENSIONS

C.1. *Variance of Signals and Constraints on Learning*

Throughout the paper, we assumed that if agent i acquires information from firm n, she privately observes a signal realization $s_i(\omega, b_n) = b_n \cdot \omega + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, 1)$. In particular, we assumed that ϵ_i has a variance of 1 and, more importantly, that it does not depend on N, the number of firms. This implies that agents are constrained in how much they can learn about the policy from the media and that this constraint is independent of the competitiveness of the market. This is in line with an interpretation of the model where the error ϵ_i is borne by the agent. For example, it arises because she has a limited time to allocate to learning about the policy. In our extension to multimedia (Appendix B.2), we retained this assumption. Agents are still endowed with a unit of time but they can split it freely across multiple firms in the market. By doing so, agents can construct signals that are better tailored to their own needs, even when these are not directly supplied by the market.

The goal of this paper has been to demonstrate how a competitive market can affect welfare simply by changing what information agents consume. However, competition may affect not only what information is supplied by firms, but also how much agents can learn from them. Here we provide two such examples. First, the level of competitiveness in the market could have an impact on how much firms invest in generating information. This could be modeled by endogenizing supply side constraints on $\|b_n\|$, which is equivalent to a change in σ_N. A priori, it is not clear whether a more competitive market could lead firms to invest more or less. Second, in a more competitive market, agents may receive *multiple* signals from different firms. This could be modeled as a decline in σ_N as N increases. Note how this is different from the multimedia model of Section B.2, where agents get a *single* signal by mixing the editorial strategies of different firms.

Without adding more structure to the model, we investigate how the results presented in Section 5 are affected when σ_N is allowed to change as a function of N. In particular, we consider a decreasing sequence of σ_N and show a counterpart of our most general result of the paper, namely Proposition 5.

We begin by fixing I and N, and let $\epsilon_i \sim \mathcal{N}(0, \sigma_N)$ for some $\sigma_N > 0$. For a fixed N, it is not surprising to see that $\sigma_N \neq 1$ only rescales the main equilibrium objects of Section 3. In particular, fix an editorial strategy b_n and a type θ_i. Following Lemma 2, the value of information b_n for type θ_i is

$$v(b_n|\theta_i) = \frac{|\theta_i \cdot b_n|}{I\sqrt{2\pi\left(\sigma_N^2 + \|b_n\|^2\right)}}.$$

Intuitively, a lower σ_N increases the value that θ_i attaches to the information from firm n. Similarly, we can follow Lemma A.2 and transform b_n and θ_i into polar coordinates to obtain the value $v_{\sigma_N}((x_n, t_n)|t_i) = \lambda_{\sigma_N}(\sqrt{x_n} + \sqrt{1-x_n}\cos(t_i - t_n))$, where $\lambda_{\sigma_N} = \frac{1}{I\sqrt{2\pi(1+\sigma_N^2)}}$.

Instead of replicating all results in the paper, we focus attention on Proposition 5 to demonstrate the impact that an N-varying σ_N has on the main takeaways of the paper.

PROPOSITION 7: *Fix a regular F. Let $(\sigma_N)_N$ be a decreasing sequence with $\sigma_N \to \sigma_\infty > 0$.*

(a) *Existence. An equilibrium exists for any $N \geq 1$ and $I \geq 1$.*
(b) *Daily-Me. Fix any t_i. As $N \to \infty$, the equilibrium value of information for type t_i, $\mathcal{V}(N|t_i)$, converges in probability to the first-best value $\bar{\mathcal{V}}_\infty := \lambda_{\sigma_\infty}\sqrt{2}$.*
(c) *Inefficiency. There exists $\bar{I}$ and $\bar{\sigma} \geq 0$ such that*
  (i) *if $\sigma_\infty > \bar{\sigma}$ and $I > \bar{I}$, $\mathcal{U}(1) > \lim_{N\to\infty}\mathcal{U}(N)$*
  (ii) *if $\sigma_\infty \leq \bar{\sigma}$ and for any I, $\mathcal{U}(1) \leq \lim_{N\to\infty}\mathcal{U}(N)$.*

Parts (a) and (b) are qualitatively identical to their counterparts in Proposition 5. Part (c) shows that the inefficiency associated with competition remains even when agents can learn more, in the sense of lower σ, in a more competitive market. It also shows that if σ_N decreases too much, the inefficiency disappears. Part (c)(ii) should be considered as a sanity check. It further highlights that information can play a positive role in our model (see the discussion at the end of Section 4.4). For example, this result shows that for any distribution of preferences F, if the society converges to the complete information benchmark, that is, if $\lim_{N\to\infty}\sigma_N = 0$, then the agents are indeed better off. That is, in this limit, the expected welfare of the typical agent is higher under perfect competition relative to a monopoly. This provides an important benchmark, illustrating how there is plenty of scope in the model for information to play a positive role. The main inefficiency identified in the model is not due to competition moving society closer to the complete information benchmark; it arises because of the trade-offs firms face in terms of which aspects of the policy to emphasize in their editorial strategies. As competition increases, firms specialize by shifting emphasis from common-interest to private-interest components of the policy.

PROOF OF PROPOSITION 7: The proof is divided into five steps that closely follow Lemmas B.1–B.5. In the interest of space, we omit the proofs that are identical up to a rescaling. We focus attention on the steps of the proof that are affected by the dependence of σ_N on N.

Step 1: Existence. In this step, we fix N. As such, the proof of Lemma B.1 applies identically.

Step 2: *Daily-Me. I*. This follows the proof of Lemma B.2. Fix $\delta > 0$ and let $\xi_1 = \frac{\delta}{2\lambda_{\sigma_\infty}}$. Let $\bar{\mathcal{V}}_{\sigma_\infty} = \max_{(x_n,t_n)} v_{\sigma_\infty}((x_n, t_n)|t_i) = \lambda_{\sigma_\infty}\sqrt{2}$. This is the highest possible value that $v_{\sigma_\infty}((x_n, t_n)|t_i)$ can achieve and, like in the baseline model, it is independent of t_i. We show that there exists $\bar{N}$ such that for all $N > \bar{N}$ and any equilibrium profile of possibly mixed editorial strategies χ, we have $\mathbb{E}_\chi(\max_n\{v_{\sigma_N}(x_n, t_n|t_i)\}) > \bar{\mathcal{V}}_{\sigma_\infty} - \delta$ for all $t_i \in T$. Suppose not. That is, suppose that for all N, there is an equilibrium profile of possibly mixed editorial strategies χ and a type $\bar{t}_i$ such that $\mathbb{E}_\chi(\max_n\{v_{\sigma_N}(x_n, t_n|t_i)\}) \le \bar{\mathcal{V}}_{\sigma_\infty} - \delta$. This implies that for all $t_j \in [\bar{t}_i - \xi_1, \bar{t}_i + \xi_1]$, $\mathbb{E}_\chi(\max_n\{v_{\sigma_N}(x_n, t_n|t_j)\}) \le \bar{\mathcal{V}}_{\sigma_\infty} - \frac{\delta}{2}$. To see this, suppose, by way of contradiction, that $\mathbb{E}_\chi(\max_n\{v_{\sigma_N}(x_n, t_n|t_j)\}) > \bar{\mathcal{V}}_{\sigma_\infty} - \frac{\delta}{2}$. Denote by $n(t_j)$ the random variable that, for each realization of χ, indicates the firm from which t_j acquires information. Note that for all $t_n \in T$, $\cos(\bar{t}_i - t_n) \ge \cos(t_j - t_n) - \xi_1$, since $\frac{d}{dt}\cos(t - t_n) \le 1$. We have that

$$
\begin{aligned}
\mathbb{E}_\chi\Big(\max_n\big\{v_{\sigma_N}\big((x_n, t_n)|t_i\big)\big\}\Big) &\ge \mathbb{E}_\chi\big(v_{\sigma_N}\big((x_{n(t_j)}, t_{n(t_j)})|t_i\big)\big) \\
&= \lambda_{\sigma_N}\mathbb{E}_\chi\big(\sqrt{x_{n(t_j)}} + \sqrt{1 - x_{n(t_j)}}\cos(\bar{t}_i - t_{n(t_j)})\big) \\
&\ge \lambda_{\sigma_N}\mathbb{E}_\chi\big(\sqrt{x_{n(t_j)}} + \sqrt{1 - x_{n(t_j)}}\big(\cos(t_j - t_{n(t_j)}) - \xi_1\big) \\
&\ge \lambda_{\sigma_N}\mathbb{E}_\chi\big(\sqrt{x_{n(t_j)}} + \sqrt{1 - x_{n(t_j)}}\cos(t_j - t_{n(t_j)})\big) - \lambda_{\sigma_N}\xi_1 \\
&\ge \mathbb{E}_\chi\Big(\max_n\big\{v_{\sigma_N}(x_n, t_n|t_j)\big\}\Big) - \lambda\xi_1 \\
&> \bar{\mathcal{V}}_{\sigma_\infty} - \frac{\delta}{2} - \lambda_{\sigma_N}\xi_1 \\
&> \bar{\mathcal{V}}_{\sigma_\infty} - \frac{\delta}{2} - \lambda_{\sigma_\infty}\xi_1 \\
&= \bar{\mathcal{V}}_{\sigma_\infty} - \delta.
\end{aligned}
$$

The first inequality holds as, in the right-hand side, agent $\bar{t}_i$ chooses the firm $n(t_j)$ that is optimal for t_j. The second inequality holds since $\cos(\bar{t}_i - t_n) \ge \cos(t_j - t_n) - \xi_1$ for all

t_n. The last inequality holds because $\lambda_{\sigma_N} < \lambda_{\sigma_\infty}$, since σ_N is decreasing. In summary, this contradicts our assumption that $\mathbb{E}_\chi(\max_n\{v(x_n, t_n|t_i)\}) \leq \bar{\mathcal{V}} - \delta$. Therefore, it must be that $\mathbb{E}_\chi(\max_n\{v(x_n, t_n|t_j)\}) \leq \bar{\mathcal{V}} - \frac{\delta}{2}$.

Fix $\bar{N}$. Note that by continuity of $v_{\sigma_N}((x_n, t_n)|t_j)$ in t_j, there exists $\xi_2^{\bar{N}} > 0$ such that for all $t_j \in [\bar{t}_i - \xi_2^{\bar{N}}, \bar{t}_i + \xi_2^{\bar{N}}]$ such that $v_{\sigma_N}((1/2, \bar{t}_i)|t_j) \geq \bar{\mathcal{V}}_{\sigma_\infty} - \frac{\delta}{4}$. Since σ_N is decreasing, this holds for all $N > \bar{N}$. Hence, $\xi_2^{\bar{N}}$ is independent of N as it increases to infinity. Let $\xi = \min\{\xi_1, \xi_2^{\bar{N}}\}$, which in turn is independent of N. We have established that for all $t_j \in [\bar{t}_i - \xi, \bar{t}_i + \xi]$,

$$\begin{aligned}\mathbb{E}_\chi\Big(\max_{n'\neq n}\big\{v_{\sigma_N}(x_{n'}, t_{n'}|t_j)\big\}\Big) &\leq \mathbb{E}_\chi\Big(\max_{n}\big\{v_{\sigma_N}(x_n, t_n|t_j)\big\}\Big) \\ &\leq \bar{\mathcal{V}}_{\sigma_\infty} - \frac{\delta}{2} < \bar{\mathcal{V}}_{\sigma_\infty} - \frac{\delta}{4} \leq v_{\sigma_N}\big((1/2, \bar{t}_i)|t_j\big). \qquad \text{(C.1)}\end{aligned}$$

Consider an arbitrary firm n that deviates from its equilibrium editorial strategy (χ_n) in favor of the pure strategy $(x_n = 1/2, t_n = \bar{t}_i)$. Its expected profits are

$$\begin{aligned}\Pi_n\big((x_n, t_n), (\chi_{n'})_{n'\neq n}\big) &= I\int_{-\pi}^{\pi} \mathbb{E}_\chi\big(\max\big\{v_{\sigma_N}\big((x_n, t_n)|t_j\big) - V_{\sigma_N}\big((x_{n'}, t_{n'})_{n'\neq n}|t_j\big), 0\big\}\big)\,dF(t_j) \\ &\geq I\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi} \mathbb{E}_\chi\big(\max\big\{v_{\sigma_N}\big((x_n, t_n)|t_j\big) - V_{\sigma_N}\big((x_{n'}, t_{n'})_{n'\neq n}|t_j\big), 0\big\}\big)\,dF(t_j) \\ &\geq I\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi} \mathbb{E}_\chi\big(v_{\sigma_N}\big((x_n, t_n)|t_j\big) - V_{\sigma_N}\big((x_{n'}, t_{n'})_{n'\neq n}|t_j\big)\big)\,dF(t_j) \\ &= I\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi} v_{\sigma_N}\big((x_n, t_n)|t_j\big) - \mathbb{E}_\chi\big(V_{\sigma_N}\big((x_{n'}, t_{n'})_{n'\neq n}|t_j\big)\big)\,dF(t_j) \\ &= I\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi} v_{\sigma_N}\big((x_n, t_n)|t_j\big) - \mathbb{E}_\chi\Big(\max_{n'\neq n}\big\{v_{\sigma_N}(x_{n'}, t_{n'}|t_j)\big\}\Big)\,dF(t_j) \\ &\geq I\int_{\bar{t}_i-\xi}^{\bar{t}_i+\xi} \bigg(\bar{\mathcal{V}}_{\sigma_\infty} - \frac{\delta}{4} - \bar{\mathcal{V}}_{\sigma_\infty} + \frac{\delta}{2}\bigg) f(t_j)\,dt_j \\ &\geq \frac{IC\delta\xi}{2}.\end{aligned}$$

Note that that the industry profits are bounded above by $I\bar{\mathcal{V}}_{\sigma_\infty}$. Then an identical argument as in the last paragraph of Lemma B.2 applies here.

Step 3: *Daily-Me. II*. This follows the proof of Lemma B.3. Fix t_i, $\epsilon > 0$, and a sequence of equilibria. For any N, denote by $(x^N_{n(t_i)}, t^N_{n(t_i)})$ the random variable specifying the information structure that agent t_i acquires in equilibrium. We want to show that for all $\delta > 0$, there exists $\bar{N}$ such that for all $N > \bar{N}$, $\Pr(\|(x^N_{n(t_i)}, t^N_{n(t_i)}) - (1/2, t_i)\| > \epsilon) < \delta$. Suppose not. Then there is $\delta > 0$ such that for all $\bar{N}$ there is $N > \bar{N}$ such that $\Pr(\|(x^N_{n(t_i)}, t^N_{n(t_i)}) - (1/2, t_i)\| > \epsilon) \geq \delta$. Let (x_n, t_n) be a realization of $(x^N_{n(t_i)}, t^N_{n(t_i)})$ such that $\|(x_n, t_n) - (1/2, t_i)\| > \epsilon$. That is, $\sqrt{(x_n - 1/2)^2 + (t_n - t_i)^2} > \epsilon$. This implies that

$$\max\big\{|x_n - 1/2|, |t_n - t_i|\big\} > \frac{\epsilon}{\sqrt{2}}.$$

Consider the difference $\bar{\mathcal{V}}_{\sigma_\infty} - v_{\sigma_N}((x_n, t_n)|t_i) = \lambda_{\sigma_\infty}\sqrt{2} - \lambda_{\sigma_N}(\sqrt{x_n} + \sqrt{1-x_n}\cos(t_n - t_i))$. Suppose $|t_n - t_i| > \frac{\epsilon}{\sqrt{2}}$. Then, since $\lambda_{\sigma_\infty} > \lambda_{\sigma_N}$,

$$\bar{\mathcal{V}}_{\sigma_\infty} - v_{\sigma_N}\big((x_n, t_n)|t_i\big) \geq \frac{\lambda_{\sigma_\infty}}{\sqrt{2}}\big(1 - \cos(t_n - t_i)\big) > \frac{\lambda_{\sigma_\infty}}{\sqrt{2}}\left(1 - \cos\left(\frac{\epsilon}{\sqrt{2}}\right)\right) =: K_1(\epsilon) > 0.$$

Conversely, suppose that $|x_n - 1/2| > \frac{\epsilon}{\sqrt{2}}$. Then

$$\begin{aligned}\bar{\mathcal{V}}_{\sigma_\infty} - v_{\sigma_N}\big((x_n, t_n)|t_i\big) &\geq \lambda_{\sigma_\infty}(\sqrt{2} - \sqrt{x_n} - \sqrt{1-x_n}) \\ &> \lambda_{\sigma_\infty}\left(\sqrt{2} - \frac{1}{2}(\sqrt{1+\epsilon\sqrt{2}} + \sqrt{1-\epsilon\sqrt{2}})\right) =: K_2(\epsilon) > 0.\end{aligned}$$

Let $K(\epsilon) = \min\{K_1(\epsilon), K_2(\epsilon)\}$. We established that for all realizations of the random variable $(x^N_{n(t_i)}, t^N_{n(t_i)})$ that satisfy $\|(x^N_{n(t_i)}, t^N_{n(t_i)}) - (1/2, t_i)\| > \epsilon$, we have $\bar{\mathcal{V}}_{\sigma_\infty} - v_{\sigma_N}((x_n, t_n)|t_i) > K(\epsilon) > 0$. This implies that

$$\Pr\big(\bar{\mathcal{V}}_{\sigma_\infty} - v_{\sigma_N}\big((x^N_{n(t_i)}, t^N_{n(t_i)})|t_i\big) > K(\epsilon)\big) \geq \delta.$$

Since δ and ϵ are independent of N, we conclude that $\mathbb{E}(v_{\sigma_N}((x^N_{n(t_i)}, t^N_{n(t_i)})|t_i))$ does not converge to $\bar{\mathcal{V}}_{\sigma_\infty}$—a contradiction.

Step 4: *Convergence of $\mathcal{U}(N)$*. In light of the previous two steps, the proof of Step 4 follows the proof of Lemma B.4.

Step 5: *Monopoly Versus Competition*. Following Lemma B.5, we compute $\mathcal{U}(1)$ and $\lim_{N\to\infty}\mathcal{U}(N)$, taking into account the role of σ_N. In both cases, details associated with the steps used for derivation can be found in the original proof. First, let $N = 1$. We have that

$$\begin{aligned}\mathcal{U}(1) &= \frac{I-1}{I}\mathbb{E}_{\omega_0}\left(\omega_0\Phi(\tilde{b}\omega_0) + \frac{\beta_F b}{\sqrt{1+b^2}}\phi(\tilde{b}\omega_0)\right) \\ &= \frac{I-1}{I}\left(\frac{\tilde{b}}{\sqrt{1+\tilde{b}^2}}\phi(0) + \frac{\beta_F b}{\sqrt{1+b^2}}\frac{1}{\sqrt{1+\tilde{b}^2}}\phi(0)\right) \\ &= \frac{I-1}{I\sqrt{2}\sqrt{1+\sigma_1^2}\sqrt{\pi}}(\sqrt{x^\star} + \beta_F\sqrt{1-x^\star}) \\ &= (I-1)\lambda_{\sigma_1}\sqrt{1+\beta_F^2}.\end{aligned} \tag{C.2}$$

Second, let $N \to \infty$. We have that

$$\begin{aligned}\lim_{N\to\infty}\mathcal{U}(N) &= \frac{I-1}{I}\mathbb{E}_{\omega,t_i,t_j}\left(\Phi\left(\frac{1}{\sqrt{2}\sigma_\infty}u_j(\omega, t_j)\right)u_i(\omega, t_i)\right) + \bar{\mathcal{V}} \\ &= \frac{I-1}{I}\mathbb{E}_{t_j}\left(\frac{1}{\sqrt{2}\sqrt{1+\sigma_\infty{}^2}}\phi(0)\right. \\ &\quad \left.+ \frac{1}{\sqrt{2}\sqrt{1+\sigma_\infty{}^2}}\beta_F\phi(0)\big(\cos(t_j)\cos(t^m) + \sin(t_j)\sin(t^m)\big)\right) + \bar{\mathcal{V}}\end{aligned}$$

$$= \frac{I-1}{I} \frac{1}{\sqrt{2}\sqrt{1+\sigma_\infty{}^2}} \phi(0)\mathbb{E}_{t_j}\big(1+\beta_F\cos(t_j - t^m)\big) + \bar{\mathcal{V}}$$
$$= \frac{I-1}{I} \frac{1}{\sqrt{2}\sqrt{1+\sigma_\infty{}^2}} \frac{1}{\sqrt{2\pi}}\big(1+\beta_F^2\big) + \lambda_{\sigma_\infty}\sqrt{2}$$
$$= \lambda_{\sigma_\infty}(I-1)\frac{1}{\sqrt{2}}\big(1+\beta_F^2\big) + \lambda_{\sigma_\infty}\sqrt{2}. \tag{C.3}$$

By Equations (C.2) and (C.3),

$$\mathcal{U}(1) - \lim_{N\to\infty}\mathcal{U}(N) = \lambda_{\sigma_1}(I-1)\sqrt{1+\beta_F^2} - \lambda_{\sigma_\infty}(I-1)\frac{1}{\sqrt{2}}\big(1+\beta_F^2\big) - \lambda_{\sigma_\infty}\sqrt{2}$$
$$= \lambda_{\sigma_1}(I-1)\sqrt{1+\beta_F^2}\left(1 - \frac{\lambda_{\sigma_\infty}}{\lambda_{\sigma_1}\sqrt{2}}\sqrt{1+\beta_F^2}\right) - \lambda_{\sigma_1}\sqrt{2}.$$

Note that for all nondegenerate distributions F, there is an associated $\beta_F \in [0,1)$. Note that $\frac{\lambda_{\sigma_\infty}}{\lambda_{\sigma_1}\sqrt{2}}\sqrt{1+\beta_F^2} = (\frac{\sqrt{1+\sigma_1^2}}{\sqrt{1+\sigma_\infty^2}\sqrt{2}})\sqrt{1+\beta_F^2}$. For any β_F, there exist a $\bar{\sigma} \geq 0$ such that the sign of $1 - (\frac{\sqrt{1+\sigma_1^2}}{\sqrt{1+\sigma_\infty^2}\sqrt{2}})\sqrt{1+\beta_F^2}$ is positive whenever $\sigma_\infty < \bar{\sigma}$ and negative otherwise. Part (ii) follows directly from the sign being negative for both terms. Part (i) relies on showing that the first term is increasing in I, as we have done in the proof of Proposition 5. *Q.E.D.*

C.2. *Policy Implementation Rule*

Throughout the paper, we assume that the policy is implemented with a probability that is equal to its approval rate. This implementation rule, combined with a finite number of agents, implies that information has instrumental value for the agents, as their approval decisions directly affect the policy outcome. At the same time, this simple implementation rule eliminates the scope for pivotal reasoning to learn about the policy. As discussed in Section 3.1.1, this substantially reduces the complexity of the agent's problem, while enabling us to focus attention on the most novel aspect of the model: the competitive supply of information.

In general, the probability that a policy is implemented or that a candidate is elected can be a nonlinear function of the behavior of individual agents within a society. The political science and political economy literature study conditions under which maximizing vote share is equivalent to maximizing the probability of winning (see Banks and Duggan (2004), Patty (2005, 2007), McKelvey and Patty (2006)). One of these is the presence of aggregate uncertainty, for example, when voting decisions are influenced by independent random perturbations. Under appropriate distributional assumptions, this aggregate uncertainty generates linearity. Below, we discuss a simple extension of our baseline model in which the policy is implemented according to the majority rule. In this extension, there is aggregate noise due to behavior of "nonpolicy" voters, whose vote is a uniform random variable that is independent of the actual policy ω. In light of this, the policy obtains a simple majority with a probability that is proportional to its approval rate among the "policy" voters, as in our baseline model.

More formally, let there be I agents that we refer to as policy voters. These voters acquire information and vote as described in Section 2. A group of $\tilde{I}$ agents, which we refer to as nonpolicy voters, also participates in the collective decision. For simplicity, assume that $\tilde{I} \geq I$ and $\tilde{I} + I$ is an odd number. Nonpolicy voters are not affected by the policy outcome (e.g., for all such voters, $\theta_i = (0, 0, 0)$ and, thus, $u(\omega, t_i) = 0$). Their vote is determined by other factors that are independent of the policy. Specifically, we assume that the approval rate among nonpolicy voters, denoted $\tilde{A}(\omega)$, is distributed according to the uniform distribution on the interval [0, 1]. Finally, suppose that the policy is implemented if it receives a simple majority of all the votes.

REMARK C.1: Under simple majority, the probability that the policy is implemented is a linear function of the approval rate among policy voters, namely $A(\omega, \theta)$.

PROOF: Fix t and suppose that the approval rate of the policy voters is $A(\omega, t)$. Under a simple majority, the policy is implemented if $IA(\omega, t) + \tilde{I}\tilde{A}(\omega) > \frac{I+\tilde{I}}{2}$. Since $\tilde{A}(\omega) \sim$ Unif[0, 1], the probability that ω is implemented is equal to $\Pr(\tilde{A}(\omega) > \frac{I+\tilde{I}-2IA(\omega,t)}{2\tilde{I}}) = \frac{\tilde{I}-I}{2\tilde{I}} + \frac{I}{\tilde{I}}A(\omega, t)$, which is a linear function of the approval rate of policy voters. *Q.E.D.*

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Co-editor Joel Sobel handled this manuscript.

Manuscript received 8 June, 2018; final version accepted 7 May, 2021; available online 14 May, 2021.